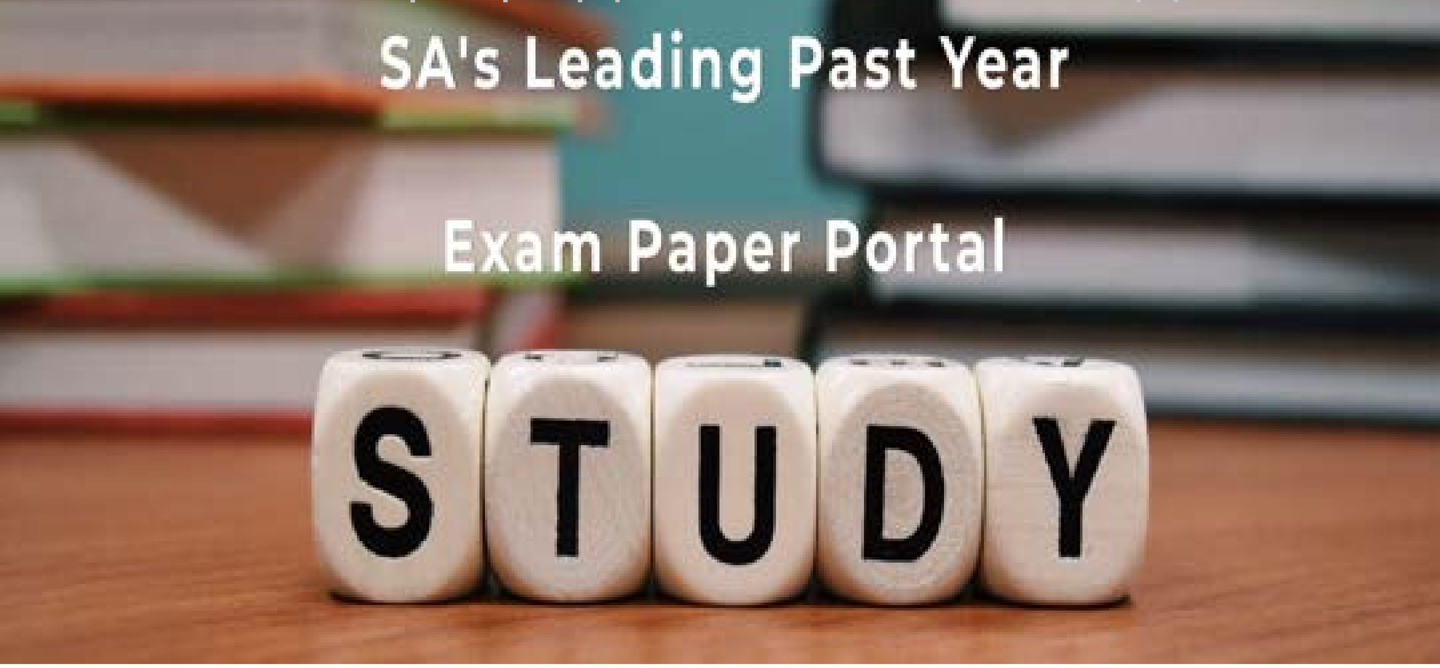

SA's Leading Past Year
Exam Paper Portal
STUDY

Confidential

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P2

MAY/JUNE 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 16 pages and a 2-page information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise

6. Diagrams are NOT necessarily drawn to scale.

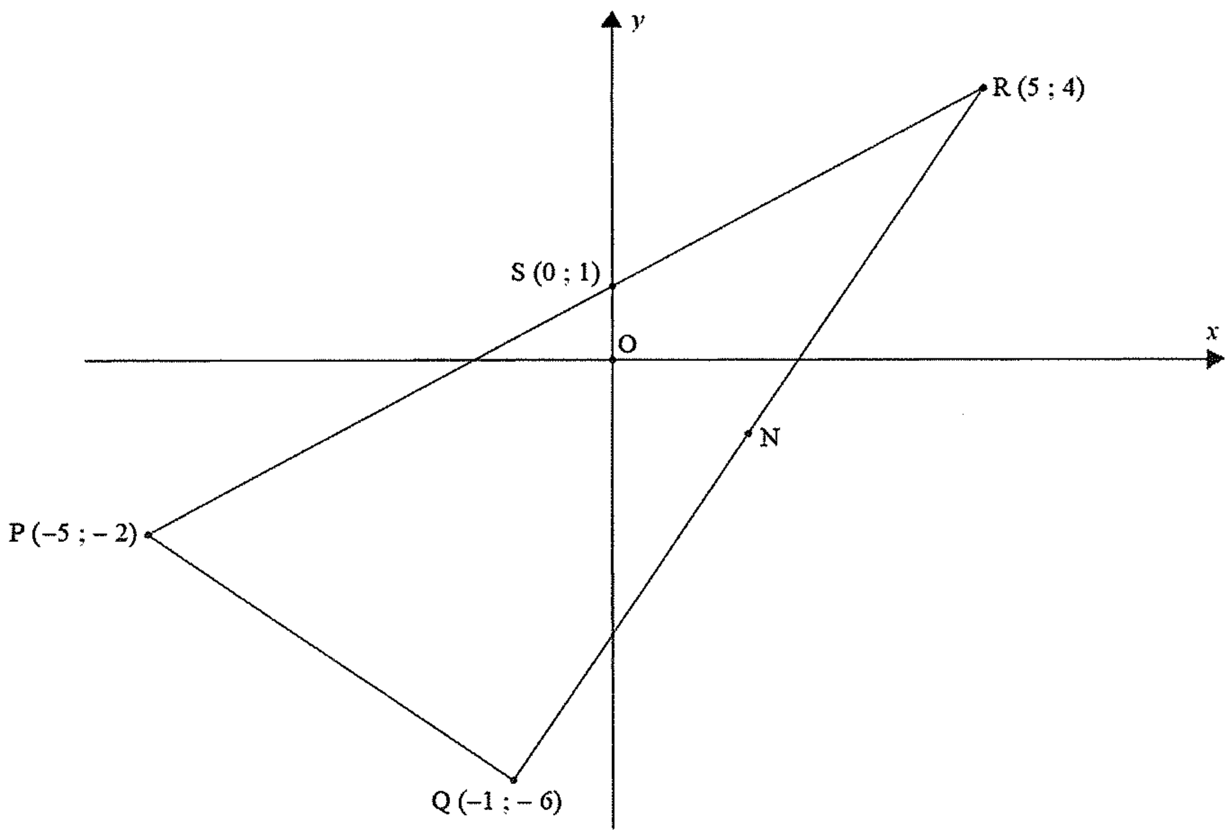
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

In the diagram below, PQR is a triangle with vertices $P(-5\ ;\ -2)$, $Q(-1\ ;\ -6)$ and $R(5\ ;\ 4)$.
N is the midpoint of RQ.
$S(0\ ;\ 1)$ is a point on the y-axis.



1.1 Determine:

1.1.1 The gradient of PQ (2)

1.1.2 The coordinates of N (2)

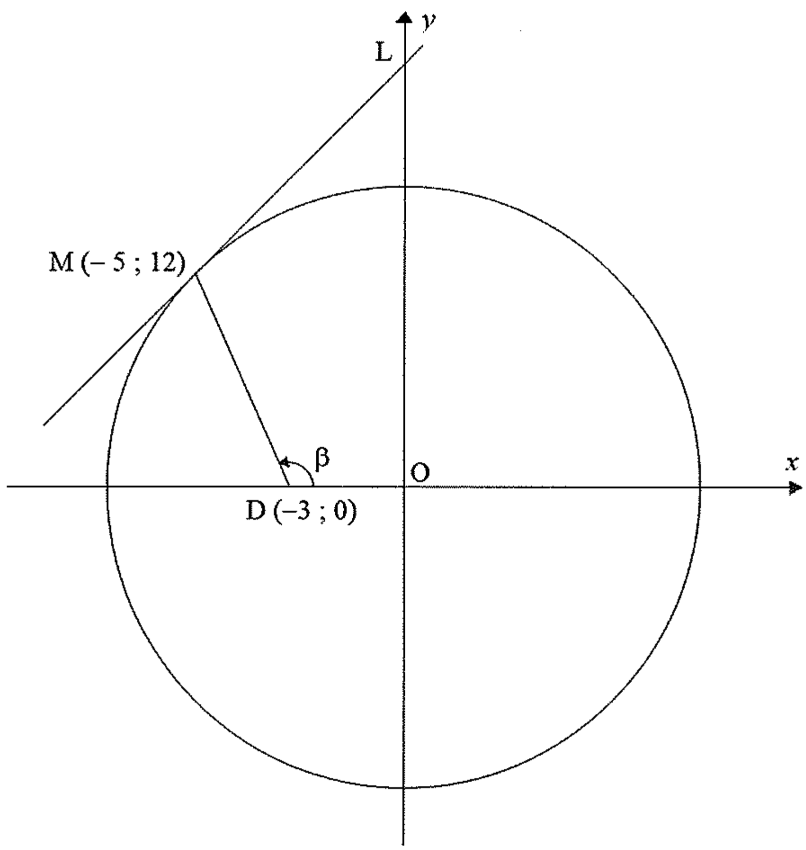
1.1.3 The equation of the line parallel to PQ passing through R (3)

1.2 Show, using analytical methods, that $\frac{PQ}{SN} = 2$ (4)

[11]

QUESTION 2

2.1 In the diagram below, O is the centre of the circle at the origin.
ML is a tangent to the circle at point M $(-5\ ;\ 12)$ and L is the y-intercept of ML.
D $(-3\ ;\ 0)$ is a point on the x-axis.
β is the angle of inclination of MD.



2.1.1 Determine the equation of the circle. (2)

2.1.2 Determine the equation of tangent ML in the form $y = \ldots$ (4)

2.1.3 Write down the coordinates of point L. (2)

2.1.4 Determine the size of β. (4)

2.2 Draw, on the grid provided in the ANSWER BOOK, the graph defined by:

$$\frac{x^2}{(3)^2} + \frac{y^2}{36} = 1$$

Clearly show ALL the intercepts with the axes. (3)

[15]

QUESTION 3

3.1 Given: $\hat{P} = 119°$ and $\hat{Q} = 61°$

Determine:

3.1.1 $\text{cosec P} \times \tan \text{Q}$ (3)

3.1.2 $\cos^2(\text{P} + 2\text{Q})$ (2)

3.2 Given: $\frac{1}{2}\tan\theta = 2$, where $\theta \in [0° ; 90°]$

Show, **without the use of a calculator**, that $\sin^2\theta + \cos^2\theta = 1$ (6)

3.3 Solve for x:

$\sin x = \tan 318°$, where $x \in [0° ; 360°]$ (4)

[15]

QUESTION 4

4.1 Given: $\dfrac{\tan(\pi + A) \cdot \cos(180° - A) \cdot \sin(360° - A)}{\sin(2\pi + A)}$

4.1.1 Simplify by reduction: $\tan(\pi + A)$ (1)

4.1.2 Simplify: $\dfrac{\tan(\pi + A) \cdot \cos(180° - A) \cdot \sin(360° - A)}{\sin(2\pi + A)}$ (5)

4.2 Complete the identity: $\cot^2 x - \text{cosec}^2 x =$ (1)

4.3 Prove the identity: $\sin x + \cos^2 x \cdot \text{cosec}\, x = \text{cosec}\, x$ (3)

[10]

QUESTION 5

Given the functions defined by $f(x) = \cos(x - 45°)$ and $g(x) = -2\sin x$, where $x \in [0° ; 360°]$

5.1 Draw sketch graphs of f and g on the same set of axes provided in the ANSWER BOOK.

Clearly indicate ALL turning points, intercepts with the axes and end-points. (7)

5.2 Write down the value of x for which g is a minimum. (1)

5.3 Write down the period of g (1)

5.4 Use the letters **A** and **B** to indicate on the graphs where $-\frac{1}{2}\cos(x - 45°) = \sin x$ (3)

5.5 Use the graphs drawn in QUESTION 5.1 to determine the values of x for which $f'(x) < 0$ (2)

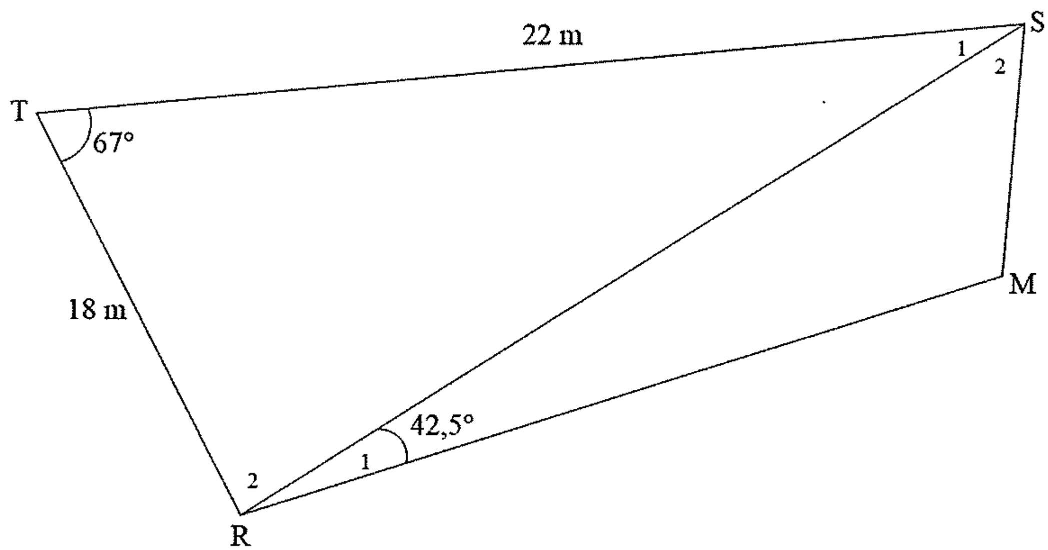
[14]

QUESTION 6

The diagram below shows a cyclic quadrilateral TSMR.

TS = 22 m and TR = 18 m

$\hat{T} = 67°$ and $\hat{R}_1 = 42{,}5°$



6.1 Determine:

6.1.1 The length of SR (3)

6.1.2 The size of $\hat{M}$ (1)

6.2 Answer the following questions with regard to ΔSMR.

6.2.1 Complete the sine rule: $\dfrac{SM}{\sin \hat{R}_1} = \dfrac{SR}{\ldots}$ (1)

6.2.2 Hence, determine the length of SM. (2)

6.3 The area of ΔSMR must be fertilised. One bag of fertiliser covers 15,178 square metres.

Determine the number of bags of fertiliser needed to cover the area of ΔSMR. (5)
[12]

Give reasons for your statements in QUESTIONS 7, 8 and 9.

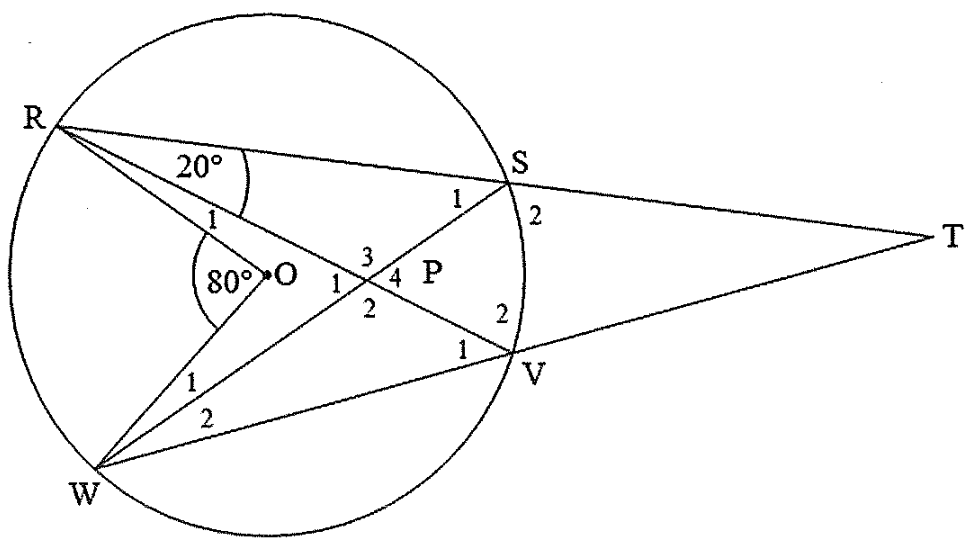
QUESTION 7

In the diagram below, O is the centre of the circle. R, S, V and W are points on the circle.

RS and WV are produced to meet at T.

RV and WS intersect at P.

$S\hat{R}V = 20°$ and $R\hat{O}W = 80°$



7.1 Determine, with reasons, the size of each of the following angles:

7.1.1 $\hat{V}_1$ (2)

7.1.2 $\hat{T}$ (2)

7.2 Show, with reasons, that SPVT is NOT a cyclic quadrilateral. (3)
[7]

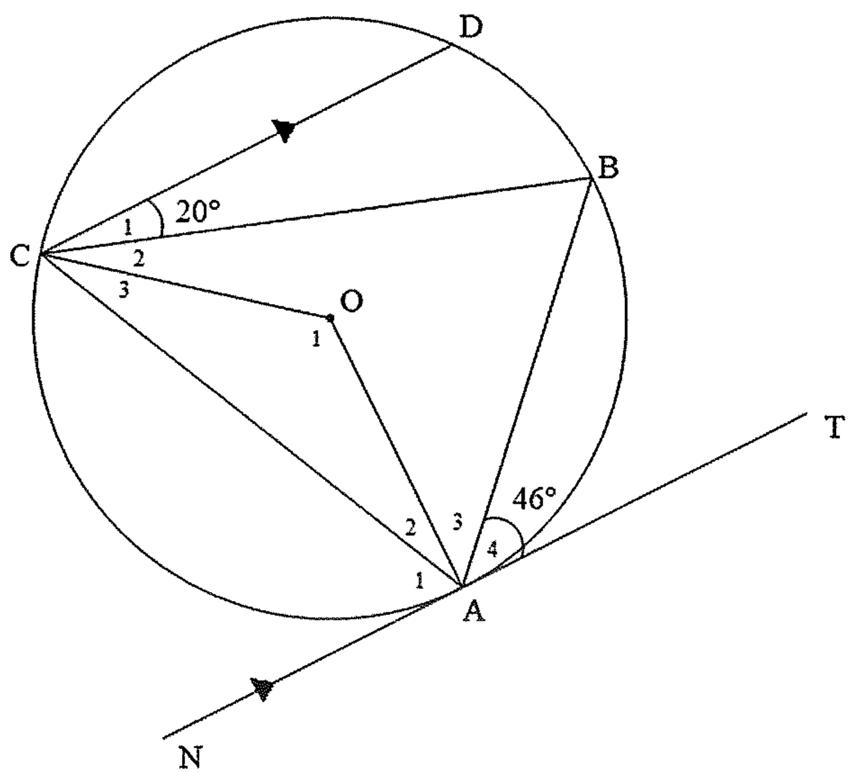
QUESTION 8

8.1 In the diagram below, O is the centre of the circle.

TN is a tangent to the circle at A.

$\hat{A}_4 = 46°$

CD || TN and $\hat{C}_1 = 20°$



Determine, with reasons, the size of each of the following angles:

8.1.1 $B\hat{C}A$ (2)

8.1.2 $\hat{A}_3$ (3)

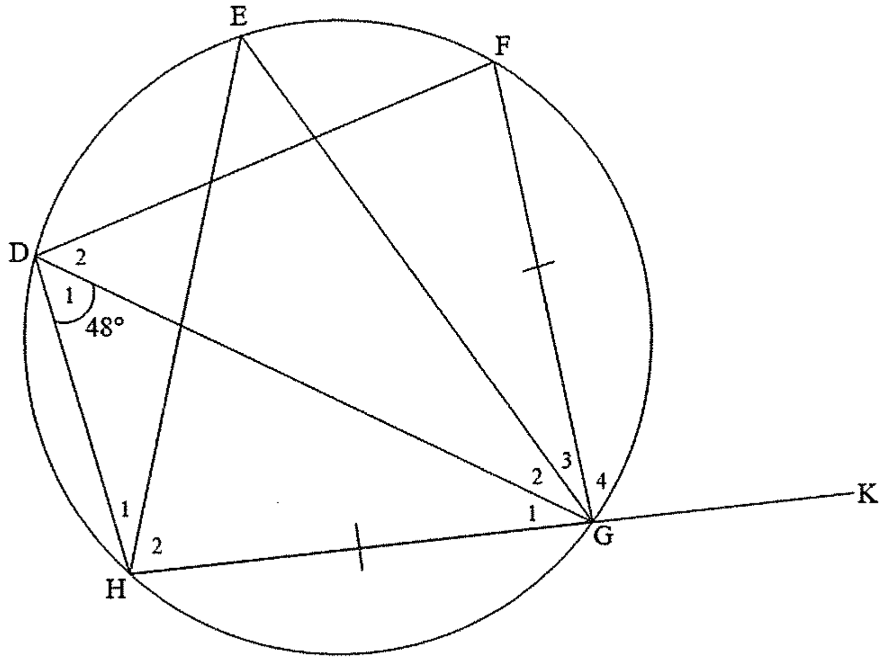
8.1.3 $\hat{A}_1$ (2)

8.1.4 $\hat{O}_1$ (3)

8.2 In the diagram below, D, H, G, F and E are points on the circle.

HG is extended to K.

$\hat{D}_1 = 48°$ and HG = FG.



Determine, with reasons, the size of each of the following angles:

8.2.1 $\hat{E}$ (2)

8.2.2 $\hat{D}_2$ (2)

8.2.3 $\hat{G}_4$ (2)
[16]

QUESTION 9

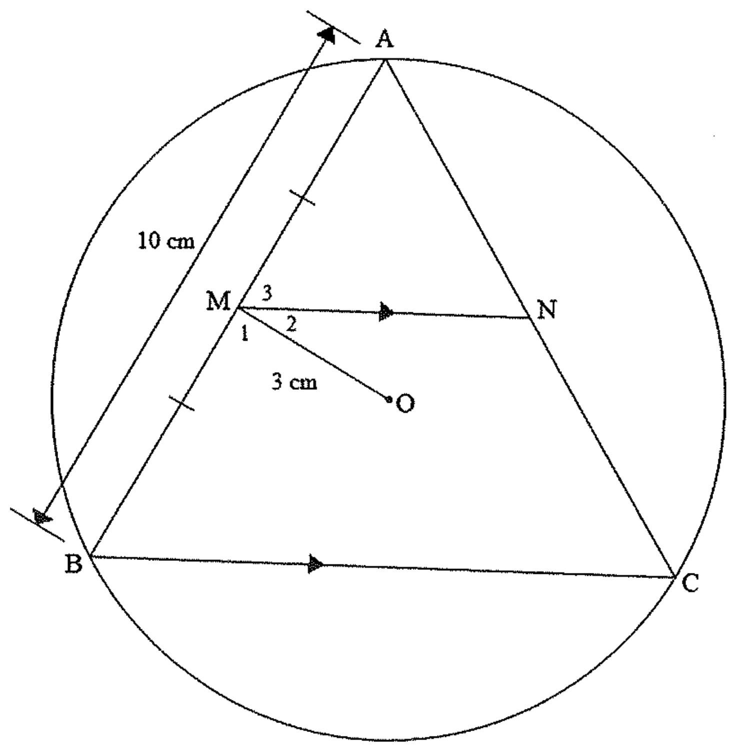
9.1 In the diagram below, O is the centre of the circle.

OM = 3 cm and bisects AB.

A, B and C are points on the circle such that AB = 10 cm.

MN || BC.

M is the midpoint of AB.



9.1.1 (a) Write down, with a reason, the size of $\hat{M}_1$ (2)

(b) Determine the length of the radius of the circle. (3)

9.1.2 If MN = 5,12 cm, write down, with a reason, the length of BC. (2)

9.2 In the diagram below, ABC is a right-angled triangle.

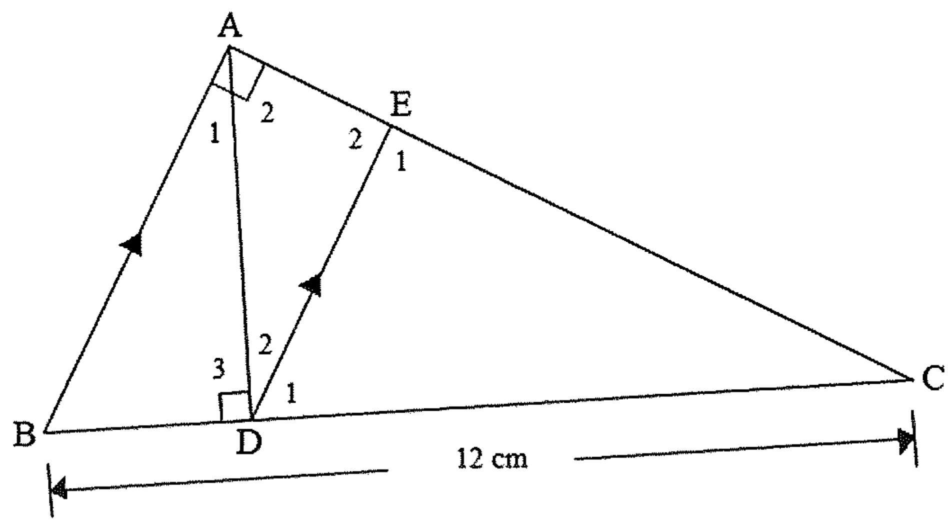
$\hat{A} = 90°$

$AD \perp BC$

$AB \parallel ED$

$CE : EA = 2 : 1$

$BC = 12$ cm



9.2.1 Prove that $\Delta ADC \parallel\!\mid \Delta BAC$ (3)

9.2.2 Show that $AC^2 = DC \times BC$ (1)

9.2.3 (a) Complete the statement and reason:

$$\frac{DC}{BC} = \frac{CE}{...} \qquad (...)$$ (2)

(b) Determine the length of DC. (2)

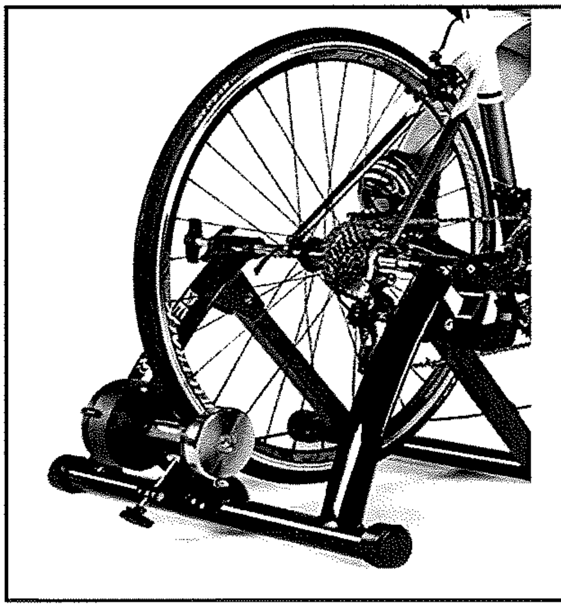
(c) Hence, determine the length of AC. (2)

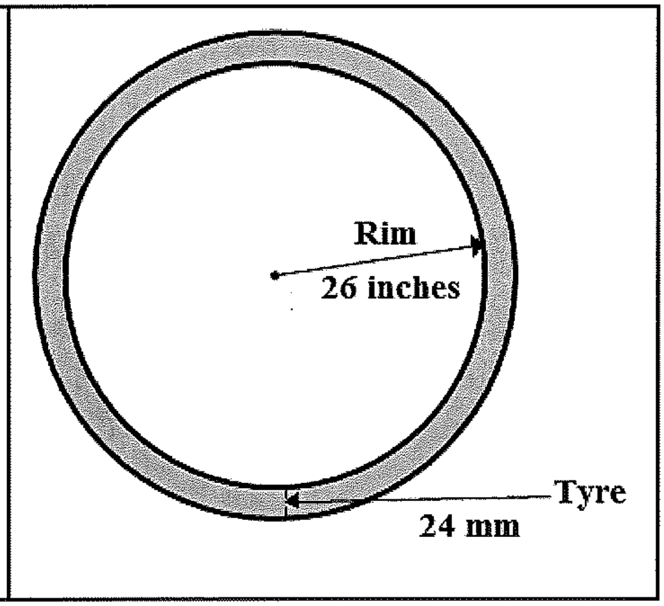
[17]

QUESTION 10

10.1 The picture and diagram below show the rear wheel of a training bicycle.

The rim has a radius of 26 inches and the thickness of the tyre is 24 mm.





10.1.1 Convert 26 inches to metres if 1 inch = 0,0254 metres. (1)

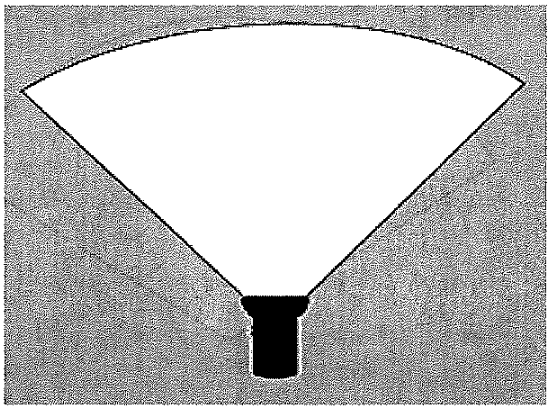
10.1.2 Calculate, in metres, the diameter of the wheel which includes the tyre. (2)

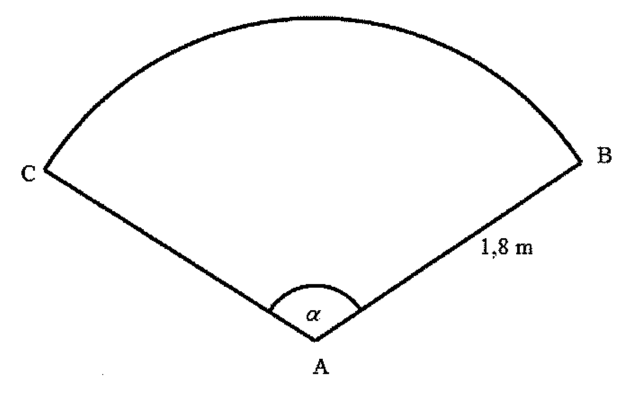
10.1.3 If the circumferential velocity of a particle on the outer edge of the wheel is 60 km/h, determine the rotational frequency of the wheel in revolutions per second. (4)

10.2 The picture and diagram below show the area covered by a beam of light cast by a torch lying on the floor.

The light beam covers a distance of 1,8 metres and the beam angle is α.

The floor area covered by the beam of light is $2,5 \text{ m}^2$.



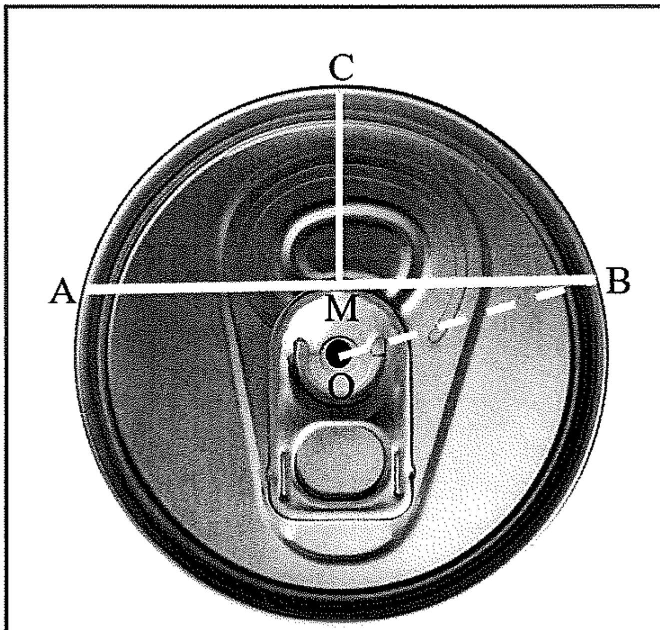


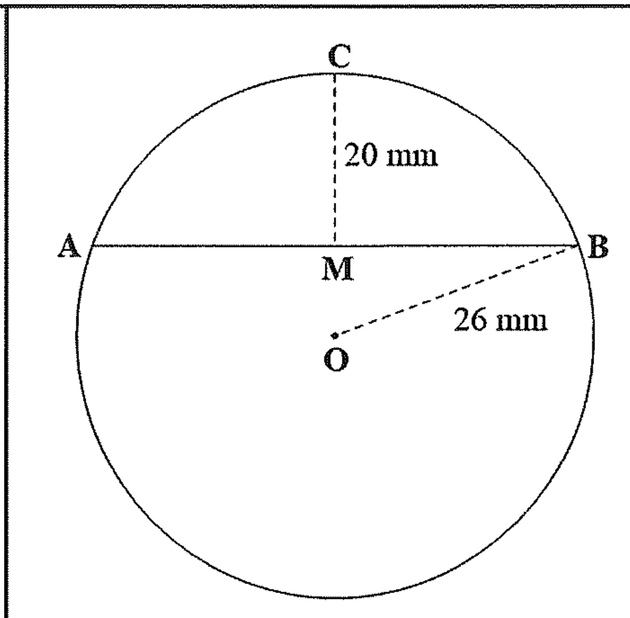
Show that α (in degrees) is an acute angle. Justify your answer. (5)

10.3 The picture and the diagram below show the circular top of a can with centre O.

The radius OB = 26 mm

AB is a chord and the height, MC, of the smaller segment, is 20 mm.





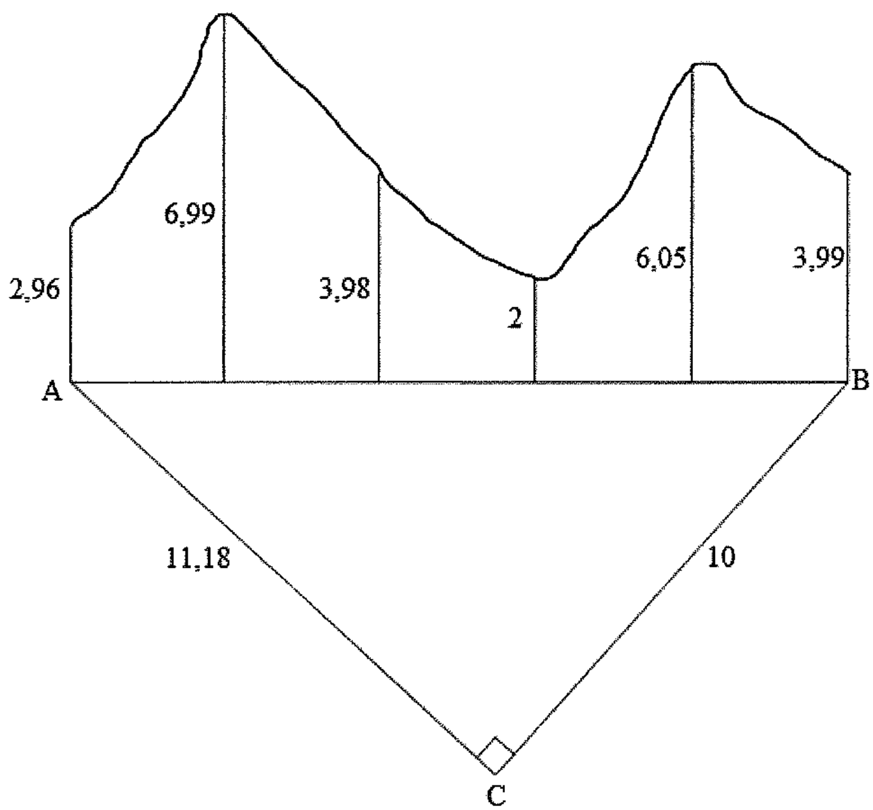
Determine the length of chord AB. (4)
[16]

QUESTION 11

11.1 The picture of an irregular figure with straight side AB is shown below.

The ordinates of this figure are 2,96 cm, 6,99 cm, 3,98 cm, 2 cm, 6,05 cm and 3,99 cm.

Right-angled ΔABC with AC = 11,18 cm and BC = 10 cm is not part of the irregular figure.



11.1.1 Calculate the length of AB to the nearest integer. (2)

11.1.2 If AB is divided into five equal parts, as shown in the diagram, determine the width of each of the equal parts. (1)

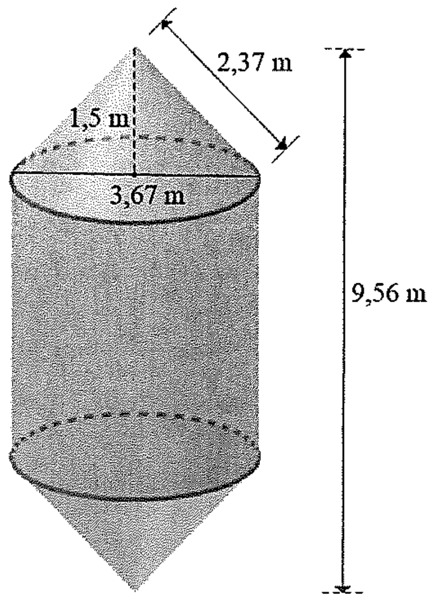
11.1.3 Hence, determine, by using the mid-ordinate rule, the area of the irregular figure. (3)

11.2 The diagram below represents a cylindrical design of a storage container, with identical cones on both ends.

The diameter of the cylindrical part is 3,67 metres and the total height is 9,56 metres.

The height of each cone is 1,5 metres.

The slant height of each cone is 2,37 metres.



The following formulae may be used:

Volume of a cylinder $= \pi r^2 h$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Total surface area of a cylinder $= 2\pi r^2 + 2\pi r h$

Total surface area of a cone $= \pi r^2 + \pi r \ell$, **where** ℓ **is the slant height of a cone.**

11.2.1 Determine:

(a) The length of the radius of the cone (1)

(b) The height of the cylinder (1)

11.2.2 Calculate the volume of the container. (3)

11.2.3 Determine whether 100 m^2 of material is sufficient to manufacture the container. (6)

[17]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad x = -\dfrac{b}{2a}$ $\quad y = \dfrac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1$ and $b > 0$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 + i)^n$ $\quad A = P(1 - i)^n$

$i_{eff} = \left(1 + \dfrac{i}{m}\right)^m - 1$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C, \ n \neq -1$ $\quad \int k x^n \, dx = k \cdot \dfrac{x^{n+1}}{n+1} + C, \ n \neq -1, \ k \neq 0$

$\int \dfrac{1}{x} dx = \ln x + C, \ x > 0$ $\quad \int \dfrac{k}{x} dx = k \cdot \ln x + C, \ x > 0, \ k \neq 0$

$\int a^x \, dx = \dfrac{a^x}{\ln a} + C, \ a > 0$ $\quad \int k a^{nx} \, dx = k \cdot \dfrac{a^{nx}}{n \ln a} + C, \ a > 0, \ k \neq 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\dfrac{x_2 + x_1}{2} ; \dfrac{y_2 + y_1}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\quad \tan\theta = m$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

area of $\Delta ABC = \dfrac{1}{2} ab \cdot \sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

π rad $= 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where r = radius and ω = angular velocity

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius and s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
$O_n = n^{th}$ ordinate and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $O_n = n^{th}$ ordinate and n = number of ordinates