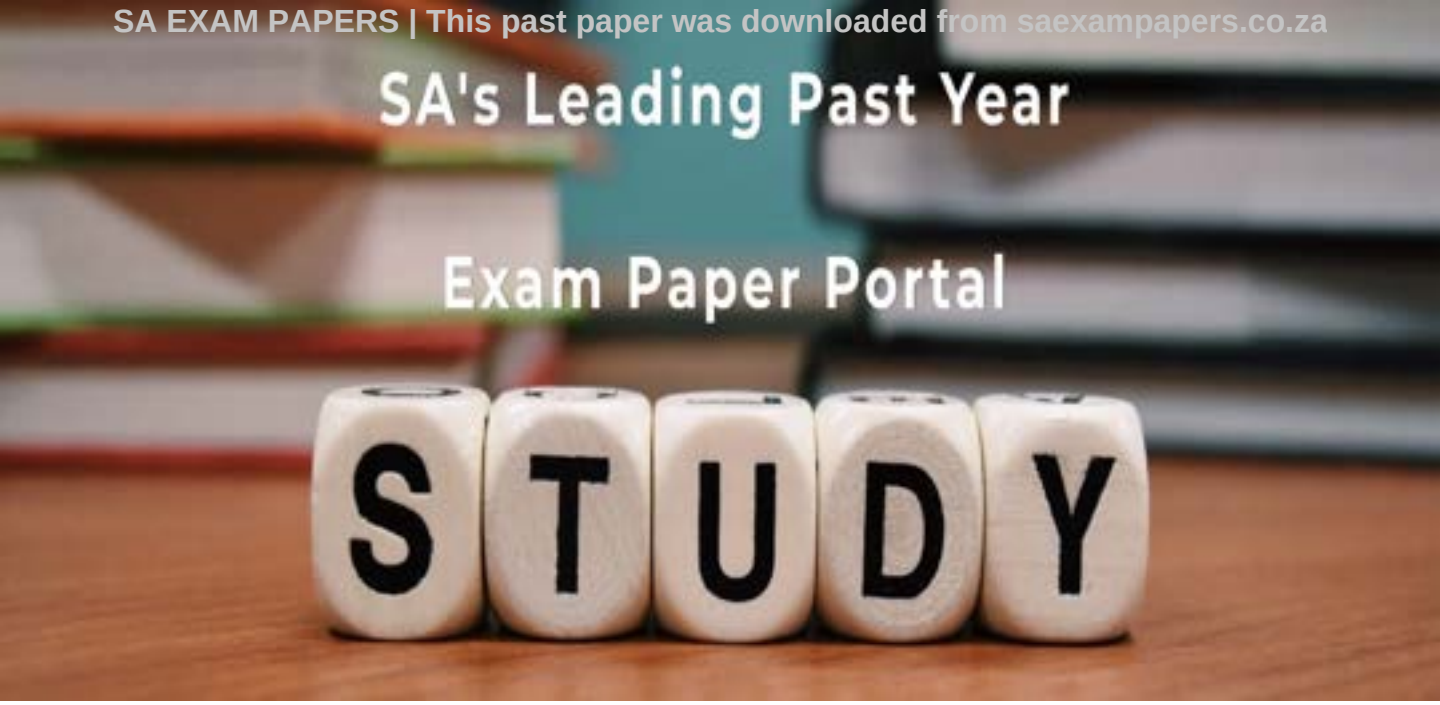


GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2024

TECHNICAL MATHEMATICS

(PAPER 1)

TECHNICAL MATHEMATICS P1

C2091E

TIME: 3 hours

MARKS: 150

X05

10 pages + a 2-page information sheet and an answer sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTIONS 4.3.1, 4.3.3 and 4.3.4 on the ANSWER SHEET provided. Hand in the ANSWER SHEET with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-10)=0$ (2)

1.1.2 $7x^2-5x-6=0$ (correct to TWO decimal places) (3)

1.1.3 $(x+2)(2x-4)\geq 0$ (3)

1.2 Solve for x and y if:

$x-2y=1$ and $x^2+y^2=9+2xy$. (6)

1.3 The velocity (v) of an object is given by $v=\sqrt{2\,gh}$, where (g) is the acceleration due to gravity and (h) is the height of the object. Two objects (A and B) are dropped from heights that differ by 10 m. The velocity of the objects when they strike the ground is 20 m/s and the acceleration is g = 9,80 m/s^2.

1.3.1 Make h the subject of the formula. (2)

1.3.2 Hence, calculate (rounded to the nearest m), the numerical value of h for both objects A and B. (3)

1.4 Given the binary numbers: $A = 1001001_2$ and $B = 1001_2$

1.4.1 Determine $A-B$. Leave your answer in binary form. (1)

1.4.2 Convert your answer in QUESTION 1.4.1 to a decimal form. (1)

[21]

QUESTION 2

2.1 Given the equation: $f(x)=x^2+3x$.

2.1.1 Determine the numerical value(s) of the discriminant. (2)

2.1.2 Hence, describe the nature of the roots of the equation. (1)

2.2 Determine the numerical value of m for which the equation $mx^2-12x+9=0$ will have equal roots. (Do not solve the equation.) (4)

[7]

QUESTION 3

3.1 Simplify the following **without the use of a calculator**. (Leave the answer with positive exponents.)

3.1.1 $-2a^0 \times x^{-3} \div x^7$ (2)

3.1.2 $\sqrt{27x^{10}} \times \sqrt{18x^{-2}}$ (3)

3.1.3 $\dfrac{5^{4x} + 2.5^{4x}}{625^x}$ (3)

3.2 Solve for x: $\log_2(1-x) + \log_2(5+x) - 3 = 0$ (5)

3.3 Given complex numbers: $z_1 = 3 + 2i$

3.3.1 Write down the conjugate of z_1 and label it z_2. (1)

3.3.2 Draw z_2 on an Argand diagram. (1)

3.3.3 Express $z_1 = 3 + 2i$ in polar form. (3)

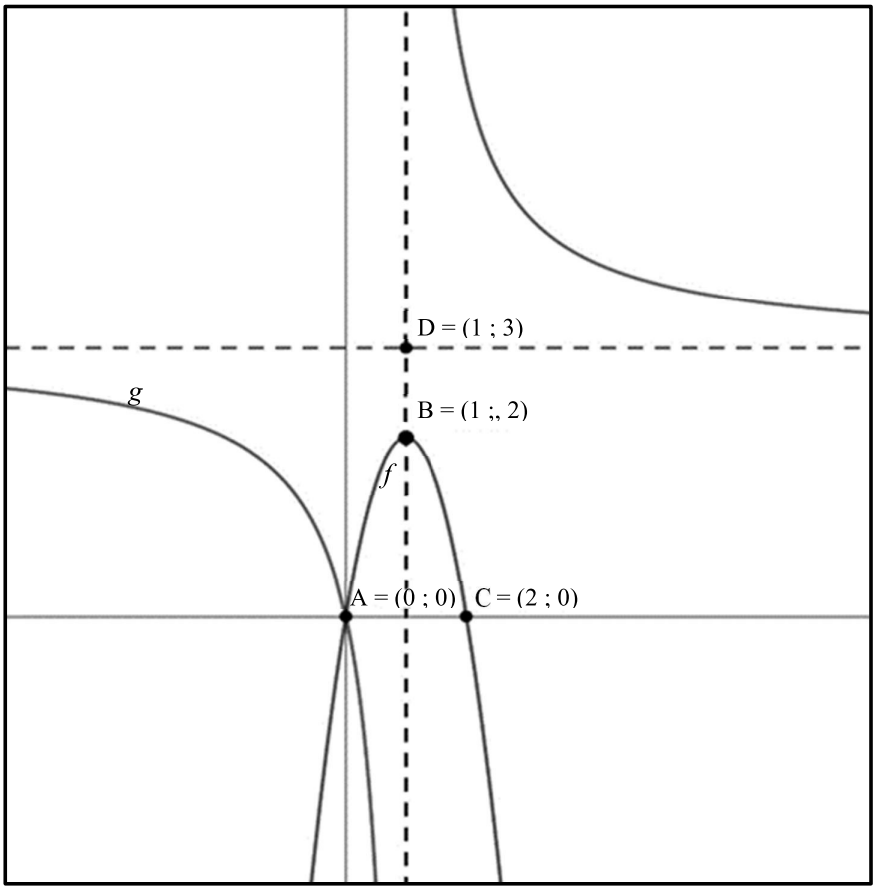
3.4 Solve for a and b if: $a(2 - 3ai) - 5 = b(25i - 1) - 2i$ (5)

[23]

QUESTION 4

4.1 The sketch below represents the functions defined by $f(x) = a(x-p)^2 + q$ and $g(x) = \frac{b}{x-r} + s$.

- The x-intercepts of $f(x)$ are $A(0;0)$ and $C(2;0)$.
- The turning point of $f(x)$ is $B(1;2)$.
- The function $g(x)$ passes through the origin.
- The point $D(1;3)$ is where the asymptotes cross.



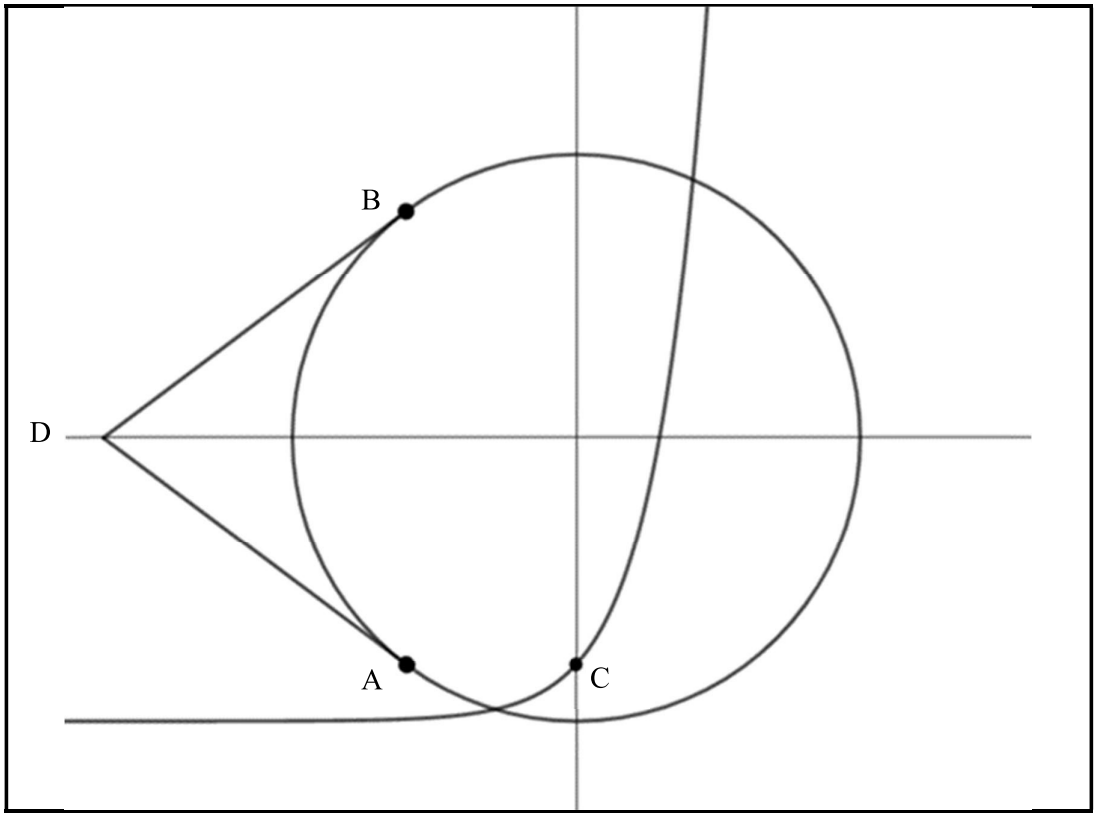
Determine:

4.1.1 The equation of the asymptotes of g (2)

4.1.2 The equation of f (3)

4.1.3 The equation of g (3)

4.2 A circle with center $O(0;0)$ has a diameter of 10 m. A horizontal tangent line at the bottom of the circle is the asymptote of $f(x) = 3^x + k$.



4.2.1 Determine the radius of the circle. (1)

4.2.2 Determine the coordinates of A and B, the two points on the circle circumference where $x = -3$. (4)

4.2.3 Write down the equation for the top half of the circle. (2)

4.2.4 Determine the equation of the asymptote of f. (1)

4.3 Given: The functions defined by $h(x) = (x-3)^2$ and $k(x) = -x + 9$

4.3.1 Draw a sketch graph of h and k on the same set of axes on the grid provided on the ANSWER SHEET. Clearly indicate the intercepts with the axes and the asymptotes. (6)

4.3.2 Determine (show ALL calculations) where $h(x) = k(x)$. (6)

4.3.3 Hence, use the graph drawn on the ANSWER SHEET and show where $h(x) = k(x)$ using the letters A and B. (2)

4.3.4 Hence, use your drawn graph on the ANSWER SHEET and shade the area where $h(x) \leq k(x)$. (1)

[31]

QUESTION 5

5.1 A motor vehicle's selling price is R390 099. The vehicle's value is reduced on the reducing balance method at 15% p.a. Calculate the resale value of the car 6 years after its purchase. (3)

5.2 An investment at 8,92% p.a. compound interest grew to R68 000. Calculate the value of the initial amount, *P*, after a period of 10 years. (3)

5.3 R12 500 is invested at a nominal interest rate of 8,5% p.a. compounded monthly.

Determine the effective interest rate. (4)

5.4 Mrs. Naidoo invested R80 000 in an account that offers the following:

- 7,5% p.a., compounded quarterly, for the first 4 years and then,
- 9,2% p.a., compounded monthly, for the next 3 years.

Calculate the total amount of money that will be in the account at the end of the 7 years, if no other transactions occur. (6)

[16]

QUESTION 6

6.1 Determine $f'(x)$ from FIRST PRINCIPLES if $f(x) = -2x + 3$. (5)

6.2 Determine:

6.2.1 y' if $y = 2x^2 - 4x + 6$ (2)

6.2.2 $\frac{dy}{dx}$ if $y = \sqrt{4x^3} + \frac{1}{x^5} - x$ (5)

6.3 A ball is thrown up in the air. The height of the ball above the ground is $h(t) = -5t^2 + 15t + 1$ metres after t seconds.

Determine the average rate of change of the height between $t = 0$ and $t = 2$ seconds, by using the following formula:

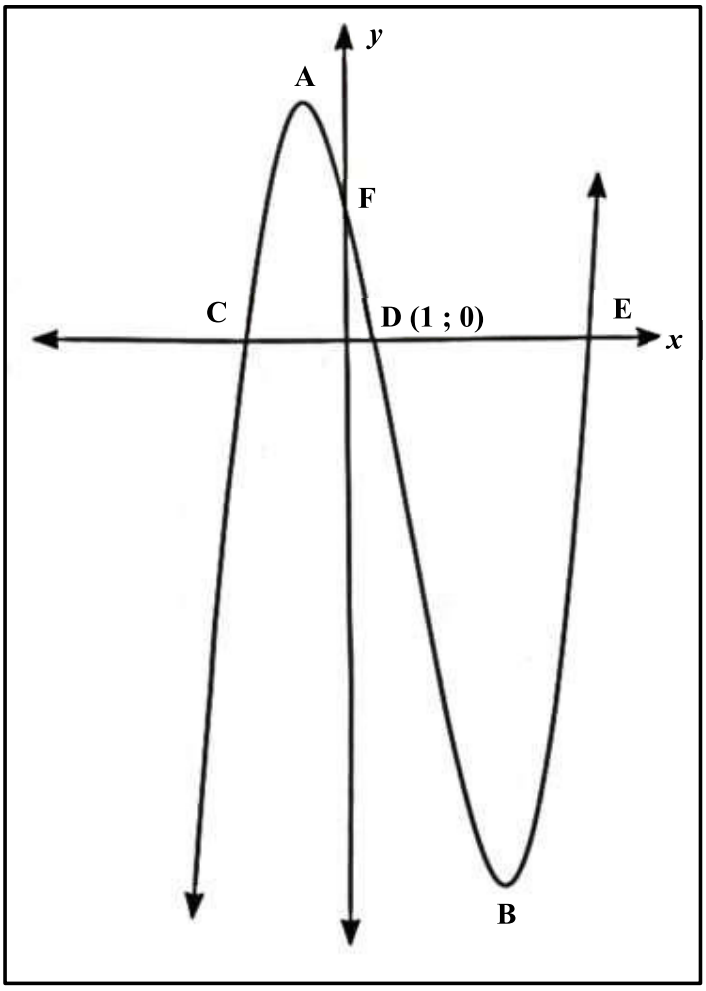
$$\text{Average rate of change} = \frac{f(b)-f(a)}{b-a}$$ (3)

[15]

QUESTION 7

The graph below represents the function defined by $f(x) = x^3 - 5x^2 - 8x + 12$.

- The x-intercepts of f are C, D(1 ; 0) and E.
- The y-intercept of f is at the point F.
- A and B are the turning points of f.



7.1 Determine the length of OF. (1)

7.2 Show that $(x - 1)$ is a factor. (2)

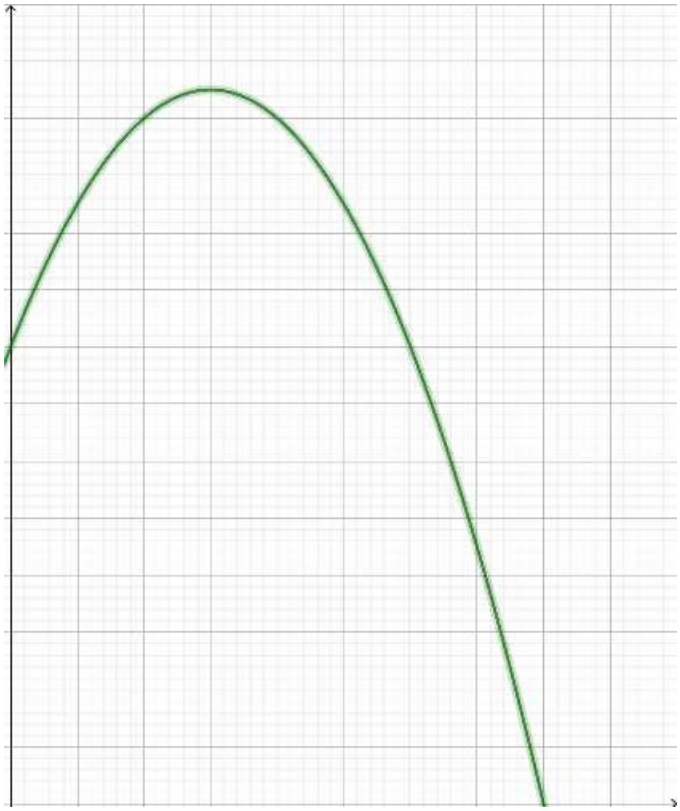
7.3 Hence, determine the coordinates of points C and E. (5)

7.4 Determine the coordinates of the turning points A and B. (7)

[15]

QUESTION 8

A ball is thrown into the air from the fifth floor of a building. The ball's projection follows the equation $h(t) = 16 + 6t - t^2$ where h represents the ball's height in metres above the ground and t, the time in seconds after the ball is thrown out of the window. The ball's projection is drawn below.



8.1 Determine the height of the fifth floor from where the ball was thrown. (2)

8.2 Determine the height of the ball, 2 seconds after it was thrown. (2)

8.3 After how many seconds did the ball reach its maximum height before it began to fall? (3)

8.4 Calculate the maximum height reached by the ball. (2)

[9]

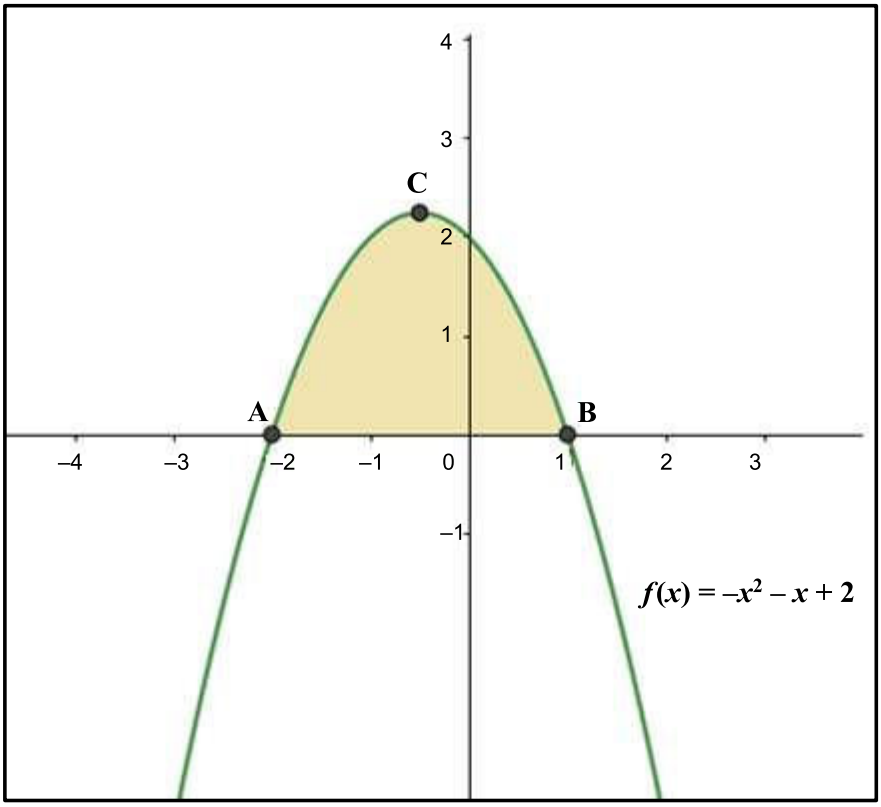
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int (x^2 + y^2)\, dc$ (3)

9.1.2 $\int \left(2x - \frac{1}{2x} - \sqrt{x} + 4^{3x}\right) dx$ (5)

9.2 The sketch below shows the shaded area bound by function f, defined by $f(x) = -x^2 - x + 2$, and the x-axis between the points where $x = -2$ and $x = 1$.



Determine the shaded area bound by function f between the points $x = -2$ and $x = 1$. Show ALL calculations. (5)

[13]

TOTAL: 150

END

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1+i)^n \qquad A = P(1-i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C \quad , \ n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C \quad , \ x > 0 \text{ and } k \in \mathbb{R} \ ; \ k \neq 0$$

$$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C \quad , \ a > 0 \ ; \ a \neq 1 \text{ and } \ k, a \in \mathbb{R} \ ; \ k \neq 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$\pi\ rad = 180°$

Angular velocity $= \omega = 2\ \pi\ n$ where n = rotation frequency

Angular velocity $= \omega = 360°\ n$ where n = rotation frequency

Circumferential velocity $= v = \pi\ D\ n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega\ r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\,\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4d\,h + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where a = length of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
$O_n = n^{th}$ ordinate and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = length of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

NAME AND SURNAME:	

GRADE:	

QUESTIONS 4.3.1, 4.3.3 and 4.3.4