education
Department of Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

TIME: 3 HOURS

SEPTEMBER 2024

MARKS: 150

This question paper consists of 14 pages, 1 information sheet and an answer book of 22 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. Unless stated otherwise, you may use an approved scientific calculator (non-programmable and non-graphical).
6. If necessary, round off answers to TWO decimal places unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

A sample of residents were asked, how many litres of water they use per week in their household.

The results were summarised in the table below.

Number of litres used	Frequency
$50 \leq x < 100$	20
$100 \leq x < 150$	30
$150 \leq x < 200$	50
$200 \leq x < 250$	100
$250 \leq x < 300$	80
$300 \leq x < 350$	70
$350 \leq x < 400$	50

1.1 Complete the table in the answer book provided. (3)

1.2 Calculate the estimate of the mean litres of water used by each household per week. (2)

1.3 Draw an ogive of the above data on the grid provided in the answer book. (2)

1.4 Use the ogive to determine the median of the data set. (1)

1.5 Comment on the skewness of the data. Give a reason for your answer. (2)

1.6 The municipality placed a restriction on the usage of water due to water shortages. The residents may not use more than 300 litres of water per household per week. How will this affect the standard deviation? Explain your answer. (2)

[12]

QUESTION 2

The leg strength and the number of leg presses done per day by a random sample of ten, eighteen-year-old boys were recorded as shown in the table below.

Number of leg presses done per day (x)	36	136	51	126	90	43	77	68	103	124
Strength of upper leg (y)	0.2	0.85	0.35	0.91	0.73	0.34	0.61	0.59	0.78	0.90

2.1 Determine the equation of the regression line and write your answer in the form. y = ... (3)

2.2 Use the regression line to predict the leg strength of the eighteen-year-old boy if he does 110 leg-presses per day. (2)

2.3 An eighteen-year-old boy does 250 leg presses per day. Can your regression line formula predict the strength of his legs? (Explain you answer). (2)

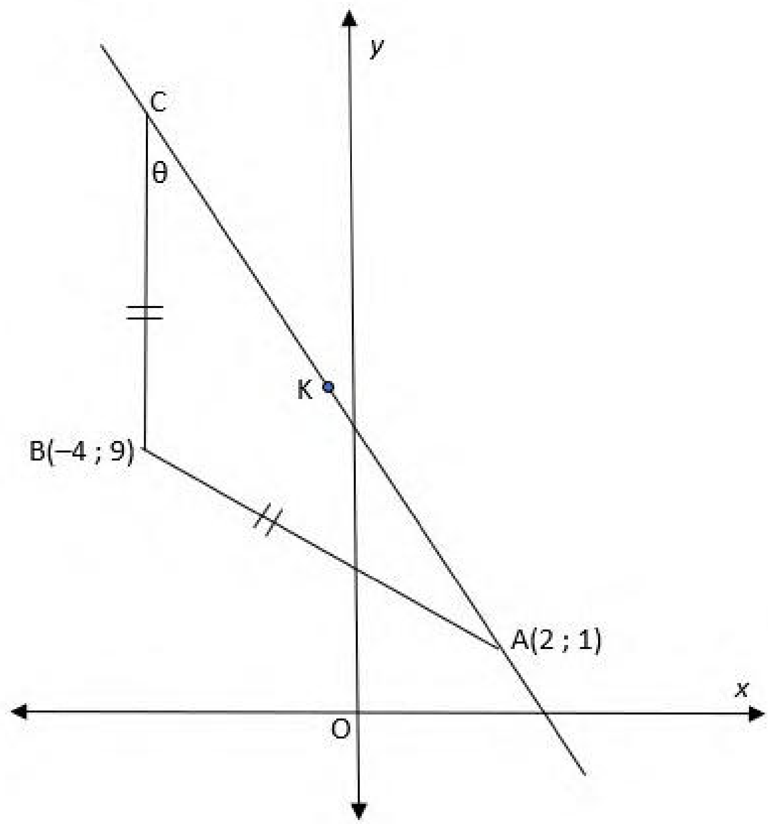
2.4 Calculate the correlation coefficient of the above data set. (2)

2.5 Use your answer in 2.4 to describe the relationship between the number of leg presses and the leg strength of an eighteen-year-old boy. (2)

[11]

QUESTION 3

In the diagram below, ABC is an isosceles triangle with A(2 ; 1) and B(–4 ; 9). AB = BC and BC is parallel to the y-axis.

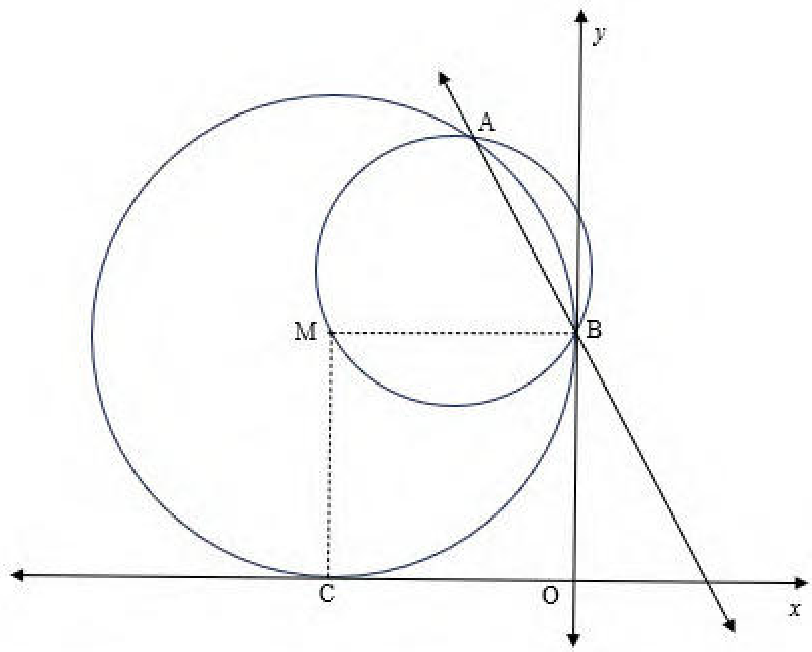


3.1 Calculate:

3.1.1	The length of AB.	(2)
3.1.2	The coordinates of C.	(2)
3.1.3	The coordinates of K, the midpoint of AC.	(2)
3.1.4	The equation of AC in the form $y = mx + c$.	(3)
3.1.5	The size of θ.	(3)
3.1.6	The area of triangle ABC.	(4)
3.1.7	The coordinates of D if ABCD is a rhombus.	(2)
		[18]

QUESTION 4

In the diagram, a circle centred at M touches the *x*-axis at C and the *y*-axis at point B. A second circle with equation $x^2 + y^2 + x - 3y + 2 = 0$ passes through A and M and intersects the circle M at A. The equation of the common chord AB is given by $y = -x + 1$.



4.1 Show that the equation of the circle centred at M, is $x^2 + y^2 + 2x - 2y + 1 = 0$ (5)

4.2 Determine the coordinates of the centre and the radius of the circle which passes through B, M and A. (4)

4.3 Calculate the coordinates of A. (5)

4.4 The straight line with equation $y = -x + k$ is a tangent to the circle with centre M.

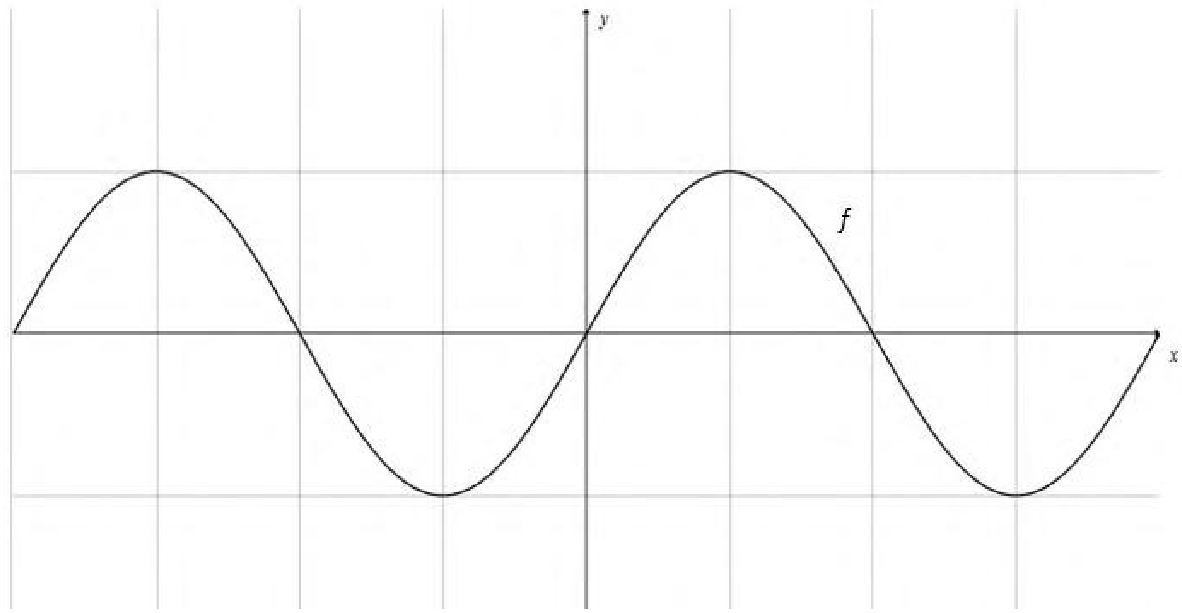
4.4.1 Show that this equation can be written as:
$2x^2 + (4 - 2k)x + (k^2 - 2k + 1) = 0$ (3)

4.4.2 Calculate the numerical value(s) of k. (5)

[22]

QUESTION 5

In the diagram below, the graph of $f(x) = \sin 2x$ is drawn for the interval $x \in [-180° ; 180°]$.



Use the graphs to answer the following questions.

5.1 Sketch the graph of $g(x) = \cos(x - 45°)$ on the same set of axes in the answer book provided. (3)

5.2 Determine the values of x in the interval $x \in [0°; 180°]$ for which.

5.2.1 $f(x) = g(x)$ (7)

5.2.2 $f(x + 30°) = g(x + 30°)$ (2)

5.2.3 $f(x) > g(x)$ (2)

5.3 Write down the period of g. (1)

5.4 Show how the graphs of f and g can be used to solve:
$\sqrt{2}\sin 2x = \cos x + \sin x$. (3)

[18]

QUESTION 6

6.1 If $\sqrt{5}\sin\theta + 2 = 0$ and $\theta \in [90°; 270°]$.

Determine without the use of a calculator the value of the following:

6.1.1 $\tan\theta$ (2)

6.1.2 $\cos 2\theta$ (2)

6.2 Simplify the following expression to ONE trigonometric ratio without the use of a calculator:

$$2\cos^2 15° - 1 + \frac{2\sin 140°}{\cos 310°}$$ (5)

6.3 If $\sin\frac{x}{2} = p$, express $\sin x - 1$ in terms of p. (4)

6.4 Prove the following:

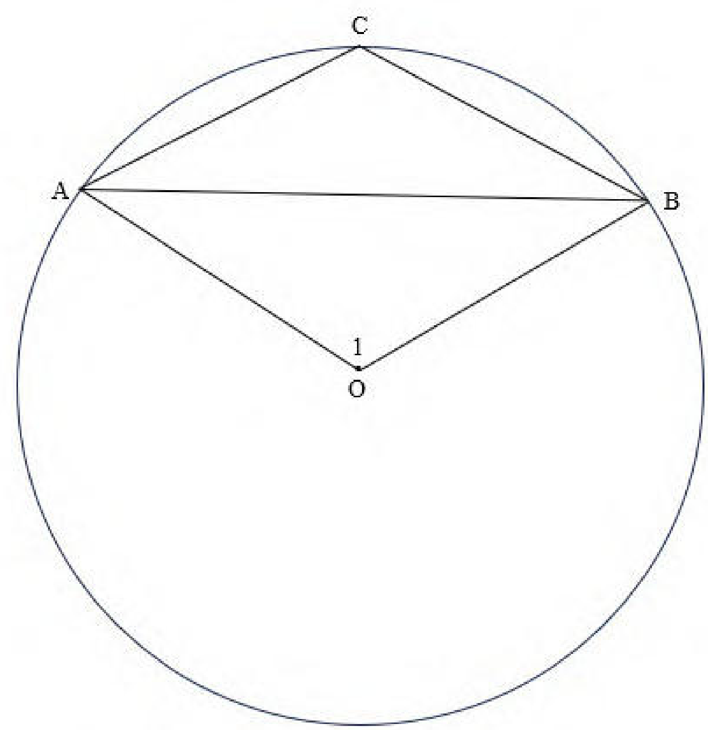
$$\frac{3\sin x + 2\sin 2x}{2 + 3\cos x + 2\cos 2x} = \tan x$$ (5)

6.5 Show that: $\frac{\sin x + \cos x}{\sin x - \cos x} = \frac{p+t}{p-t}$ if $\tan x = \frac{p}{t}$ (5)

[23]

QUESTION 7

In the diagram below, O is the centre of the circle with radius equal to r units.
AC = CB = t units and $A\hat{O}B = 2\theta$.



7.1 Determine AB in terms of r and θ (3)

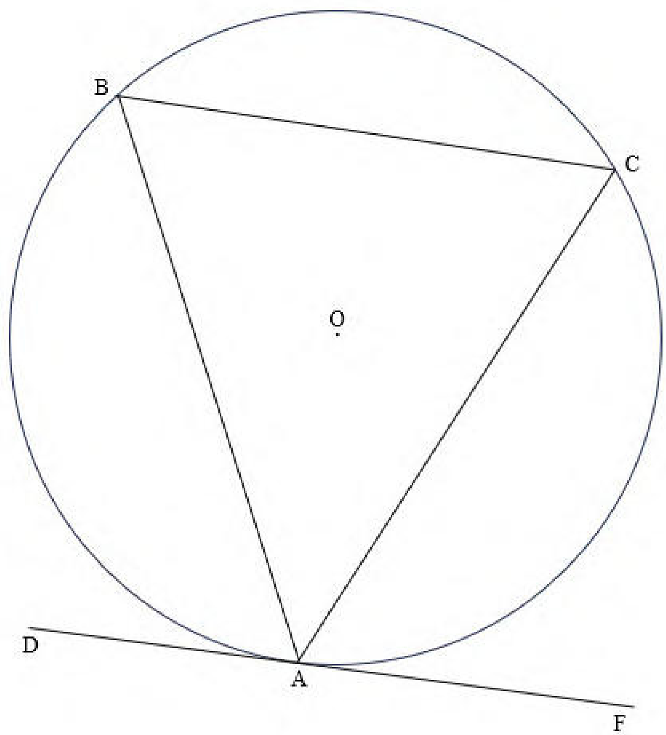
7.2 Show that $\frac{r}{t} = \sqrt{2(1 - \cos\theta}$ (4)

7.3 If $\theta = 30°$, determine the area of ΔAOB, in terms of r (2)
[9]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

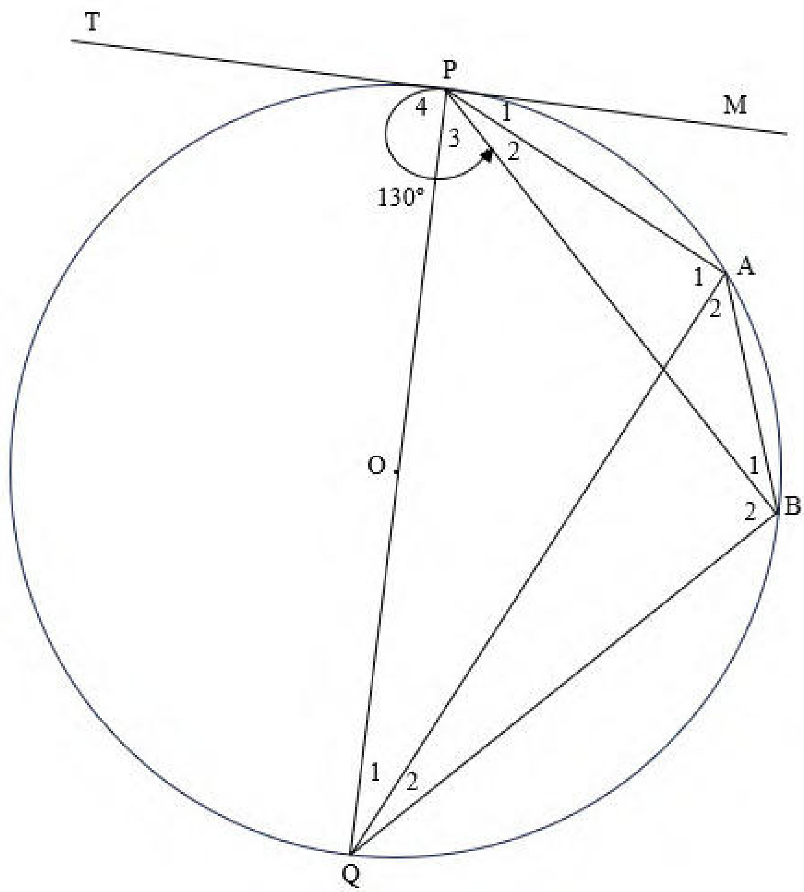
QUESTION 8

8.1 In the diagram, chords AB, BC and CA are drawn in the circle with centre O. DAF is a tangent to the circle at A,



Prove the theorem which states that $C\hat{A}F = A\hat{B}C$ (5)

8.2. In the diagram below, TPM is a tangent to circle with centre O and $T\hat{P}B = 130°$.
POQ is a diameter and ABQP is a cyclic quadrilateral.



Determine with reasons the size of the following.

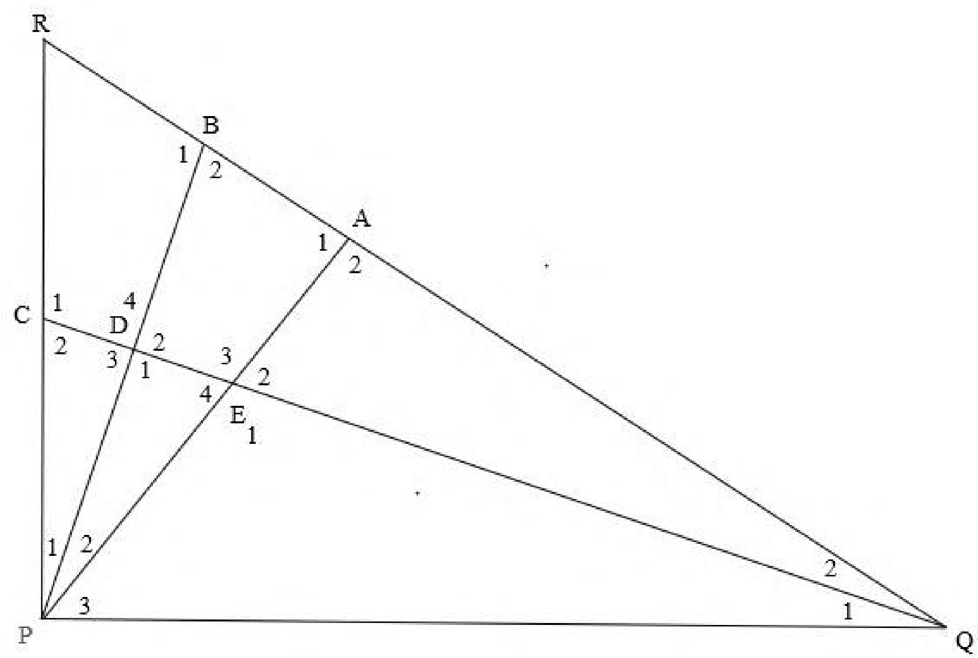
8.2.1 $Q\hat{P}B$ (3)

8.2.2 $P\hat{Q}B$ (3)

8.2.3 $P\hat{A}B$ (2)
[13]

QUESTION 9

9.1 In the diagram below, triangle QRP is drawn with $RP \perp QP$ and $PA \perp QR$. QC intersects PA and PB at E and D respectively. $Q_1 = Q_2$ and $P_1 = P_2$

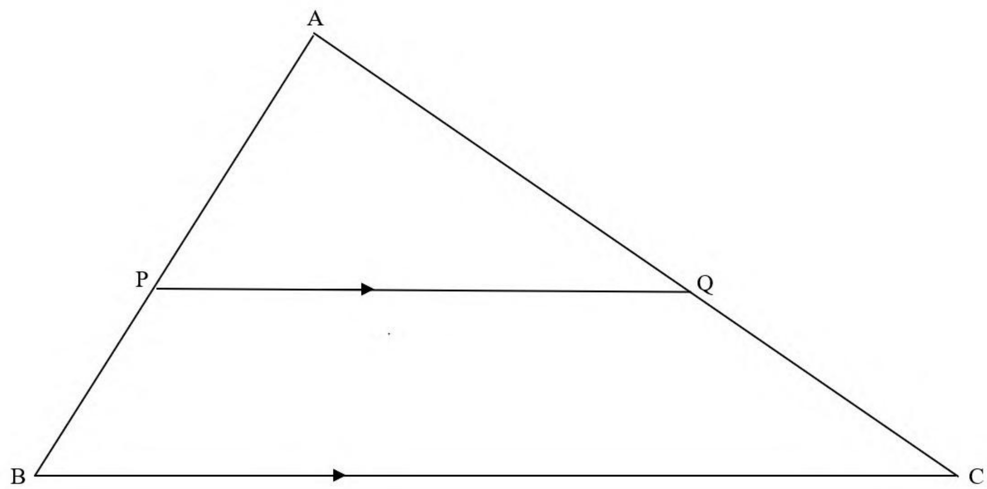


Prove:

9.1.1 BP is a tangent to circle PQE. (6)

9.1.2 ADPQ is a cyclic quadrilateral (3)

9.2 In the diagram ΔABC is drawn with PQ ∥ BC. PQ : BC = 4 : 7

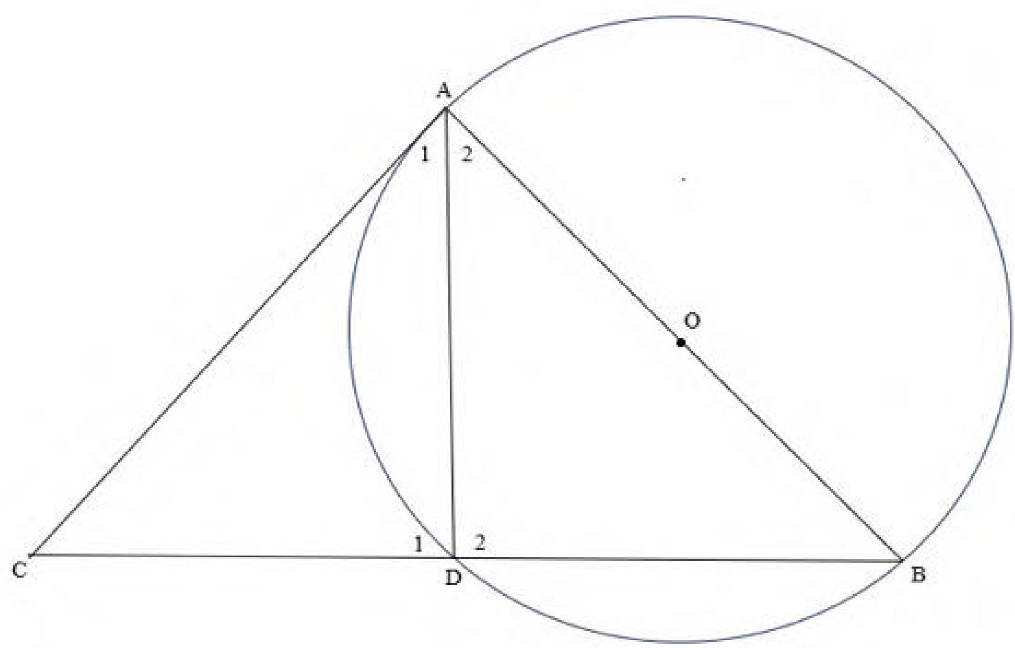


Prove $\Delta APQ = \frac{16}{49}\Delta ABC$ (5)

[14]

QUESTION 10

In the diagram below, CA is a tangent to the circle O with diameter AOB. Points A, D and B are points on the circumference of the circle. BD is produced to C and AD intersects CB at D. It is further given that CD : DB = 2 : 3.



Prove

10.1 $\text{AD}^2 = 6x^2$ (6)

10.2 $\sqrt{\text{AD}^2 + \text{AC}^2 + \text{AB}^2} = x\sqrt{31}$ (4)

[10]

TOTAL: 150

INFORMATION SHEET:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}; -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{n}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$area\ \Delta ABC = \frac{1}{2} ab . \sin C$$

$\sin(\alpha + \beta) = \sin\alpha . \cos\beta + \cos\alpha . \sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha . \cos\beta - \cos\alpha . \sin\beta$

$\cos(\alpha + \beta) = \cos\alpha . \cos\beta - \sin\alpha . \sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha . \cos\beta + \sin\alpha . \sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha . \cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$