KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

MARCH 2025

MEMO

NATIONAL SENIOR CERTIFICATE

GRADE 12

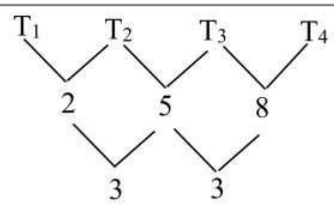
MARKS: 100

These marking guidelines consist of 11 pages.

QUESTION 1

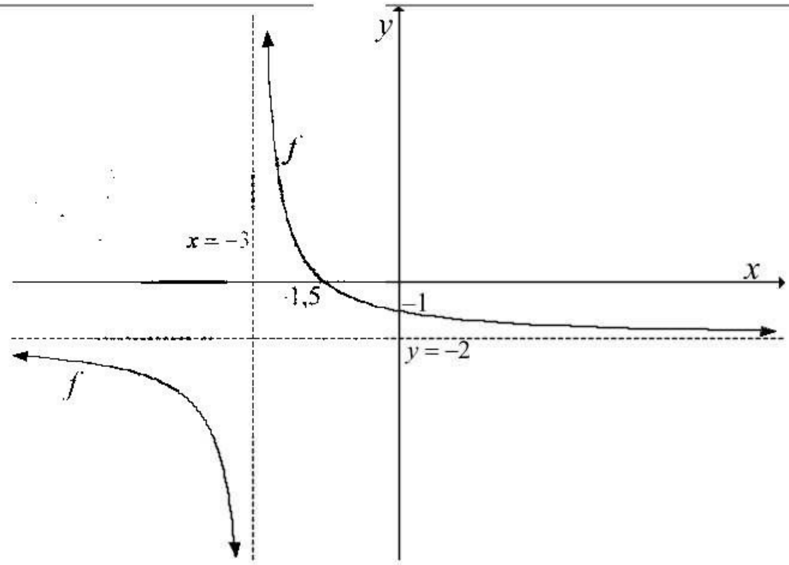
1.1.1	$S_{21} = 3(21)^2 - 5(21)$ $= 1218$	✓A substitution ✓A answer (2)
1.1.2	$S_{22} = 3(22)^2 - 5(22)$ $= 1342$ $T_{22} = S_{22} - S_{21}$ $= 1342 - 1218$ $= 124$	✓A value of S_{22} ✓CA answer (2)
1.1.3	$8162 = 3n^2 - 5n$ $3n^2 - 5n - 8162 = 0$ $n = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-8162)}}{2(3)}$ $n = 53$ or $n = -\frac{154}{3}$ N/A 53 terms have to be added	✓A equating ✓A standard form ✓CA substitution ✓CA answer (53 only) (4)
1.2.1	The even-numbered terms form an AS with $a = 7$ and $d = 5$. T_{39} of the AS $= a + (n-1)d$ $= 7 + (39-1)5$ $= 197$	✓A substitution ✓CA answer (2)
1.2.2	Sum of the 52 odd-numbered terms $= 52 \times 7 = 364$ Sum of the 51 even-numbered terms $= \frac{n}{2}[2a + (n-1)d]$ $= \frac{51}{2}[2(7) + (51-1).5]$ $= 6732$ Sum of first 103 terms $= 364 + 6732$ $= 7096$	✓A 52×7 ✓CA substitution ✓CA sum of AS ✓CA answer (4)
		[14]

QUESTION 2

2.1.1	$9\ ;\ -27\ ;\ 81$	✓A 9 ✓A $-27\ ;\ 81$ (2)
2.1.2	$r=-3$	✓A answer (1)
2.1.3	No, the series will not converge $r<-1$ **OR** r does not lie between -1 and 1.	✓CA answer (no) ✓CA motivation (2)
2.1.4	$a=9x$ $r=-3$ $n=12$ $S_n=\dfrac{a\left(r^n-1\right)}{r-1}$ $=\dfrac{9x\left((-3)^{12}-1\right)}{-3-1}$ $=-1\,195\,740x$	✓A $n=12$ ✓CA substitution ✓CA answer (3)
2.2	 2^{nd} difference = 3 $a=\dfrac{\text{2nd difference}}{2}=\dfrac{3}{2}$ $3a+b=2$ $b=2-3\left(\dfrac{3}{2}\right)=-\dfrac{5}{2}$ $T_{29}=\dfrac{3}{2}(29)^2-\dfrac{5}{2}(29)+c=1166$ $c=-23$ $\therefore T_n=\dfrac{3}{2}n^2-\dfrac{5}{2}n-23$ $T_1=\dfrac{3}{2}(1)^2-\dfrac{5}{2}(1)-23=-24$	✓A 2^{nd} difference = 3 ✓CA value of a ✓CA value of b ✓CA substitution in T_{29} ✓CA value of c ✓CA answer (6)
		[14]

QUESTION 3

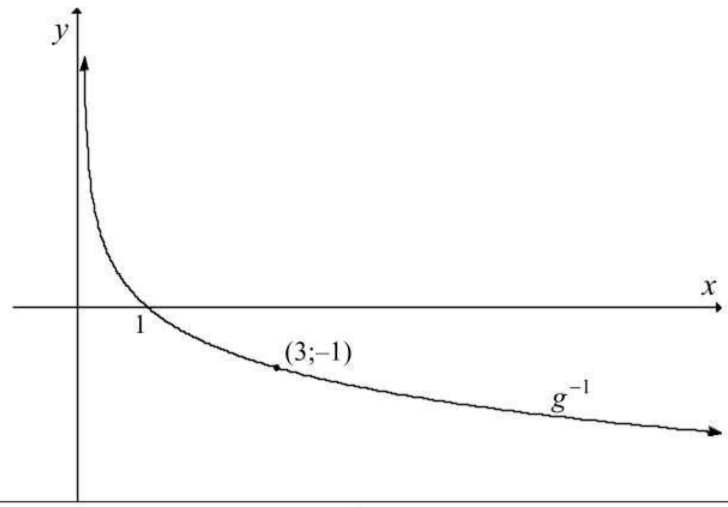
3.1.1	$f(x)=\frac{2x+3}{-x-3}$ $=\frac{2x+3}{-(x+3)}$ $=\frac{-(2x+3)}{x+3}$ $=\frac{-(2x+6-3)}{x+3}$ $=\frac{-[2(x+3)-3]}{x+3}$ $=\frac{-2(x+3)}{x+3}+\frac{3}{x+3}$ $\therefore f(x)=\frac{3}{x+3}-2$ **OR** $f(x)=\frac{2x+3}{-x-3}$ $=-2+\frac{-3}{-x-3}$ $=-2+\frac{-3}{-(x+3)}$ $=-2+\frac{3}{x+3}$	✓A $-(x+3)$ ✓A $\frac{-[2(x+3)-3]}{x+3}$ (2) **OR** ✓A $-2+\frac{-3}{-x-3}$ (through the method of long division) ✓A $-(x+3)$ (2)
3.1.2	For y-intercept, let $x=0$: $y=\frac{3}{x+3}-2$ $y=\frac{3}{0+3}-2=-1$ For x-intercept, let $y=0$: $\frac{3}{x+3}-2=0$ $\frac{3}{x+3}=2$ $x=-\frac{3}{2}$	✓A $y=-1$ ✓A $\frac{3}{x+3}-2=0$ ✓A $x=-\frac{3}{2}$ (3)

3.1.3		✓CA intercepts ✓A asymptotes ✓A shape (3)
3.1.4	$y = -x + c$ Substitute $(-3\ ;\ -2)$: $-2 = -(-3) + c$ $c = -5$ $y = -x - 5$	✓A equation of straight line with a gradient of – 1 ✓CA answer (2)
3.1.5 (a)	$x \in R,\ \ x \neq -3$	✓A $x \in R$ ✓CA $x \neq -3$ (2)
3.1.5 (b)	$-3 < x \leq -\dfrac{3}{2}$	✓CA ✓CA $-3 < x \leq -\dfrac{3}{2}$ (2)

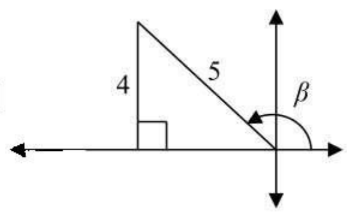
3.2.1	At R: $3x+6=0$ $x=-2$ $R(-2\ ;\ 0)$	✓A x-coordinate ✓A y-coordinate (2)
3.2.2	From symmetry, the x-coordinate at S = 1 $y=a(x+2)(x-1)$ $y=ax^2+ax-2a$ Substitute $\left(-\frac{1}{2};9\right)$: $9=a\left(-\frac{1}{2}\right)^2+a\left(-\frac{1}{2}\right)-2a$ $9=-\frac{9}{4}a$ $\therefore a=-4$ $\therefore y=-4x^2-4x+8$ **OR** $y=a(x+p)^2+q$ $y=a\left(x+\frac{1}{2}\right)^2+9$ Subst. $S(1\ ;\ 0)$ OR Subst. $R(-2\ ;\ 0)$: $:0=a\left(1+\frac{1}{2}\right)^2+9$ $0=a\left(-2+\frac{1}{2}\right)^2+9$ $\therefore -9=a\left(\frac{9}{4}\right)$ $a=-4$ $\therefore y=-4\left(x+\frac{1}{2}\right)^2+9$	✓CA $y=a(x+2)(x-1)$ ✓CA substitution ✓CA value of a ✓CA answer….. (4) **OR** ✓A $y=a\left(x+\frac{1}{2}\right)^2+9$ ✓CA substitution ✓CA value of a ✓CA answer (4)
		[20]

QUESTION 4

4.1.1	f^{-1}: $y=\sqrt{3x}$ f: $x=\sqrt{3y}$ $x^2=3y$ $y=\frac{x^2}{3}$; $x\geq 0$	✓A swopping x and y ✓A $y=\frac{x^2}{3}$ ✓A $x\geq 0$ (3)
4.1.2	$0\leq x\leq 3$ **OR** $x\in[0\ ;\ 3]$	✓A ✓A answer (2)

4.2.1	$g:\ y=\left(\frac{1}{3}\right)^x$ **OR** $g:\ y=3^{-x}$ $g^{-1}:\ x=\left(\frac{1}{3}\right)^y$ $g^{-1}:\ x=3^{-y}$ $\therefore y=\log_{\frac{1}{3}} x$ $\therefore y=-\log_3 x$	✓A swopping x and y ✓A answer (2)
4.2.2		✓CA shape ✓A x-intercept ✓CA coordinates of one more point (3)
4.2.3	$y=a\left(\frac{1}{3}\right)^x+7$ Substitute $(-2\ ;10)$: $10=a\left(\frac{1}{3}\right)^{-2}+7$ $9a=3$ $a=\frac{1}{3}$	✓A substitution ✓CA answer (2)
4.2.4	$h:\ y=\left(\frac{1}{3}\right).\left(\frac{1}{3}\right)^x+7$ From $y=\left(\frac{1}{3}\right)^{x+1}+7$ to $y=\left(\frac{1}{3}\right)^x$: Translation of 1 unit to the right and 7 units downwards. Answer only: full marks	✓A $y=\left(\frac{1}{3}\right)^{x+1}+7$ ✓A 1 unit to the right ✓A 7 units downwards (3)
		[15]

QUESTION 5

5.1.1	$\sin\beta = \frac{4}{5}$ $x^2 = r^2 - y^2$ [Pythagoras] $= 5^2 - 4^2$ $= 9$ $x = -3$ $\therefore \cos\beta = \frac{-3}{5}$  	✓A $\sin\beta = \frac{4}{5}$ ✓A x – value ✓CA answer (3)
5.1.2	$\cos 2\beta = 2\cos^2\beta - 1$ $= 2\left(\frac{-3}{5}\right)^2 - 1$ $= 2\left(\frac{9}{25}\right) - 1$ $= \frac{-7}{25}$ **OR** $\cos 2\beta = \cos^2\beta - \sin^2\beta$ $= \left(\frac{-3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$ $= \frac{9}{25} - \frac{16}{25}$ $= \frac{-7}{25}$ **OR** $\cos 2\beta = 1 - 2\sin^2\beta$ $= 1 - 2\left(\frac{4}{5}\right)^2$ $= 1 - 2\left(\frac{16}{25}\right)$ $= \frac{-7}{25}$	✓A double ∠ expansion ✓CA substitution ✓CA answer (3) **OR** ✓A double ∠ expansion ✓CA substitution ✓CA answer (3) **OR** ✓A double ∠ expansion ✓A substitution ✓CA answer (3)

5.1.3	$\sin 3\beta = \sin(2\beta + \beta)$ $= \sin 2\beta \cos\beta + \cos 2\beta \sin\beta$ $= 2\sin\beta\cos\beta\cos\beta + \cos 2\beta\sin\beta$ $= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)\left(\frac{-3}{5}\right) + \left(\frac{-7}{25}\right)\left(\frac{4}{5}\right)$ $= \frac{72}{125} - \frac{28}{125}$ $= \frac{44}{125}$	✓A compound ∠ expansion ✓A double ∠ expansion ✓CA substitution ✓CA answer (4)
5.2	$\frac{\sin(-180° - \theta)\tan(180° - \theta)\cos(-\theta)}{\cos^2(90° + \theta) + 3\sin^2\theta}$ $= \frac{\sin\theta . -\tan\theta . \cos\theta}{(-\sin\theta)^2 + 3\sin^2\theta}$ $= \frac{\sin\theta . -\frac{\sin\theta}{\cos\theta} . \cos\theta}{4\sin^2\theta}$ $= -\frac{\sin^2\theta}{4\sin^2\theta}$ $= -\frac{1}{4}$	✓A $\sin\theta$ ✓A $-\tan\theta$ ✓A $\cos\theta$ ✓A $-\sin\theta$ ✓A $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ✓CA answer (6)
5.3	$\frac{1}{8}(1 - \cos 4x)$ $= \frac{1}{8}[1 - \cos(2 \times 2x)]$ $= \frac{1}{8}[1 - (1 - 2\sin^2 2x)]$ $= \frac{1}{8}[2\sin^2 2x]$ $= \frac{1}{4}[\sin^2 2x]$ $= \frac{1}{4}(2\sin x\cos x)^2$ $= \frac{1}{4}(4)(\sin^2 x . \cos^2 x)$ $= \sin^2 x . \cos^2 x$ **OR**	✓A $\frac{1}{8}[1 - \cos(2 \times 2x)]$ ✓A $\frac{1}{8}[1 - (1 - 2\sin^2 2x)]$ ✓A $\frac{1}{4}[\sin^2 2x]$ ✓A $\frac{1}{4}(2\sin x\cos x)^2$ ✓A $\frac{1}{4}(4)(\sin^2 x . \cos^2 x)$ (5) **OR**

	$\frac{1}{8}(1-\cos 4x)$ $=\frac{1}{8}[1-\cos(2\times 2x)]$ $=\frac{1}{8}[1-(2\cos^2 2x-1)]$ $=\frac{1}{8}[2-2\cos^2 2x]$ $=\frac{1}{8}[2(1-\cos^2 2x)]$ $=\frac{1}{8}[2\sin^2 2x]$ $=\frac{1}{4}[\sin^2 2x]$ $=\frac{1}{4}(2\sin x\cos x)^2$ $=\frac{1}{4}(4)(\sin^2 x.\cos^2 x)$ $=\sin^2 x.\cos^2 x$	✓A $\frac{1}{8}[1-\cos(2\times 2x)]$ ✓A $\frac{1}{8}[1-(2\cos^2 2x-1)]$ ✓A $\frac{1}{4}[\sin^2 2x]$ ✓A $\frac{1}{4}(2\sin x\cos x)^2$ ✓A $\frac{1}{4}(4)(\sin^2 x.\cos^2 x)$ (5)
		[21]

QUESTION 6

6.1	$\cos(x-45°)=\sin 2x$ $\cos(x-45°)=\cos(90°-2x)$ $x-45°=90°-2x+k.360°$ or $x-45°=360-(90°-2x)+k.360°$ $3x=135°+k.360°$ $\quad$ $x-45°=270°+2x+k.360°$ $x=45°+k.120°,\ k\in\mathbb{Z}$ $\quad$ $-x=315°+k.360°$ $x=-315°-k.360°$ $x=45°+k.360°;\ \ k\in\mathbb{Z}$ **OR** $\cos(x-45°)=\sin 2x$ $\sin[90°-(x-45°)]=\sin 2x$ $-x+135°=2x+k.360°$ or $-x+135°=180°-2x+k.360°$ $3x=135°+k.360°$ $\quad$ $x=45°+k.360°$ $x=45°+k.120°,\ k\in\mathbb{Z}$ $\quad$ $x=45°+k.360°,\ k\in\mathbb{Z}$	✓A co-ratio ✓A both equations ✓CA $45°+k.120°$ ✓A $k\in\mathbb{Z}$ (4) **OR** ✓A co-ratio ✓A both equations ✓CA $45°+k.120°$ ✓A $k\in\mathbb{Z}$ (4)
6.2.1	$360°$	✓A answer (1)
6.2.2 (a)	$(-135°\ ;\ -1)$	✓A x-coordinate ✓A y-coordinate (2)
6.2.2 (b)	$\left(-75°\ ;\ -\frac{1}{2}\right)$	✓CA x-coordinate ✓CA y-coordinate (2)
6.2.3	$90°<x<135°$ OR $x\in(90°\ ;\ 135°)$	✓A✓A answer (2)
6.2.4	$\frac{1}{\sqrt{2}}(\cos x+\sin x)$ $=\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x$ $\cos 45°.\cos x+\sin 45°\sin x$ $=\cos(x-45°)$ $\therefore$ To solve for x if $\sin 2x\geq\cos(x-45°)$: $x=45°$ or $x\in[165°;180°]$ **OR** $x=45°$ or $165°\leq x\leq 180°$	✓A $\cos 45°.\cos x+\sin 45°\sin x$ ✓A compound ∠ identity ✓A $x=45°$ ✓CA ✓CA $x\in[165°;180°]$ or $165°\leq x\leq 180°$ (5)
		[16]

TOTAL: 100