GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2024

10612

MATHEMATICS

(PAPER 2)

MATHEMATICS: Paper 2

TIME: 3 hours

MARKS: 150

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PREPARATORY EXAMINATION 2024

CANDIDATE'S NAME	
DATE	
TEACHER	
SUBJECT NAME	MATHEMATICS (10612)

BOOK NUMBER		OF		BOOKS
PAPER NUMBER	2			

ANSWER ALL THE QUESTIONS IN THE QUESTION PAPER.

MARKER							MODERATOR'S INITIALS IN RELEVANT BLOCK							
Question	Marks			Marker's Code & Initials	Marks									
1														
2														
3														
4														
5														
6														
7														
8														
9														
10														
				TOTAL										

RE-MARK/RE-CHECK				
Question	Marks			Initials
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
TOTAL				

READ THE INSTRUCTIONS ON THE NEXT PAGE.

TIME: 3 hours

MARKS: 150

34 pages + 1 information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

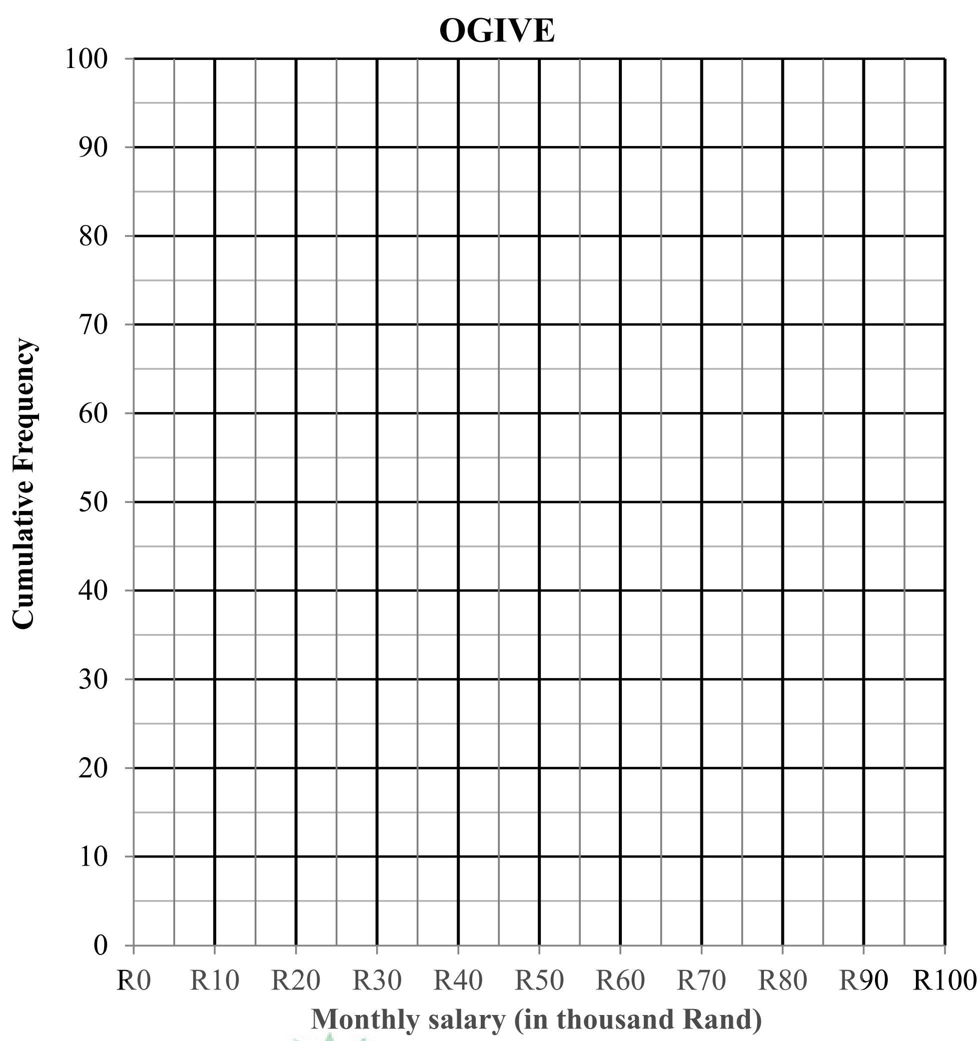
1. This question paper consists of 10 questions. Answer ALL questions in the spaces provided.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will NOT necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round-off answers correct to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An INFORMATION SHEET with formulae is included at the end of the question paper.
8. No pages may be torn from this question paper.
9. Candidates may not retain a question paper or remove it from the examination room. Question papers must be returned to the invigilator at the end of the examination session.
10. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.
11. Indicate the questions you have answered by drawing a circle around the relevant numbers on the front cover of the question paper where marks are to be recorded.
12. Draw a neat line through any work/rough work that must not be marked.
13. In the event that you use the additional space provided:
    - 13.1 Write down the number of the question.
    - 13.2 Leave a line and rule off after your answer.
14. Write neatly and legibly.

QUESTION 1

The table below shows the monthly salaries of 100 employees at Adams Law.

Monthly salary (in thousand Rand)	Number of employees	Cumulative frequency
R0 $< x \leq$ R10	3	
R10 $< x \leq$ R20	4	
R20 $< x \leq$ R30	13	
R30 $< x \leq$ R40	20	
R40 $< x \leq$ R50	21	
R50 $< x \leq$ R60	12	
R60 $< x \leq$ R70	12	
R70 $< x \leq$ R80	8	
R80 $< x \leq$ R90	5	
R90 $< x \leq$ R100	2	

1.1 Draw an ogive (cumulative frequency graph) to represent the data given in the table.



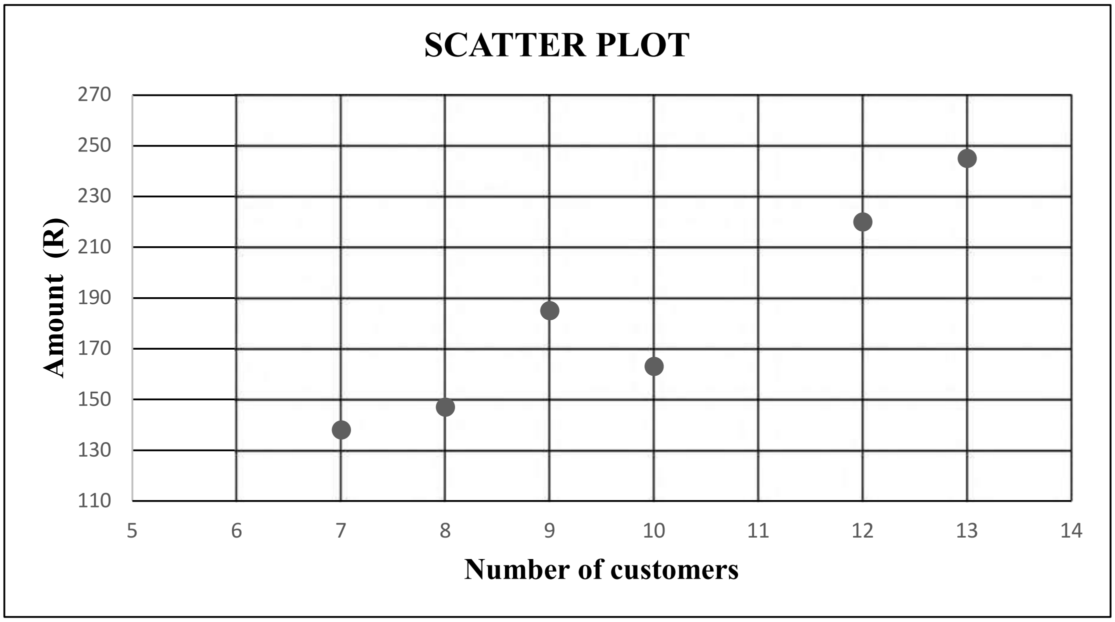
(4)

1.2	Use the ogive to determine the number of employees that receive less than R35 000 per month.	(1)
1.3	Determine the median of the data.	(1)
1.4	The monthly salaries of the 5 employees in the interval $R80\,000 < x \leq R90\,000$ are given below: R84 000 | R85 000 | R87 000 | R89 000 | R89 000 Two employees from the interval R70 000 $< x \leq$ R80 000 are promoted and receive monthly salaries of R83 000 and R84 000 respectively.	
1.4.1	Determine the mean monthly salary in the interval R80 000 $< x \leq$ R90 000.	(1)
1.4.2	Calculate the standard deviation in this interval.	(1)
1.4.3	Determine the percentage of employees whose monthly salary lie within one standard deviation of the mean.	(3)
		[11]

QUESTION 2

During a netball tournament, there are several stalls selling food and refreshments. In a period of 10 minutes at 6 stalls, the following was observed:

Stall	Number of customers	Amount of money spent (R)
1	10	163
2	7	138
3	9	185
4	12	220
5	8	147
6	13	245

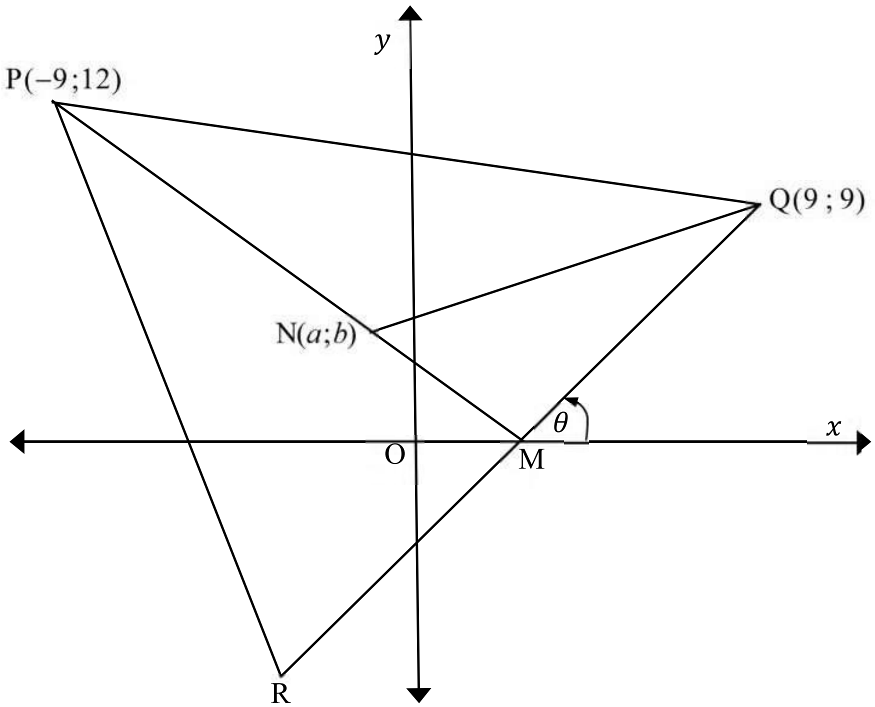


2.1	Determine the equation of the least squares regression line of the data.	(3)
2.2	Predict the amount of money spent by 11 customers.	(2)

2.3	Determine the correlation coefficient of the data.	(1)
2.4	The organisers of the event think that there is a very weak positive correlation between the number of customers and the amount of money received at stalls. Motivate whether you agree or not.	(1)
2.5	At another stall, 6 customers spent a total amount of R195. If this point is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations.	(2)
		[9]

QUESTION 3

In the diagram below, P(–9 ; 12), Q(9 ; 9) and R are vertices of ΔPQR. M is the midpoint of QR and N(a ; b) is a point on PM in the second quadrant. The equation of QR is given by $2y - 3x + 9 = 0$. The angle of inclination of QR is θ.

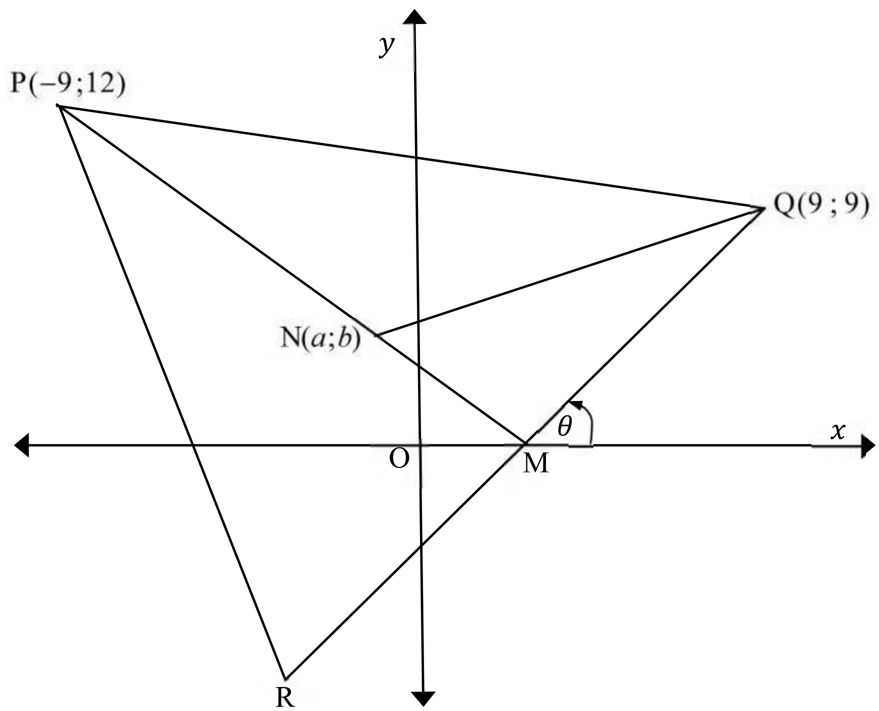


3.1	Calculate the coordinates of M, the x-intercept of line PM.	
		(2)

3.2	Determine the equation of PM in the form $y = mx + c$.	
		(4)
3.3	Calculate the size of θ.	
		(2)
3.4	Show that $b = 3 - a$, if P, N and M are collinear.	
		(1)

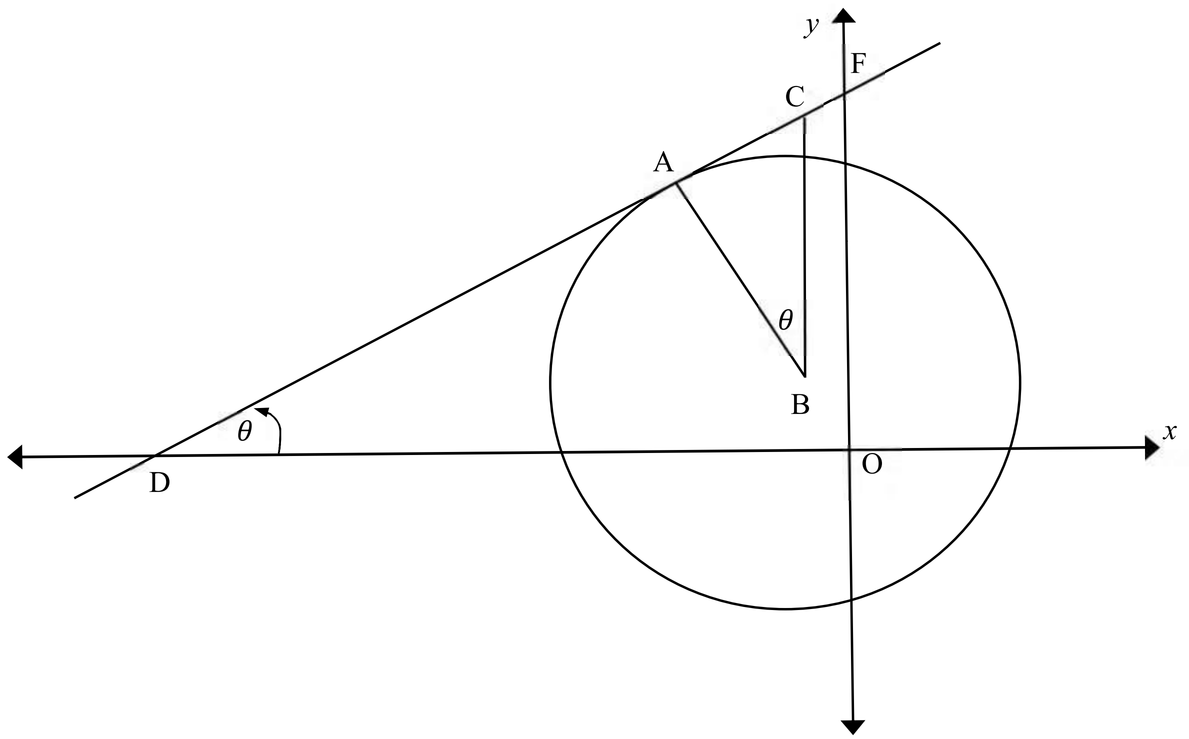
3.5	Hence, determine the value of a and b if $\text{NQ} = 5\sqrt{5}$ units.	(5)
3.6	Determine the equation of a circle having centre at O, the origin, and passing through point R.	(4)

3.7	The acute angle between the lines QR and the line with equation $y = mx + 4$ is $45°$. Determine the possible value(s) of m.	
		(4)
		[22]



QUESTION 4

In the diagram, the equation of the circle centred at B is given by $(x+1)^2+(y-1)^2=20$.
DF is a tangent to the circle at A with D and F, the x- and y-intercepts respectively. $C(-1;6)$ is a point on DF with BC parallel to the y-axis. $C\hat{B}A = A\hat{D}O = \theta$.



4.1	Write down the coordinates of B.	(1)
4.2	Show that $AC=\sqrt{5}$.	(3)

4.3	Write down the value of $\tan\theta$.	(1)
4.4	Show that the equation of AB is given by $y = -2x - 1$.	(3)

4.5	Determine the coordinates of A.	
		(4)
4.6	Calculate the ratio of the area of ΔABC to the area of ΔODF. Simplify your answer.	
		(6)
		[18]

QUESTION 5

5.1	Simplify $\dfrac{\sin^2(180°+x).\sin(-x)}{-\sin(90°+x).\tan x}-1$ to a single trigonometric term.	(6)
5.2	Given: $\cos(A-B)=\cos A\cos B+\sin A\sin B$	
5.2.1	Use the above identity to deduce that $\sin(A+B)=\sin A\cos B+\cos A\sin B$.	(3)
5.2.2	**Without using a calculator,** simplify the following: $\cos 420°\cos 15°+\sin 300°\cos 105°$	(5)

5.3	Given: $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)$	
5.3.1	Prove that $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)=\frac{\cos 2x}{\cos^2 x}$.	(3)
5.3.2	For what value(s) of x in the interval $x \in (0°\ ;\ 180°)$ is $\tan^2 x\left(\frac{1}{\tan^2 x}-1\right)$ undefined?	(1)
5.4	Determine the general solution of the equation $\cos 2x = \cos x$.	(6)

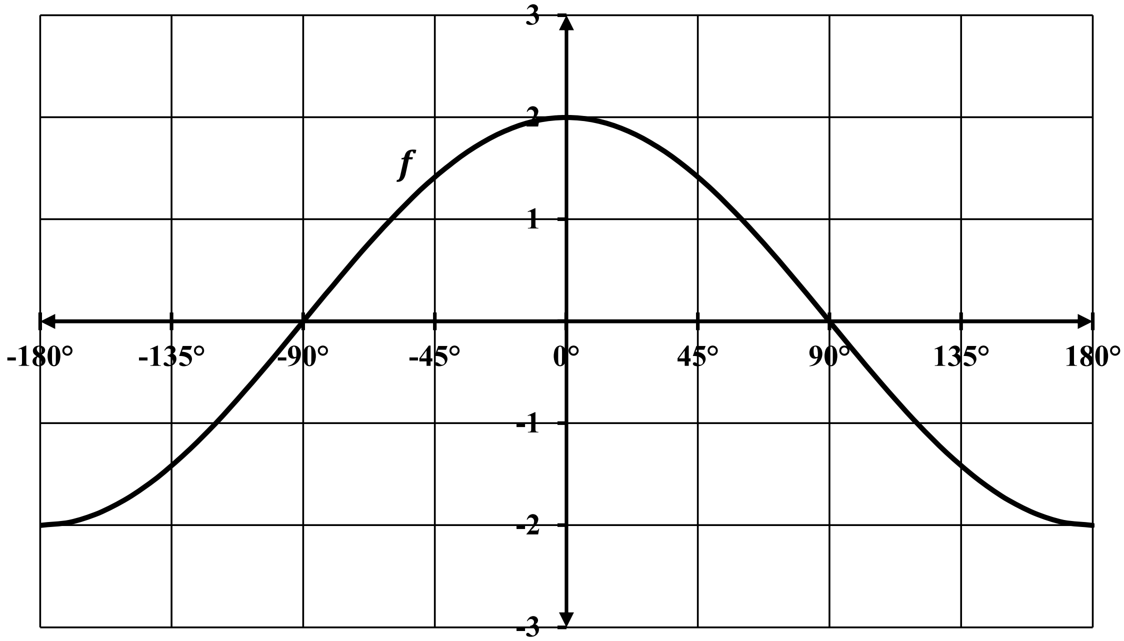
5.5	Given that $\sin\theta = \frac{1}{2}$, calculate the numerical value of $\sin 3\theta$, **without using a calculator**.	
		(7)

[31]

QUESTION 6

In the diagram below, the graph of $f(x) = 2\cos x$ is drawn for the interval $x \in [-180°;180°]$.

6.1 On the grid below, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180°\ ;\ 180°]$. Clearly show all intercepts with the axes and the turning point(s) of the graph.



(3)

6.2 Write down the period of $2g(x+10°)$.

(1)

6.3 Use the graphs to determine the value(s) of x in the interval $x \in [0°;180°]$ for which $f(x).g(x) \leq 0$.

(2)

6.4 Write down the maximum value of $f(x) - g(x)$.

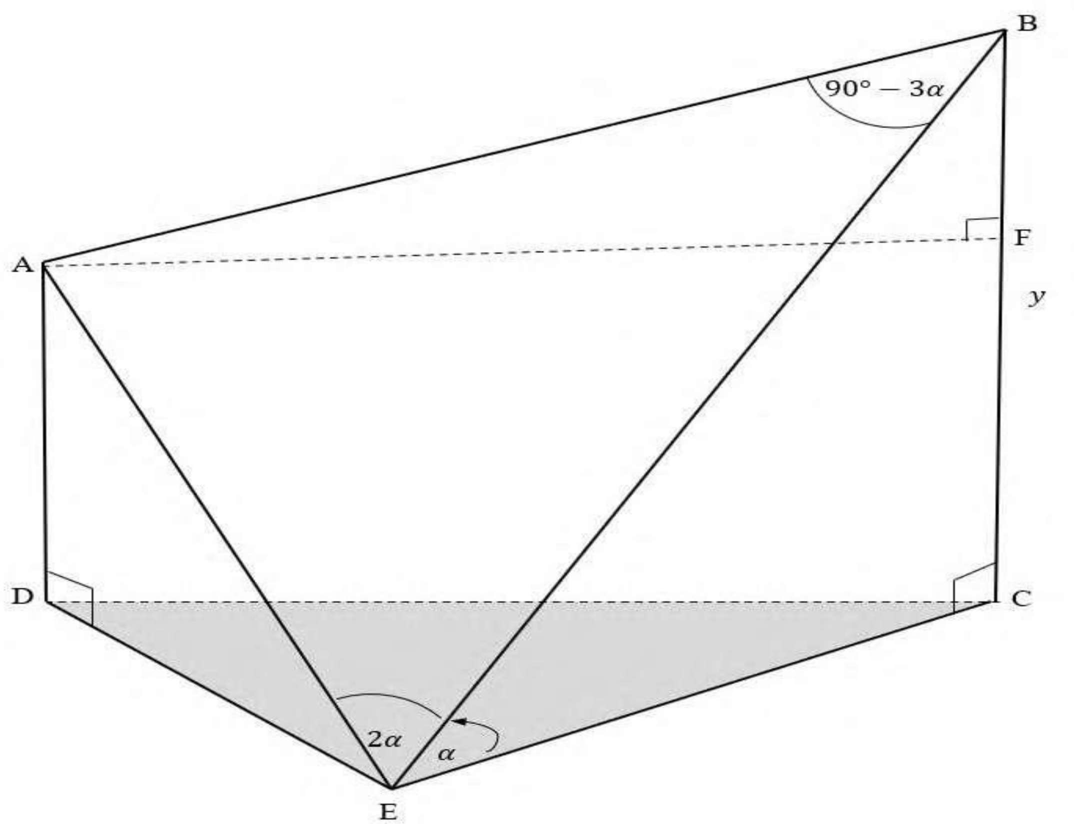
(1)

6.5	Determine the range of $y = 2^{2\cos x + 2}$	
		(2)
		[9]

QUESTION 7

The diagram below shows two vertical poles, AD and BC. Point E lies on the same horizontal plane as bases D and C of poles AD and BC.

$A\hat{E}B = 2\alpha$; $B\hat{E}C = \alpha$; $A\hat{B}E = 90° - 3\alpha$ and $BC = y$ metres

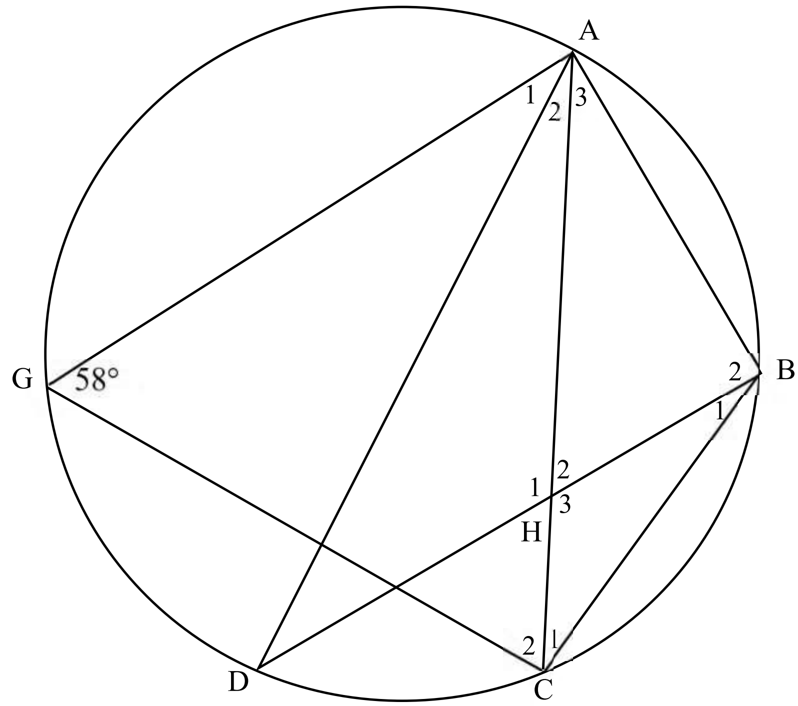


7.1	Determine BE in terms of α and y.	
		(2)

7.2	Show that AB = $2y$.	
		(5)
7.3	It is further given that $AF = \frac{7}{4}BC$. Determine $B\hat{A}F$, the angle of elevation of B from A. Give your answer to the nearest degree.	
		(3)
		[10]

QUESTION 8

In the diagram below, the circle passes through points A, B, C, D and G. AD is the diameter of the circle. BD and AC intersect at H and $A\hat{G}C = 58°$.



8.1 Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{B}_2$

8.1.2 $\hat{B}_1$

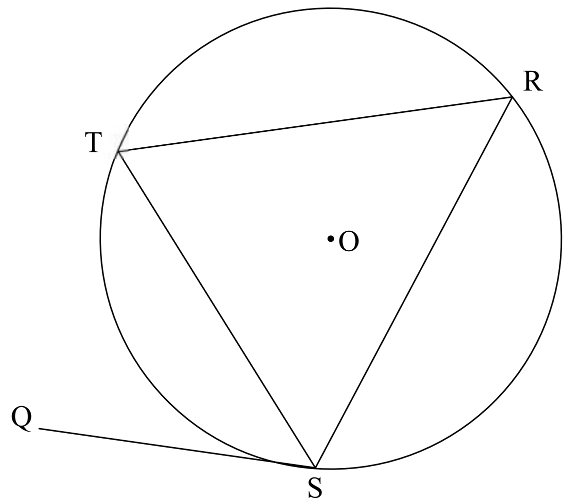
8.1.3 $\hat{A}_2$

8.1.1		
		(2)
8.1.2		
		(2)
8.1.3		
		(2)

8.2	If it is given that AB = BC. Prove that AB is a tangent to the circle passing through A, H and D.	
		(3)
		[9]

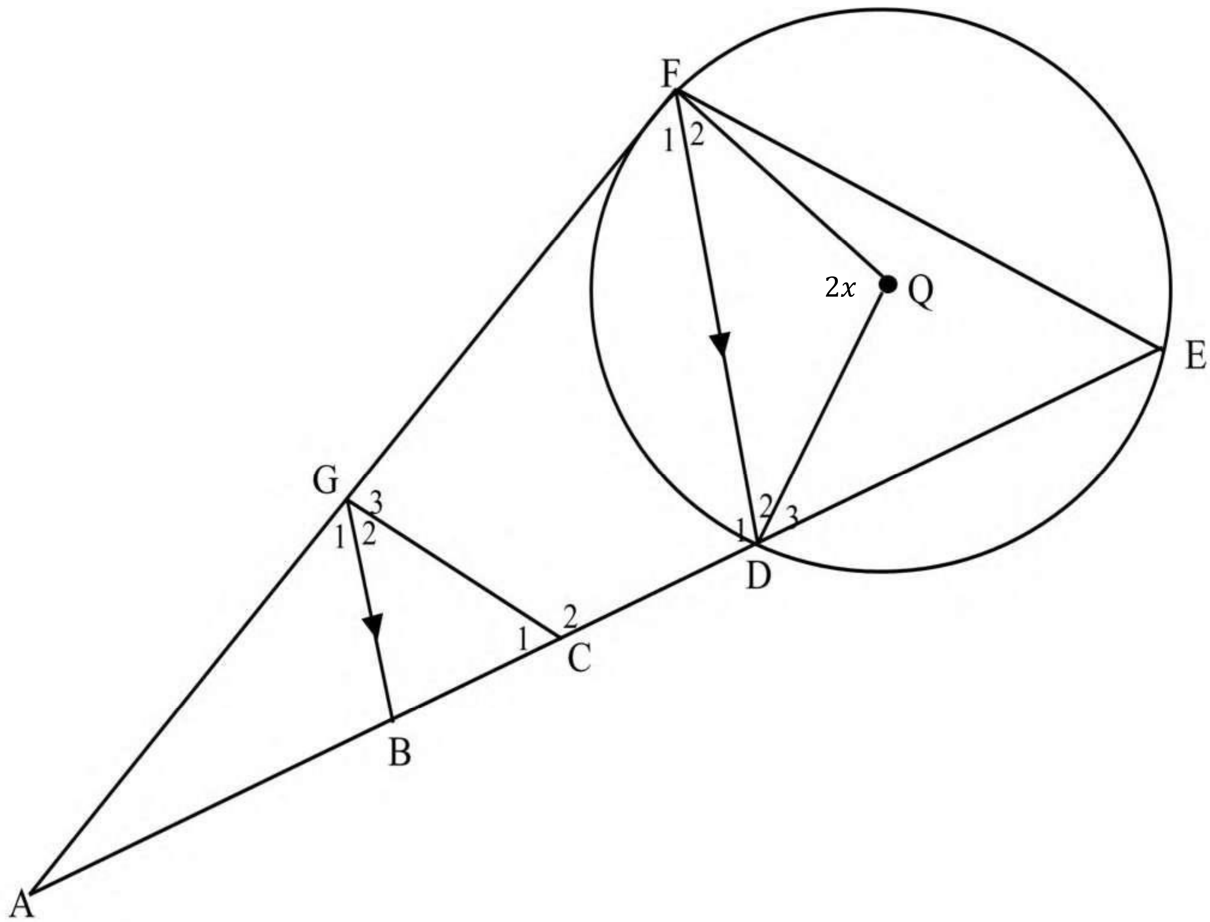
QUESTION 9

9.1 In the diagram below, O is the centre of the circle. Points S, T and R lie on the circle.
QS is a tangent to the circle at S.



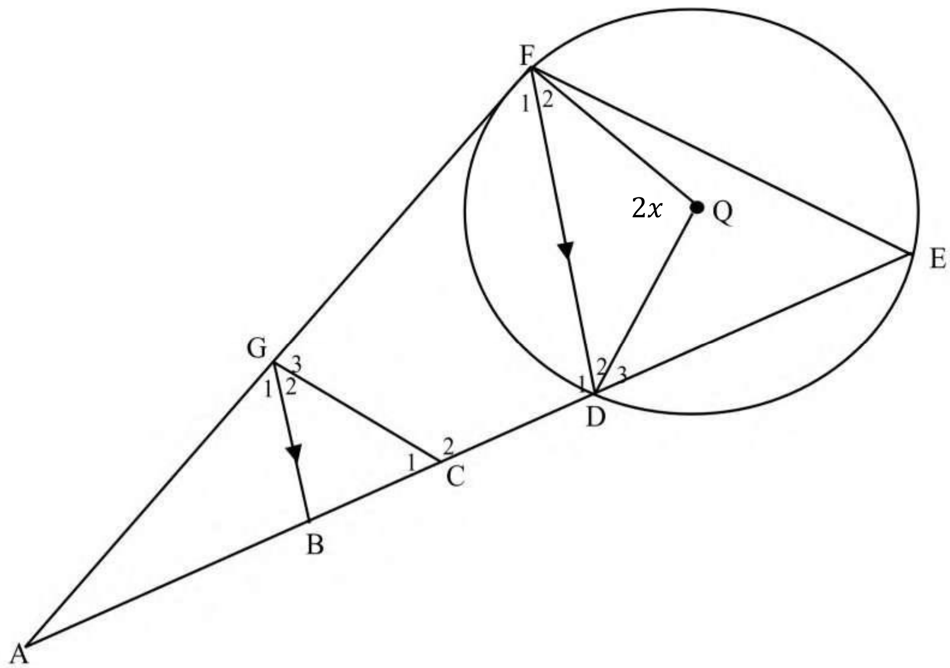
Use the diagram above to prove that the angle between the tangent and the chord is equal to the angle in the alternate segment, that is, prove that $Q\hat{S}T = \hat{R}$.	
	(6)

9.2 In the diagram below, D, E and F are points on the circle centred at Q. AGF is a tangent to the circle at F. ED is produced to meet the tangent at A. B and C are points on AE such that GB || FD. GC is joined. GCDF is a cyclic quadrilateral and $F\hat{Q}D = 2x$.



9.2.1	Give a reason why $\hat{E}$ is equal to x.	
		(1)

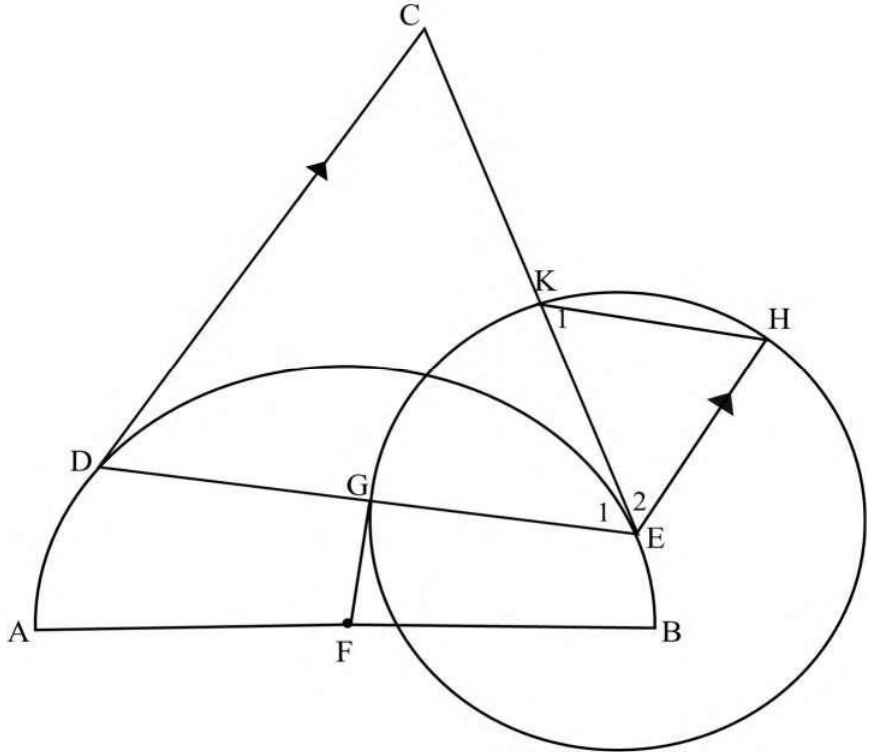
9.2.2	Prove that GC ∥ FE.	
		(3)
9.2.3	Prove that $\frac{AB}{BD} = \frac{AC}{CE}$.	
		(3)



9.2.4	It is further given that AB = 12 units, CE = 22,5 units and 5AG = 4GF. Determine the length of CD.	
		(5)
		[18]

QUESTION 10

In the diagram below, K, H and G are points on the circle and FG is a tangent to the circle with centre E. Semi-circle with centre F is drawn. Points D and E lie on the semi-circle. CD and CE are tangents to the semi-circle at D and E respectively. CD || HE.



10.1	Give a reason why DC = EC.	(1)
10.2	Prove that $\Delta\,DCE \,|||\, \Delta\,HEK$.	(6)

10.3	Prove that DG = GE.	
		(3)
10.4	Show that $2HE^2 = DC \times HK$.	
		(3)
		[13]

Additional space

Additional space

Additional space

TOTAL: 150

END

INFORMATION SHEET

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

InΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$