Province of the
EASTERN CAPE
EDUCATION

Iphondo leMpuma Kapa: Isebe leMfundo
Provinsie van die Oos Kaap: Department van Onderwys
Porafensie Ya Kapa Botjahabela: Lefapha la Thuto

NATIONAL
SENIOR CERTIFICATE

IBANGA 12

SEPTEMBA 2024

IMATHEMATIKA P2

AMANQAKU: 150

IXESHA: 3 Iiyure

Eli phepha lemibuzo linamaphepha ayi-14 lidibene nephepha
leenkcukacha nencwadi yokuphendulela enamaphepha ayi-21.

IMIYALELO NEENKCUKACHA

Funda imiyalelo elandelayo ngocoselelo phambi kokuphendula imibuzo.

1.   Eli phepha lemibuzo linemibuzo eyi10.

2.   Phendula YONKE imibuzo kwiNCWADI EKHETHEKILEYO YOKUPHENDULELA
        enikiweyo.

3.   Bonisa ngokucacileyo ZONKE iikhalityhuleyishini, iidayagram, iigrafu, njl
        ozisebenzisileyo ukubonisa iimpendulo.

4.   Iimpendulo kuphela AZINYANZELEKANGA ukunikwa amanqaku apheleleyo.

5.   Ungayisebenzisa ikhalityhuleyitha esayentifikhi (engaprogranywanga nengenagrafikhi),
        ngaphandle kokuba uxelelwe ngeny'indlela.

6.   Xa kunyanzelekile, sondeza iimpendulo kwiindawo EZIMBINI zedesimali,
        ngaphandle kokuba uxelelwe ngeny'indlela.

7.   Iidayagram AZIZOTYWANGA ngokwesikeyile.

8.   Iphepha leenkcukacha elineefomyula lifakiwe ekugqibeleni kwephepha lemibuzo.

9.   Bhala ngokucocekileyo nangokucacileyo.

UMBUZO 1

1.1   Inani leelitha zedizili ezithengwe ngabaqhubi beeloli abayi15 kwisitishi sepetroli arekhodwe ngokulandelayo.

82	64	55	50	41
71	78	88	98	96
63	66	80	84	88

    1.1.1   Bhala imode.   (1)

    1.1.2   Bhala irange.   (1)

    1.1.3   Khalityhuleyitha imean.   (2)

    1.1.4   Khalityhuleyitha istandard deviation semean.   (1)

    1.1.5   Fumana bangaphi abaqhubi beeloli abathenge iilitha zedizili ezingaphantsi kwe-one standard deviation semean.   (3)

1.2   Imean weight yabantu abayi8 abangena kwilift yi75 kg. Ilift inobunzima obungadluli kwi1 000 kg.

    Bangaphi abantu abanongena kwilift ngokucingela imean weight ihlale ingu75 kg?   (4)
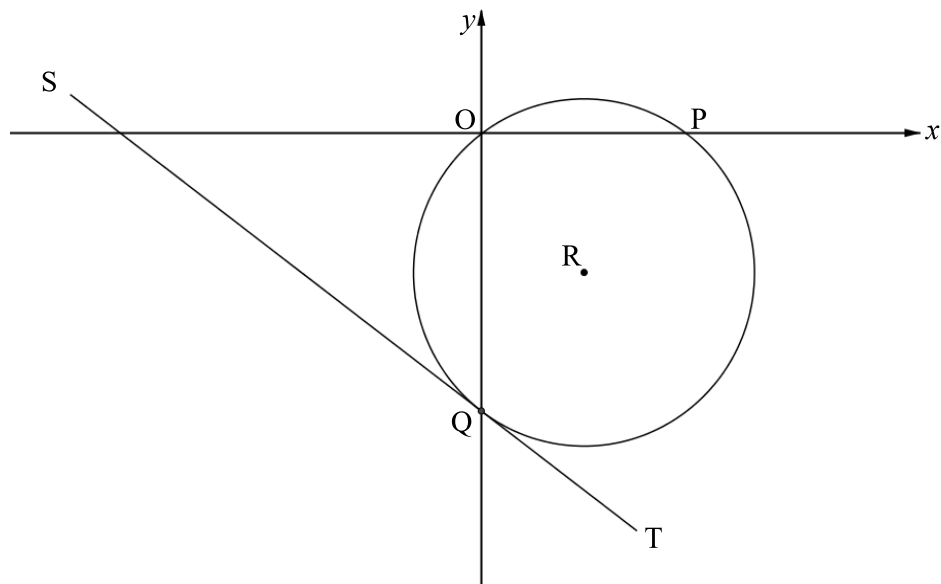
    [12]

UMBUZO 2

Iziphumo zebanga lesi8 zeemvavanyo ezimbini lulunye lunamanqaku ayi50 zidweliswe ngezantsi.

UVAVANYO A (*x*)	39	33	35	44	37	40	24	31	30	5
UVAVANYO B (*y*)	41	45	48	40	47	42	37	44	43	24

2.1   Fumana ioutlier kwitheyibhile enikiweyo.   (1)

2.2   Fumana i-ikhweyizhini yeleast squares regression line.   (3)

2.3   Sebenzisa i-ikhweyizhini yeleast squares regression line ukuqikelela inqaku loVAVANYO B xa umfundi efumene amanqaku ayi14 kuVAVANYO A. Sondeza impendulo yakho kwiwhole namba ekufutshane.   (2)

2.4   Phawula ngestrength secorrelation phakathi koVAVANYO A noVAVANYO B.   (2)
    [8]

UMBUZO 3

Ikhwadrilatherali AEBC izotyiwe. Iikho-odineyithi zikaB ngu (3 ; 4). G, O noF (5 ; 0) zii
x-intasephuthi zeelayini AC, AB noBC ngokulandelelanayo. uH (1 ; – 4) yipoyinti kulayini
AC. $A\hat{B}C = \theta$. Area of $\Delta OBF = 12$ square units ze inclination kalayini AB ibengu 45°.
 $HC = 2AH$

3.1   Khalityhuleyitha ubude bukaBF. Shiya impendulo yakho ikweyona surd form ilula.   (2)

3.2   Khalityhuleyitha igradiyenti kaBF.   (2)

3.3   Khalityhuleyitha isayizi ka θ .   (3)

3.4   Pruva u $HF \| AB$.   (4)

3.5   Kunikwe ukuba, u EC ubhayisekhitha u AB perpendicularly. Loluphi uhlobo
        lwekhwadrilatherali u AEBC?   (1)

3.6   Ngoko, okanye kungenjalo khalityhuleyitha ubude bukaAC.   (4)

3.7   Khalityhuleyitha ieriya yekhwadrilatherali AOFC.   (3)
                                                                                           [19]

UMBUZO 4

4.1 Kwidayagram engezantsi, u R ngumbindi wesekile OPQ. Upoyinti Q yi y-intasephuthi yesekile. SQT yithanjenti yesekile kuQ. I-ikhweyizhini kaSQT ngu $y = -\dfrac{3}{4}x - 8$.
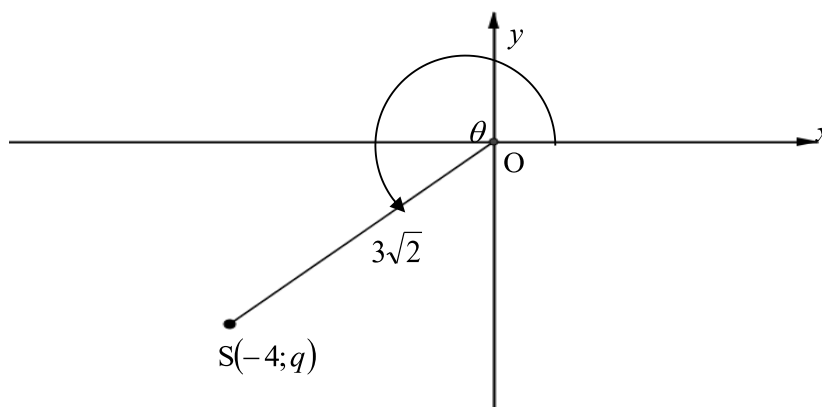

4.1.1 Khalityhuleyitha iikho-odineyithi zikaQ. (2)

4.1.2 Fumana i-ikhweyizhini kaQR ngokwefom $y = mx + c$. (3)

4.1.3 Khalityhuleyitha iikho-odineyithi zikaP, i x-intasephuthi yelayiniQR. (2)

4.1.4 Khalityhuleyitha iikho-odineyithi zikaR ongumbindi wesekile. (3)

4.1.5 Bhala i-ikhweyizhini yesekile enombindi kuR ngokwefom:
$(x - a)^2 + (y - b)^2 = r^2$. (3)

4.1.6 Ukuba u $y = k$ yithanjenti yesekile, fumana i/iivelyu zika k. (3)

4.2 Khalityhuleyitha imaximum length yerediyasi yesekile eneikhweyizhini
$x^2 + y^2 - 2x\sin\theta - 4y\sin\theta = -2$. (5)

[21]

UMBUZO 5

5.1 Kwidayagram engezantsi, upoyinti $S(-4;q)$ nereflex angle θ babonisiwe. U O yipoyinti kwiorijini. $OS = 3\sqrt{2}$.

Ungasebenzisi khalityhuleyitha, fumana ivelyu ka:

5.1.1 q (2)

5.1.2 $\sin(\theta + 45^\circ)$ (4)

5.1.3 $\cos(2\theta - 360^\circ)$ (4)

5.2 Simplifaya okulandelayo ungasebenzisi khalityhuleyitha:

$$\frac{\sin(90^\circ - \theta).\cos 480^\circ + \cos(180^\circ - \theta)}{\cos\theta.\sin 150^\circ - \tan 180^\circ}$$ (5)

5.3 Pruva ukuba u $\dfrac{\cos x}{\sin 2x} - \dfrac{\cos 2x}{2\sin x} = \sin x$ (5)

5.4 Unikiwe: $\dfrac{\cos 60^\circ}{\sin x} - \dfrac{\sin 60^\circ}{\cos x} = 2$

5.4.1 Bonisa ukuba i-ikhweyizhini $\dfrac{\cos 60^\circ}{\sin x} - \dfrac{\sin 60^\circ}{\cos x} = 2$ ingabhalwa njengo

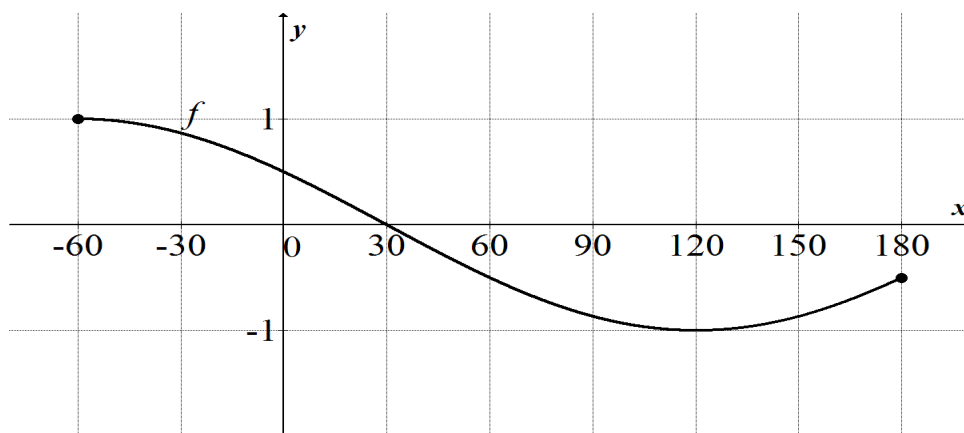$\cos(x + 60^\circ) = \cos(90^\circ - 2x)$ (3)

5.4.2 Ngoko, okanye kungenjalo, fumamna igeneral solution ka

$\dfrac{\cos 60^\circ}{\sin x} - \dfrac{\sin 60^\circ}{\cos x} = 2$ (4)

5.5 Unikiwe ukuba u $\cos 22,5^\circ = \dfrac{a}{c}$ no $a^2 + b^2 = c^2$.

Ngoncedo lwedayagram, okanye kungenjalo, bonisa ukuba u $\dfrac{2ab}{c^2} = \dfrac{\sqrt{2}}{2}$. (5)

[32]

UMBUZO 6

Igrafu ka $f(x) = -\sin(x - 30^\circ)$ izotywe kwi-intavali ka $x \in \left[-60^\circ\,;\,180^\circ \right]$.
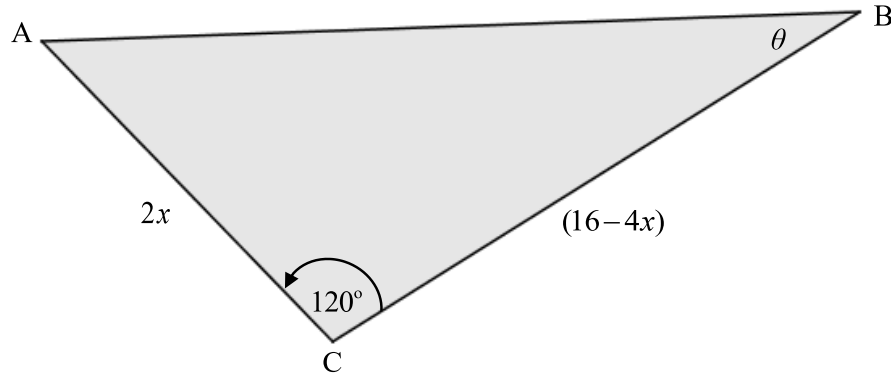

Sebenzisa igrafu ukuphendula imibuzo elandelayo.

6.1   Bhala iperiod ka f.   (1)

6.2   Bhala iminimum veliyu ka f.   (1)

6.3   Fumana irange ka $f(x) + 1$.   (2)

6.4   Zeziphi iivelyu zika x apho igrafu ka f i-inkhrisayo, apho u $x \in \left[-60^\circ\,;\,180^\circ \right]$?   (2)

6.5   Igrafu ka f ishenxiswe 60° ukuya ngasekunene yaze yariflekthwa kwi x-axis
        ukwenza igrafu entsha ka g. Fumana i-ikhweyizhini ka g ngeyona fom ilula.   (3)

6.6   Zoba igrafu ka g kwakule seti ye-axes. Bonisa ngokucacileyo ii-intasephuthi kwii-
        axis neeturning points kwi-intavali zika $x \in \left[-60^\circ\,;\,180^\circ \right]$.   (3)
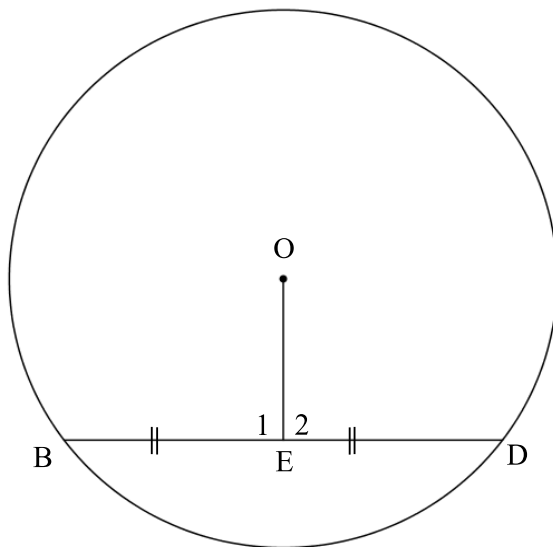
                                                                                                              [12]

UMBUZO 7

Ku ΔABC ngezantsi, uAC = 2x, BC = (16 – 4x), $\hat{C} = 120^{\circ}$, $\hat{B} = \theta$.

7.1   Fumana ieriya ka ΔABC ngokethem zika x, ungasebenzisi khalityhuleyitha.   (3)

7.2   Zeziphi i/iiveliyu zika x apho ieriya ka ΔABC iba yimaximum?   (3)

                                                              [6]

UMBUZO 8

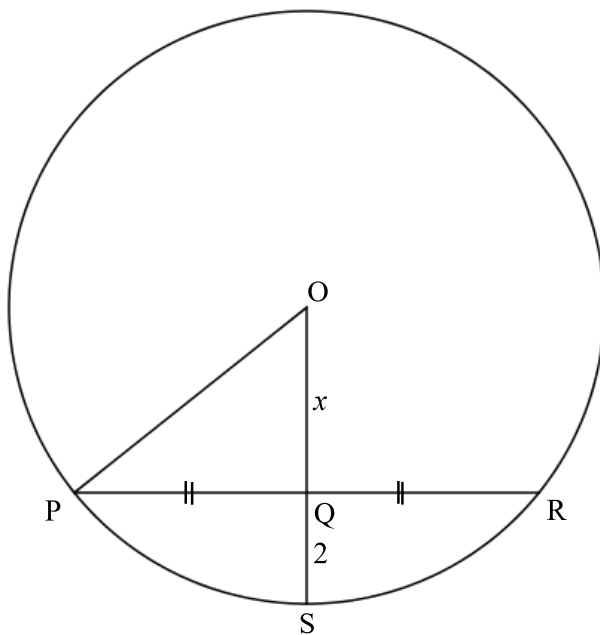8.1   Kwidayagram engezantsi, uO ngumbindi wesekile. U BD yichord yesekile. U E
        yimidipoyinti yechord BD.

Sebenzisa idayagram ekwiNCWADI YOKUPHENDULELA ukupruva ithiyorem ethi:
Ilayini ezotywe ukusuka embindini wesekile ebhayisekhitha ichord iperpendicular
kwichord.

Ngamanye amazwi, pruva ukuba: $OE \perp BD$.   (5)

8.2   Kwidayagram engezantsi, uO ngumbindi wesekile. U Q yimidipoyinti kachord PR.
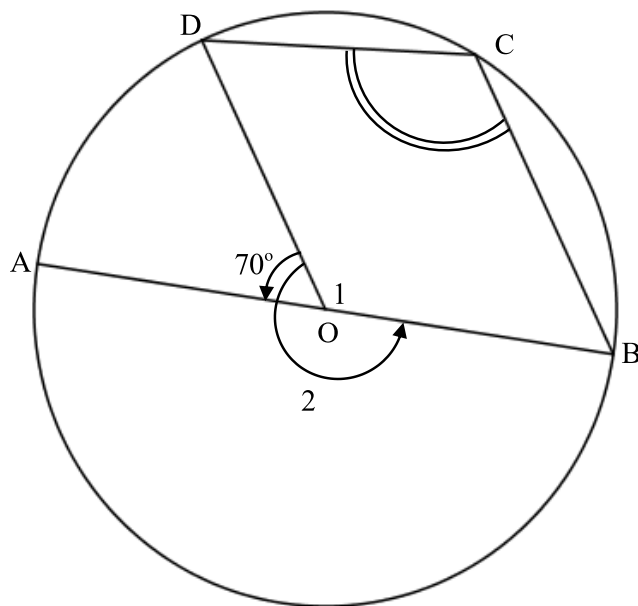   U OQS yirediyasi yesekile. U PR = 8 yunithi, uOQ = x yunithi ze uQS = 2 yunithi.

8.2.1   Fumana, unike nezizathu, isayizi ka $O\hat{Q}P$.   (2)

8.2.2   Khalityhuleyitha ubude bukaPO.   (5)

                                                            [12]

UMBUZO 9

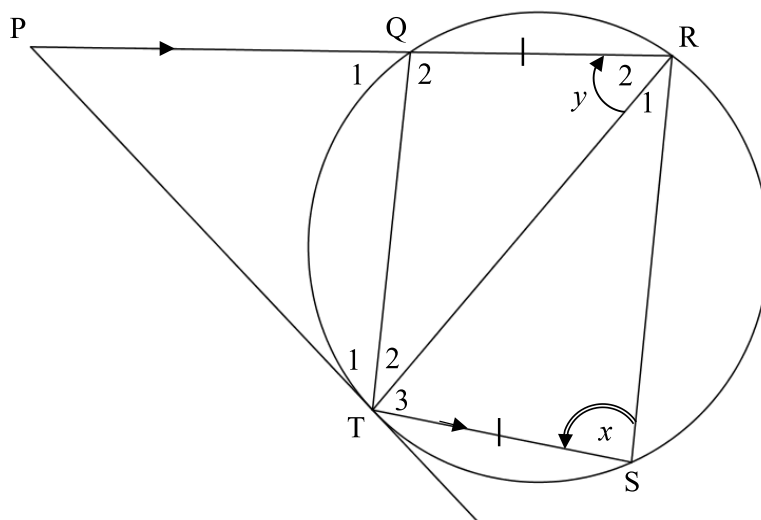9.1　　A, B, C noD ziipoyinti kwisekhamfarenisi yesekile enombindi onguO. U AOB yidayametha yesekile. $A\hat{O}D = 70^{\circ}$.

Khalityhuleyitha isayizi ka $\hat{C}$, unike nezizathu.　　　　　　　　　　　(5)

9.2   U PT yithanjenti kwisekile kuT. PR‖TS ze uPQR abe sisitreyithi layini. U Q, uR

noS ziipoyinti kwisekhamfarenisi yesekile. u $\hat{R}_2 = y$ aze u $\hat{S} = x$. U QR = TS.
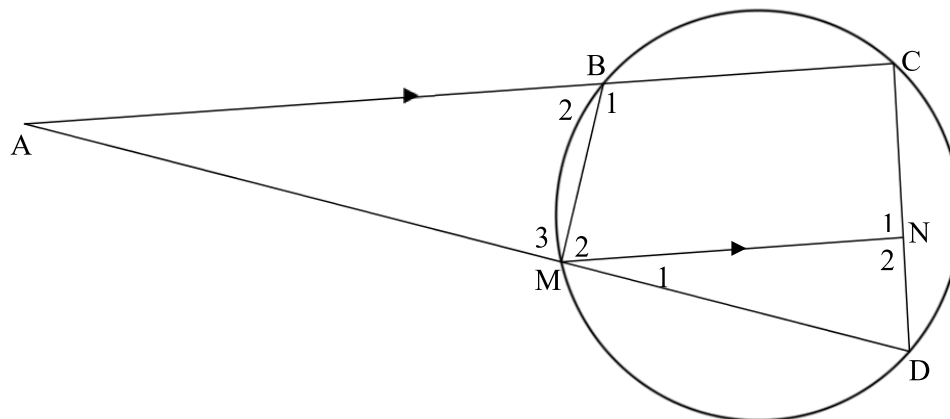

9.2.1   Xela, unike nezizathu, ezinye iiengile eZIMBINI apho inye ilingana no y.   (4)

9.2.2   Fumana, unike nezizathu, enye iengile elingana no $\hat{T}_2$.   (2)

9.2.3   Pruva ukuba uTR yidayametha yesekile.   (4)

[15]

UMBUZO 10

U BCDM yicyclic quadrilateral. Iichord MD noBC zandisiwe zaya kudibana kupoyinti A. U N yipoyinti kuCD. U $AC \parallel MN$ ze $AM = CD$.

AC = 36 yunithi, AD = 48 yunithi aze uBM = 24 yunithi.

10.1   Pruva ukuba u $\triangle ABM \parallel\mid\mid \triangle ADC$.   (4)

10.2   Pruva ukuba u $CD^2 = BM \times AC$.   (3)

10.3   Khalityhuleyitha ubude bukaCN.   (6)

                                                                                                              [13]

                                                                            EWONKE: 150

IPHEPHA LEENKCUKACHA: IMATHEMATIKA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad\qquad A = P(1 - ni) \qquad\qquad A = P(1 - i)^n \qquad\qquad A = P(1 + i)^n$$

$$F = \frac{x\left[(1 + i)^n - 1\right]}{i} \qquad\qquad P = \frac{x\left[1 - (1 + i)^{-n}\right]}{i}$$

$$T_n = a + (n - 1)d \qquad\qquad S_n = \frac{n}{2}\left(2a + (n - 1)d\right)$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a\left(r^n - 1\right)}{r - 1} \quad ; \quad r \neq 1 \qquad\qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$Ku\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} \qquad P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A\ okanye\ B) = P(A) + P(B) - P(A\ noB)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$