KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2024

MEMO

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 150

These marking guidelines consist of 14 pages.

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	**A mark for a correct statement (A statement mark is independent of a reason.)**
R	**A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)**
S/R	**Award a mark if the statement AND reason are both correct.**

QUESTION 1

Penalise only once for incorrect rounding in Question 1.

1.1.1	Mean $= \frac{165500}{12}$ $= R13792$ Also accept: 13,79 thousand rand Answer only: Full marks If answer is given as 13,79 instead of R13 792, penalise in 1.1.1, but don't penalise again for this mistake in 1.1.2, 1.2 and 1.5.	✓A 165 500 in numerator ✓CA answer (2)
1.1.2	Standard deviation = R4 404	✓A answer (1)
1.2	R13 792 +R4 404 = R18 196 2 employees earn a salary more than one standard deviation above the mean. Answer only: Full marks	✓CA R18 196 ✓CA 2 employees (2)
1.3	$a = 8,45$ $b = 0,45$ $\hat{y} = 0,45x + 8,45$ Answer only: Full marks	✓A correct a value ✓A correct b value ✓ CA answer (3)
1.4	$r = 0,94$	✓A answer (1)
1.5	$\hat{y} = 0,45(30) + 8,45$ $\hat{y} = 21,95$ $\therefore$ R21 950 **OR** R21 804 (calculator)	✓CA substitution ✓CA answer (2) **OR** ✓✓ CA CA (2)

1.6	Yes. $r = 0{,}94$ implies a strong correlation between employee experience and monthly salary and therefore a prediction would be reliable. **OR** Yes. $r = 0{,}94$, which is close to 1, and therefore implies a strong correlation between employee experience and monthly salary and therefore a prediction would be reliable.	✓CA answer ✓CA justification (2) **OR** ✓CA answer ✓CA justification (2)
		[13]

QUESTION 2

2.1	5500	✓A answer (1)
2.2	$Q_1 = 29$ (accept 28 – 29) $Q_3 = 39$ (accept 38 – 39) $IQR = 10$ (accept 9 – 11)	✓A value of Q_1 ✓A value of Q_3 ✓CA answer (3)
2.3	$\frac{650}{5500}$ (accept 620 – 700) $= 11{,}82\%$ (accept 11,27% – 12,73%)	✓A numerator in range 620 to 700 ✓CA answer (2)
		[6]

QUESTION 3

3.1	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2-4}{-6-0}$ $= 1$	✓A substitution ✓CA answer (2)
3.2	$m_{CD} = m_{AB} = 1$ $y = mx + c$ Substitute $(10\,;-1)$ and $m_{CD} = 1$: $-1 = 1(10) + c$ $c = -11$ $y = 1x - 11$	✓CA $m_{CD} = 1$ ✓CA substitution of gradient and point ✓CA answer (3)

3.3	Midpoint of AC is the same as the midpoint of BD [diagonals of parm. bisect each other] $\therefore$ Midpoint of AC $= M\left(\frac{-6+10}{2}\ ;\ \frac{-2-1}{2}\right)$ $= M\left(2\ ;\ \frac{-3}{2}\right)$ Answer only: Full marks **OR** $C(4\ ;-7)$ $\therefore$ Midpoint of AC $= M\left(\frac{0+4}{2}\ ;\ \frac{4-7}{2}\right)$ $= M\left(2\ ;\ \frac{-3}{2}\right)$	✓A midpoint of BD ✓CA x coordinate ✓CA y-coordinate (3) **OR** ✓A coordinates of C ✓CA x coordinate ✓CA y-coordinate (3)
3.4	$C(4\ ;-7)$	✓CA x coordinate ✓CA y-coordinate (2)
3.5	$m_{AB} = 1$ $\tan A\hat{F}G = 1$ $A\hat{F}G = 45°$ $m_{AD} = \frac{-1-4}{10-0}$ $= -\frac{1}{2}$ $\tan A\hat{H}J = -\frac{1}{2}$ $A\hat{H}J = 153,43°$ $B\hat{A}D = 153,43° - 45°$ [exterior $\angle$ of ΔHAF] $= 108,43°$ $\therefore B\hat{C}D = 108,43°$ [opp $\angle$ s of a parm.]	✓CA $\tan A\hat{F}G = 1$ ✓CA $A\hat{F}G = 45°$ ✓A $m_{AD} = -\frac{1}{2}$ ✓CA $A\hat{H}J = 153,43°$ ✓CA $B\hat{A}D = 108,43°$ ✓CA $B\hat{C}D = 108,43°$ (6)

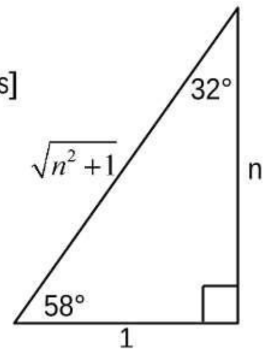
	OR $CD=\sqrt{(10-4)^2+(-1+7)^2}=6\sqrt{2}$ $BC=\sqrt{(-6-4)^2+(-2+7)^2}=5\sqrt{5}$ $BD=\sqrt{(-6-10)^2+(-2+1)^2}=\sqrt{257}$ $BD^2=BC^2+CD^2-2.BC.CD.\cos B\hat{C}D$ $(\sqrt{257})^2=(5\sqrt{5})^2+(6\sqrt{2})^2-2.(5\sqrt{5}).(6\sqrt{2}).\cos B\hat{C}D$ $\therefore \cos B\hat{C}D=\frac{(5\sqrt{5})^2+(6\sqrt{2})^2-(\sqrt{257})^2}{2.(5\sqrt{5}).(6\sqrt{2})}$ $B\hat{C}D=108,43°$	**OR** ✓CA length of CD ✓CA length of BC ✓A length of BD ✓A use of cosine rule ✓CA substitution into cosine rule ✓CA answer (6)
		[16]

QUESTION 4

4.1.1	$r^2=OJ^2=2^2+(-1)^2$ $r=\sqrt{5}$	✓A substitution ✓A length of OJ (2)
4.1.2	$OK=OJ+JK=\sqrt{5}+2\sqrt{5}=3\sqrt{5}$ $(3\sqrt{5})^2=(a-0)^2+(-3-0)^2$ $45=a^2+9$ $a^2=36$ $a=-6$ or $a=6$ N/A **OR** $OJ=\sqrt{5}$ $\therefore JK=2\sqrt{5}$ $(2\sqrt{5})^2=(a-2)^2+(-3+1)^2$ $20=a^2-4a+4+4$ $a^2-4a-12=0$ $(a-6)(a+2)=0$ $a=6$ or $a=-2$ N/A	✓A length of OK ✓A substitution ✓A a^2 subject of formula (3) **OR** ✓A length of JK ✓A substitution ✓A standard form (3)
4.1.3	$(x-6)^2+(y+3)^2=20$	✓A $(x-6)^2+(y+3)^2$ ✓CA $=20$ (2)

4.1.4	Substitute $(10;-4)$: $(10-6)^2+(-4+3)^2$ $=17$ $17<20$, $\therefore$ the point lies inside the circle	✓CA substitution ✓CA $17<20$ ✓CA conclusion (3)
4.1.5	$KO=\sqrt{5}+2\sqrt{5}=3\sqrt{5}$ In ΔPOR and ΔPKS: 1. $\hat{P}=\hat{P}$ [common] 2. $P\hat{R}O=P\hat{S}K$ [$=90°$; tangent $\perp$ radius] 3. $P\hat{O}R=P\hat{K}S$ [remaining $\angle$s] $\Delta POR \mid\mid\mid \Delta PKS$ [$\angle\angle\angle$] $\frac{PO}{PK}=\frac{OR}{KS}$ [$\mid\mid\mid$ Δs] $=\frac{OR}{2OR}=\frac{1}{2}$ $\therefore PO=\frac{1}{2}PK$ $PO=OK=3\sqrt{5}$ $PK=2(3\sqrt{5})=6\sqrt{5}$ $P\hat{S}K=90°$ [radius $\perp$ tangent] $PS^2=PK^2-KS^2$ [Theorem of Pythagoras] $=(6\sqrt{5})^2-(2\sqrt{5})^2$ $=160$ $\therefore PS=\sqrt{160}=4\sqrt{10}$	✓ CA length of KO ✓A $PO=\frac{1}{2}PK$ ✓ CA length of PK ✓ CA substitution in Theorem of Pythagoras ✓CA answer (5)
4.2.1	$x^2-4x+4+y^2+5y+\frac{25}{4}=-d+4+\frac{25}{4}$ $(x-2)^2+\left(y+\frac{5}{2}\right)^2=-d+\frac{41}{4}$ Centre $\left(2;-\frac{5}{2}\right)$ Answer only: Full marks	✓A completing the square ✓A $(x-2)^2+\left(y+\frac{5}{2}\right)^2$ ✓CA x coordinate ✓CA y coordinate (4)
4.2.2	diameter = 24 units, $\therefore$ radius =12 units $-d+\frac{41}{4}=144$ $d=-\frac{535}{4}$	✓A radius =12 units ✓CA equating ✓CA answer (3)
		[22]

QUESTION 5

5.1.1	$\tan 58° = n$ $r^2 = x^2 + y^2$ [Theorem of Pythagoras] $r^2 = 1^2 + n^2$ $r = \sqrt{n^2+1}$ $\therefore \sin 58° = \frac{n}{\sqrt{1+n^2}}$  	✓A subst. in Theorem of Pythagoras ✓A $r = \sqrt{n^2+1}$ ✓CA answer (3)
5.1.2	$\sin 296°$ $= -\sin 64°$ $= -\sin 2(32°)$ $= -2\sin 32° \cos 32°$ $= -2\cos 58° \sin 58°$ $= -2\left(\frac{1}{\sqrt{1+n^2}}\right)\left(\frac{n}{\sqrt{1+n^2}}\right)$ $= \frac{-2n}{1+n^2}$	✓A $-\sin 64°$ ✓CA expansion ✓CA co-functions ✓CA answer (4)
5.1.3	$\cos 2°$ $= \cos(60° - 58°)$ $= \cos 60° \cos 58° + \sin 60° \sin 58°$ $= \frac{1}{2} \times \frac{1}{\sqrt{1+n^2}} + \frac{\sqrt{3}}{2} \times \frac{n}{\sqrt{1+n^2}}$ $= \frac{1+\sqrt{3}n}{2\sqrt{1+n^2}}$ **OR** $\cos 2°$ $= \cos(32° - 30°)$ $= \cos 32° \cos 30° + \sin 32° \sin 30°$ $= \frac{n}{\sqrt{1+n^2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{1+n^2}} \times \frac{1}{2}$ $= \frac{\sqrt{3}n+1}{2\sqrt{1+n^2}}$	✓A $\cos(60° - 58°)$ ✓A expansion ✓CA answer (3) **OR** ✓A $\cos(32° - 30°)$ ✓A expansion ✓CA answer (3)

5.2.1	LHS $=\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$ $=\frac{2\sin^2 x}{2\sin x\cos x}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=\text{RHS}$	✓A $1-2\sin^2 x$ ✓A $2\sin x\cos x$ ✓A simplification (3)
	OR	**OR**
	LHS $=\frac{\sin^2 x+\cos^2 x-(2\cos^2 x-1)}{2\sin x\cos x}$ $=\frac{\sin^2 x-\cos^2 x+1}{2\sin x\cos x}$ $=\frac{\sin^2 x-\cos^2 x+\sin^2 x+\cos^2 x}{2\sin x\cos x}$ $=\frac{2\sin^2 x}{2\sin x\cos x}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=\text{RHS}$	✓A $2\cos^2 x-1$ ✓A $2\sin x\cos x$ ✓A simplification (3)
	OR	**OR**
	LHS $=\frac{\sin^2 x+\cos^2 x-(\cos^2 x-\sin^2 x)}{2\sin x\cos x}$ $=\frac{2\sin^2 x}{2\sin x\cos x}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=\text{RHS}$	✓A $\cos^2 x-\sin^2 x$ ✓A $2\sin x\cos x$ ✓A simplification (3)

5.2.2	$\tan 15°$ $=\frac{1-\cos 2(15°)}{\sin 2(15°)}$ $=\frac{1-\cos 30°}{\sin 30°}$ $=\frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $=\left(1-\frac{\sqrt{3}}{2}\right)\times\frac{2}{1}$ $=2-\sqrt{3}$	✓A $\frac{1-\cos 2(15°)}{\sin 2(15°)}$ ✓A substitution of special angle values ✓CA answer (3)
5.3	$\sin(360°+x).\cos(90°+x)-\frac{\sin x}{\cos(-x).\tan(360°-x)}$ $=\sin x.(-\sin x)-\frac{\sin x}{\cos x.(-\tan x)}$ $=-\sin^2 x+1$ $=\cos^2 x$	✓A $\sin x$ ✓A $-\sin x$ ✓A $\cos x$ ✓A $-\tan x$ ✓CA 1 ✓CA answer (6)
5.4	$\cos 2x-\frac{1}{3}=\frac{1}{3}\sin x$ $1-2\sin^2 x-\frac{1}{3}=\frac{1}{3}\sin x$ $3-6\sin^2 x-1=\sin x$ $6\sin^2 x+\sin x-2=0$ $(3\sin x+2)(2\sin x-1)=0$ $\sin x=-\frac{2}{3}$ $\therefore x=221,81°+k.360°$ or $x=318,19°+k.360°$, $k\in Z$ **or** $\sin x=\frac{1}{2}$ $\therefore x=30°+k.360°$ or $x=150°+k.360°$, $k\in Z$	✓A $1-2\sin^2 x$ ✓A standard form ✓CA factors ✓CA $x=221,81°+k.360°$ or $x=318,19°+k.360°$ ✓CA $x=30°+k.360°$ or $x=150°+k.360°$ ✓A $k\in Z$ (6)
5.5	$\sin(2x+30°)+k=3$ $\sin(2x+30°)=3-k$ $\sin(2x+30°)<-1$ or $\sin(2x+30°)>1$ $3-k<-1$ or $3-k>1$ $k>4$ or $k<2$	✓A $\sin(2x+30°)=3-k$ ✓A $\sin(2x+30°)<-1$ or $\sin(2x+30°)>1$ ✓CA $3-k<-1$ or $3-k>1$ ✓CA $k>4$ ✓CA $k<2$ (5)
		[33]

QUESTION 6

6.1	$b = \frac{1}{2}$	✓A answer (1)
6.2	period = 360°	✓A answer (1)
6.3	$A(30°;1)$	✓A 30° ✓A 1 (2)
6.4	$x = 160°$	✓A answer (1)
6.5	$-3 \leq y \leq 1$ **OR** $y \in [-3\,;\,1]$	✓✓ AA (2) **OR** ✓✓ AA (2)
		[7]

QUESTION 7

7.1	$\frac{\sin y}{2b} = \frac{\sin x}{b}$ $\sin y = \frac{2b\sin x}{b}$ $\sin y = 2\sin x$ **OR** $b\sin y = 2b\sin x$	✓A substitution in sine rule ✓A $\sin y = \frac{2b\sin x}{b}$ OR $b\sin y = 2b\sin x$ (2)
7.2	$\frac{AB}{BC} = \tan\theta$ $\therefore AB = BC.\tan\theta$ $\hat{D} = 180° - (x+y)$ $BC^2 = BD^2 + CD^2 - 2BD.CD\cos\hat{D}$ $BC^2 = (2b)^2 + b^2 + 2(2b)(b)\cos[180° - (x+y)]$ $BC^2 = (2b)^2 + b^2 + 2(2b)(b)\cos(x+y)$ $BC^2 = 5b^2 + 4b^2\cos(x+y)$ $BC^2 = b^2(5 + 4\cos(x+y))$ $BC = b\sqrt{(5+4\cos(x+y))}$ $\therefore AB = b\tan\theta\sqrt{(5+4\cos(x+y))}$	✓A $\frac{AB}{BC} = \tan\theta$ ✓A $AB = BC.\tan\theta$ ✓A $\hat{D} = 180° - (x+y)$ ✓A substitution in cosine rule ✓A $+\cos(x+y)$ ✓A simplification ✓A taking square root on LHS and RHS (7)
7.3	$AB = 54{,}8\tan 42{,}6°\sqrt{5+4\cos(31°+75{,}84°)}$ $AB = 98{,}76$ metres	✓A substitution ✓A answer (2)
		[11]

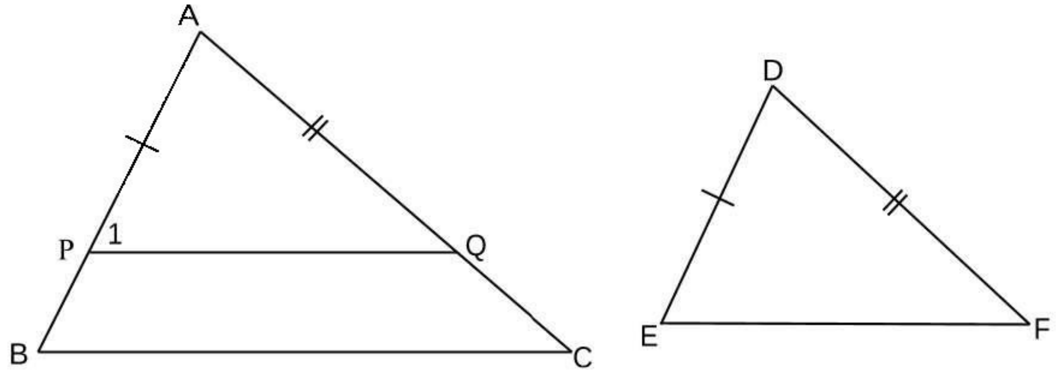
QUESTION 8

8.1.1	$\hat{A}_1 = \frac{1}{2}(C\hat{O}E)$ [$\angle$ at centre = $2\times\angle$ at circumference] $= 68°$	✓R ✓A answer (2)
8.1.2	$\hat{E}_1 = \hat{A}_1$ [tan chord theorem] $= 68°$	✓R ✓CA answer (2)
8.1.3	$B\hat{C}E = \hat{E}_1$ [alt $\angle$s; DF || CA] $= 68°$	✓R ✓CA answer (2)
8.1.4	$\hat{G} = 180° - B\hat{C}E$ [opp. $\angle$s of cyclic quad] $= 112°$	✓R ✓CA answer (2)
8.2	$B\hat{E}D = 90°$ [radius $\perp$ tangent] $= \hat{B}_1$ [co-interior $\angle$s ; DF || CA] $\therefore AB = \frac{1}{2}AC$ [line from centre $\perp$ to chord] $= 7$ units **OR** $B\hat{E}D = 90°$ [radius $\perp$ tangent] $\therefore \hat{E}_2 = 22°$ $\therefore \hat{B}_1 = 180° - (B\hat{C}E + \hat{E}_2)$ [sum of $\angle$s of ΔBCE] $= 180° - (68° + 22°)$ $= 90°$ $\therefore AB = \frac{1}{2}AC$ [line from centre $\perp$ to chord] $= 7$ units	✓S✓R ✓ S/R ✓ R ✓A answer (5) **OR** ✓S✓R ✓ S/R ✓ R ✓A answer (5)
		[13]

QUESTION 9

9.	In ΔQVS:	
	$\frac{QU}{UV}=\frac{QT}{TS}$ [prop. theorem; UT‖VS] OR [line ‖ one side of Δ]	✓S ✓R
	$=\frac{5}{2}$	
	$=\frac{5k}{2k}$	
	$\therefore 5k=2x$; or: $x=\frac{5}{2}k$. And: $3x=\frac{15}{2}k$	✓ x i.t.o. k
	In ΔUPR:	
	$\frac{PS}{PR}=\frac{UV}{UR}$ [prop. theorem; UP‖VS] OR [line ‖ one side of Δ]	✓S
	$=\frac{2k}{\frac{15}{2}k}$	
	$=\frac{4}{15}$	✓S
	$\therefore \frac{PS}{SR}=\frac{4}{11}$	✓answer
		(6)
		[6]

QUESTION 10

<table>
<tr><td>10.1</td><td colspan="2">
</td></tr>
<tr><td></td><td>Construct AP = DE and AQ = DF
In ΔAPQ and ΔDEF:
1. AP = DE [from construction]
2. AQ = DF [from construction]
3. $\hat{A} = \hat{D}$ [given]
$\therefore \Delta APQ \equiv \Delta DEF$ [SAS]
$\therefore \hat{P}_1 = \hat{E}$ [from $\equiv \Delta s$]
But: $\hat{B} = \hat{E}$ [given]
$\therefore \hat{P}_1 = \hat{B}$
$\therefore PQ \parallel BC$ [corresponding $\angle$s are =]
$\therefore \frac{AP}{AB} = \frac{AQ}{AC}$ [prop. theorem; PQ ∥ BC]
$\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [DE = AP; DF = AQ]</td><td>✓construction
✓ $\Delta APQ \equiv \Delta DEF$
✓ $\hat{P}_1 = \hat{E}$
✓S✓R
✓S/R
(6)</td></tr>
<tr><td>10.2.1</td><td>In ΔMKL and ΔMNP:
1. $\frac{MK}{MN} = \frac{40}{16} = 2,5$
2. $\frac{KL}{NP} = \frac{25}{10} = 2,5$
3. $\frac{ML}{MP} = \frac{30}{12} = 2,5$
$\therefore \Delta MKL \parallel\!| \Delta MNP$ [sides of Δs in proportion]</td><td>✓S
✓S
✓S
✓R
(4)</td></tr>
</table>

10.2.2	$N\hat{P}M = \hat{L}$ $\therefore$ KLNP is a cyclic quadrilateral **OR** $P\hat{N}M = \hat{K}$ $\therefore$ KLNP is a cyclic quadrilateral	[from ||| Δs] [converse: ext. $\angle$ of cyclic quadrilateral] OR [ext. $\angle$ of quad = int. opp. $\angle$] [from ||| Δs] [converse: ext. $\angle$ of cyclic quadrilateral] OR [ext. $\angle$ of quad = int. opp. $\angle$]	✓S ✓R ✓R (3) **OR** ✓S ✓R ✓R (3)
10.3.1	In ΔBCE and ΔADE: 1. $\hat{E}_1 = \hat{E}_3$ 2. $\hat{C}_1 = \hat{D}_2$ 3. $\hat{B} = \hat{A}$ $\therefore \Delta BCE \|\|\| \Delta ADE$ $\therefore \frac{BC}{CE} = \frac{AD}{DE}$ $\therefore BC = \frac{AD.CE}{DE}$	 [vertically opp. $\angle$s] [$\angle$s in the same segment] [sum of $\angle$s of Δs] [$\angle\angle\angle$] [from ||| Δs]	✓ selecting triangles ✓S ✓S/R ✓ $\hat{B} = \hat{A}$ OR [$\angle\angle\angle$] ✓R ✓S/R (5)
10.3.2	In ΔADE and ΔBDC: 1. $\hat{D}_2 = \hat{D}_1$ 2. $\hat{A} = \hat{B}$ 3. $\hat{E}_3 = B\hat{C}D$ $\therefore \Delta ADE \|\|\| \Delta BDC$ $\therefore \frac{AD}{BD} = \frac{DE}{CD}$ $\therefore AD.CD = DE.BD$ $= DE.(DE + BE)$ $= DE^2 + DE.BE$	 [given] [$\angle$s in the same segment] [sum of $\angle$s of Δs] [$\angle\ \angle\ \angle$] [from ||| Δs]	✓ selecting triangles ✓S/R ✓ R ✓ S ✓ substitute DE + BE (5)
			[23]

TOTAL: 150