KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS PRE PREPARATORY PAPER 2

August 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, a diagram sheet and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 10 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   Write neatly and legibly.

QUESTION 1

A group of four-year-old children were given the same puzzle to complete. The time taken (in minutes) for each child to complete the puzzle was recorded. The results recorded are shown in the table below.

TIME TAKEN (t) (IN MUNUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

1.1	How many children completed the puzzle?	(1)
1.2	Calculate the estimated mean time taken to complete the puzzle.	(2)
1.3	Complete the cumulative frequency column in the table given in the diagram sheet	(2)
1.4	Draw a cumulative frequency graph (ogive) to represent the data on the grid provided	(3)
1.5	Use the graph to determine the median time taken to complete the puzzle.	(2)

[10]

QUESTION 2

Learners who scored a mark below 50% in Mathematics test were selected to use a computer based programme as a part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as percentage) scored by 15 of these learners in both tests is given in the table below.

Learner	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
Test 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
Test 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1	Determine the equation of the least squares regression line.	(3)
2.2	A learner's mark in the first test was 15 out of a maximum of 50 marks.	
	2.2.1 Write down the learner's mark for this test as a percentage.	(1)
	2.2.2 Predict The learners mark for the second test. Give your answer to the nearest integer	(2)
2.3	For the 15 learners above, the mean mark is 45,67% and the standard deviation is 13,88. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. When the marks of the last question are added, the new mean mark is 50.67%.	
	2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total?	(2)
	2.3.2 What is the total mark of the last question	(2)

[10]

Please turn over

QUESTION 3

In the diagram, $A(-3\ ;\ 4)$, B and C are vertices of $\triangle ABC$. AB is produced to T. D and E are the $x-$ and y-intercepts of AC respectively. E is the midpoint of AC and the angle of inclination of AC is α. The equation of AB is $y = kx + 3$ and the equation of AC is $y = -2x - 2$.
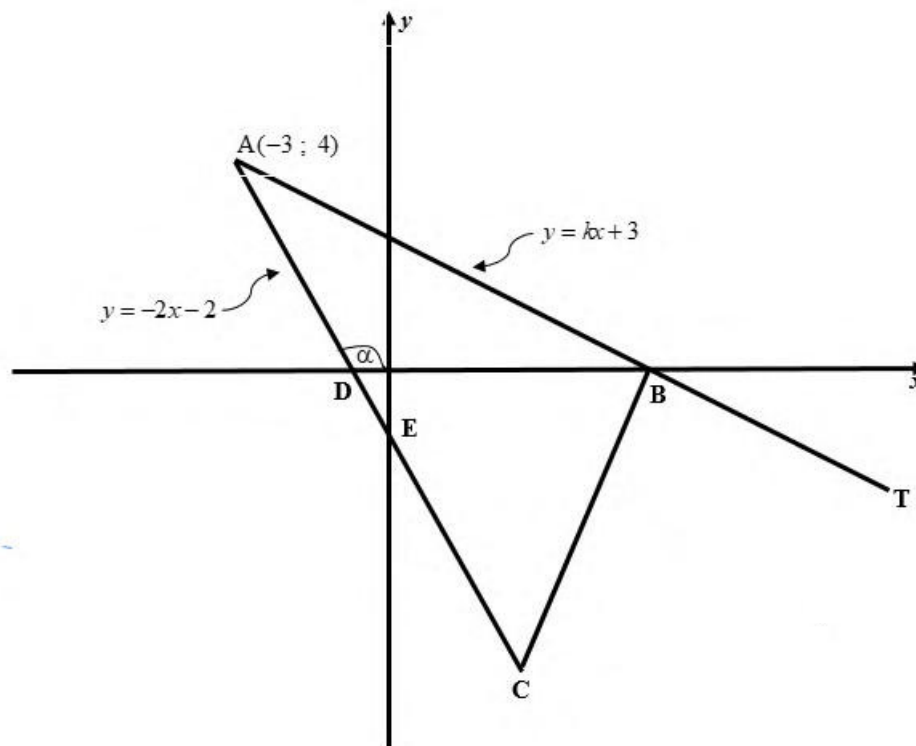

3.1	Show that $k = -\dfrac{1}{3}$.	(1)
3.2	Calculate the coordinates of B, the x-intercept of line AT.	(2)
3.3	Calculate the coordinates of C.	(4)
3.4	Determine the equation of the line parallel to BC and passing through $T(15\ ;-2)$. Write your answer in the form $y = mx + c$.	(3)
3.5	Calculate the size of $B\hat{A}C$.	(5)
3.6	It is further given that the length of AC is $8\sqrt{10}$ units, calculate the value of $\dfrac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ATC}$.	(5)

[20]

QUESTION 4

In the diagram below, the circle centred at N(2 ; 3) passes through A($-1 : -1$) and C. BA and BC are tangents to the circle at A and C respectively, with BC parallel to the y
-axis.
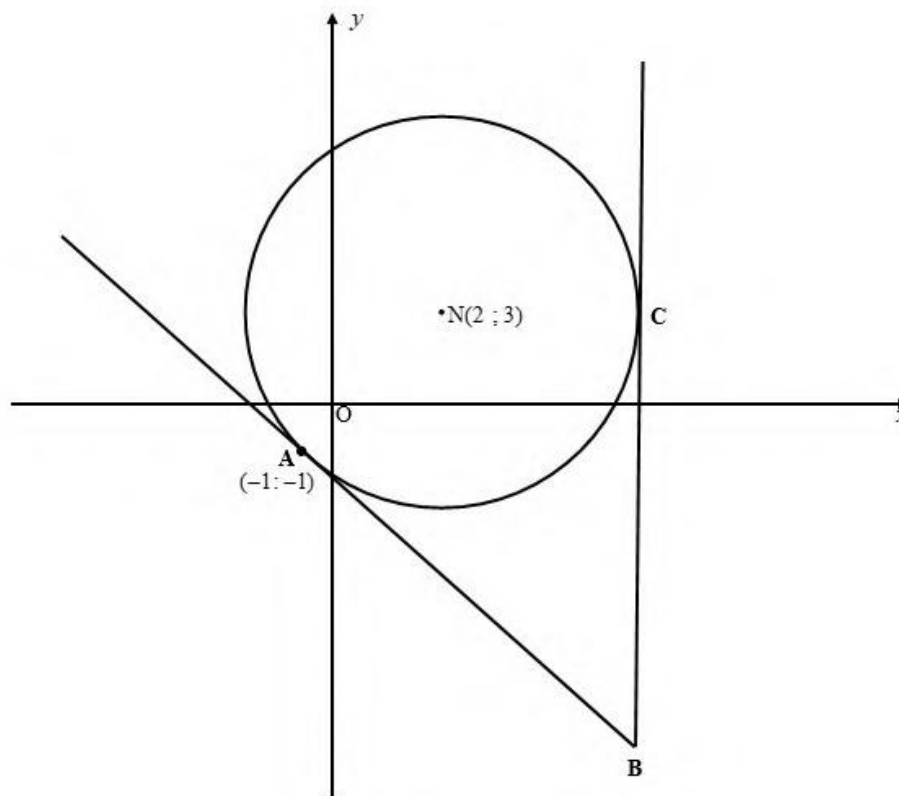
4.1   Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.   (3)
4.2   Write down the coordinates of C.   (2)
4.3   Determine the equation of the tangent AB in the form $y = mx + c$.   (5)
4.4   Determine the length of BC.   (3)
4.5   Determine the equation of the circle centered at A that has both the x- and y-axis   (2)
        as tangents.
4.6   If another circle with centre M(6 ; -5) and radius 4 units is drawn. Determine   (5)
        whether the circles will INTERSECT or NOT.

                                                                                                      [20]

QUESTION 5

5.1   If cos34° = p, WITHOUT using a calculator, determine the following in terms of p.
        5.1.1   Sin 64°   (3)

        5.1.2   cos 68°   (2)

        5.1.3   sin 17°   (3)

        5.1.4 $2\sin^2 28°$ (3)

5.2   Simplify each of the following without using a calculator. Show all Calculations

5.2.1 $\dfrac{\sin110°.\tan60°}{\cos540°.\tan250°.\sin380}$ (7)

5.2.2. $\left(1-\sqrt{2}\sin22,5°\right)\left(\sqrt{2}\sin22,5°+1\right)$ (4)

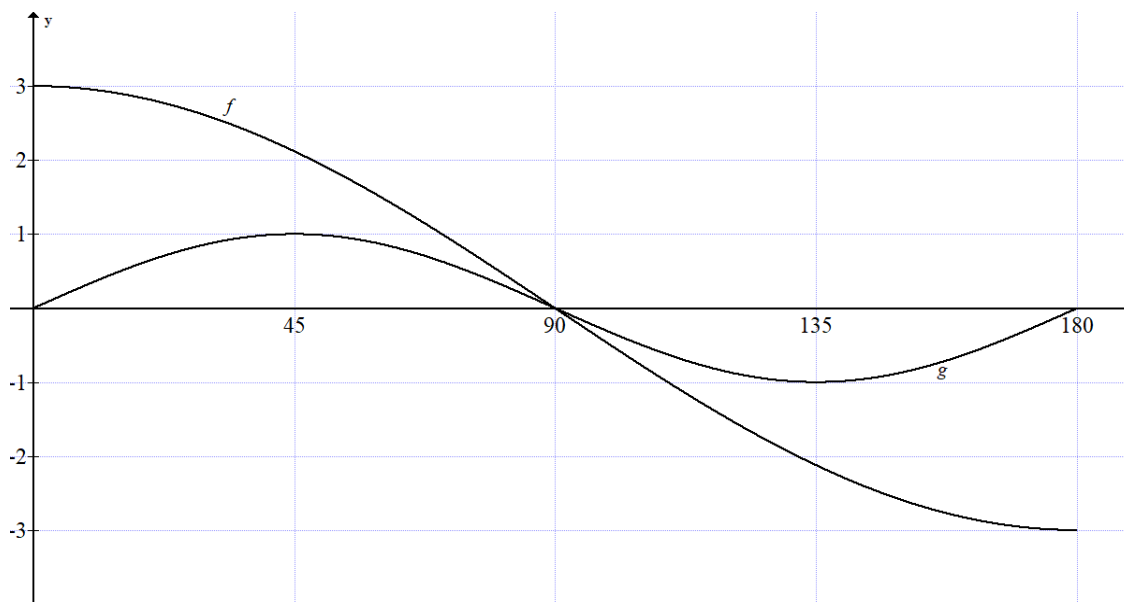5.3   Given the expression: $\dfrac{\cos2x\tan x}{\sin^2 x}$

5.3.1   For which value(s) of x in the interval $x\in\left[0°;180°\right]$, will this expression   (3)
          be undefined?

5.3.2   Prove that $\dfrac{\cos2x\tan x}{\sin^2 x}=\dfrac{\cos x}{\sin x}-\tan x$ (5)

                                                                                                      [30]

QUESTION 6

In the diagram below, the graphs of $f(x)=a\cos x$ and $g(x)=\sin bx$ are drawn for the interval $x\in\left[0°\ ;\ 180°\right]$.

6.1   Write down the values of a and b (2)
6.2   Write down the period of f (1)
6.3   Write down the range of $g(x)+3$ (2)
6.4   For which values of x, in the given interval, is $f(x).g'(x)>0$ (3)
6.5   When the graph of g is shifted $q°$ to the left, it coincides with the function   (3)
        $y-\cos^2 x=-\sin^2 x$. Determine the value of q.

                                                                                                      [11]

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly North of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M, and R lie in the same horizontal plane.
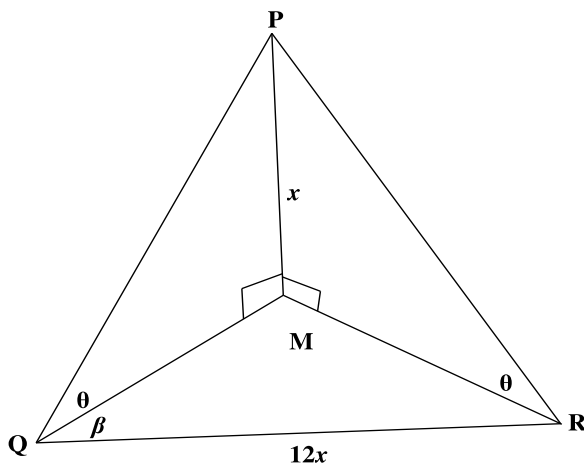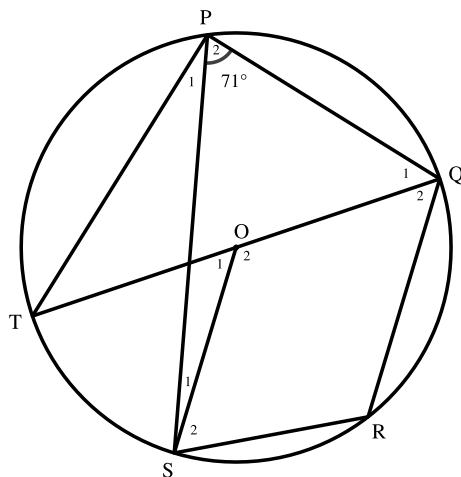

7.1   Write QM in terms of x and θ (2)

7.2   Prove that $\tan\theta = \dfrac{\cos\beta}{6}$ (4)

7.3   If $\beta = 40°$ and QM = 60 metres, calculae the height of the lighthouse to the nearest metre.   (3)

                                                        [09]

QUESTION 8

8.1   In the diagram, O is the centre of the circle. PQRS is a cyclic quadrilateral and TQ is the diameter of the circle. Chord PQ and radius OS are drawn. $\hat{P} = 71°$.
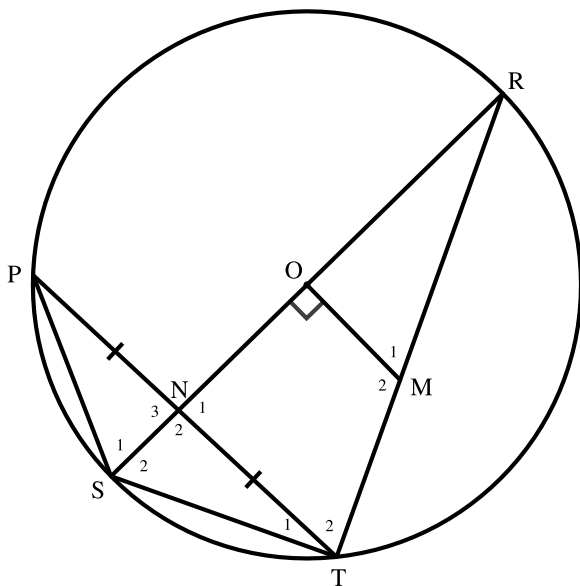

         Determine, giving reasons, the sizes of the following angles:

8.1.1 $\hat{R}$ (2)

8.1.2 $\hat{P}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

8.2   In the diagram, O is the centre of a circle PSTR and SOR is a diameter. N, the midpoint of chord PT, lies on SOR. M is a point on TR such that $OM \perp SR$.

Prove the following, giving reasons:
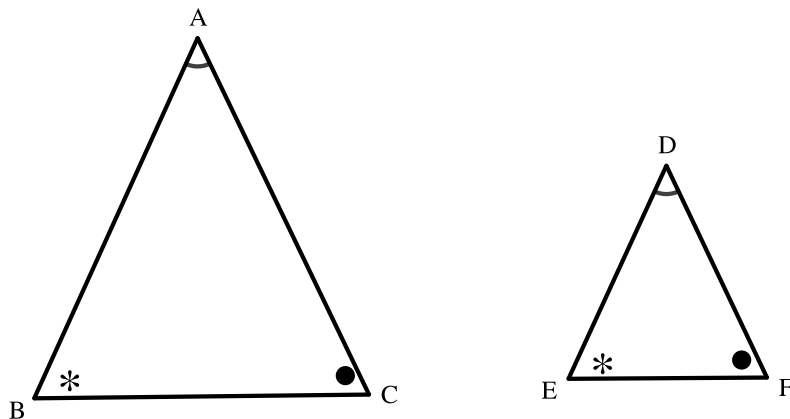
8.2.1   TSOM is a cyclic quadrilateral.   (2)

8.2.2   PT // OM.   (3)

8.2.3 $\hat{S}_1 = \hat{M}_1$ (4)

[15]

QUESTION 9

9.1   In the diagram, $\triangle ABC$ and $\triangle DEF$ are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion. i.e. $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ (6)

9.2   In the diagram, $\triangle ABC$ is drawn. E and F are points on AC and AB respectively such that
$\dfrac{AE}{EC} = \dfrac{3}{2}$ and $\dfrac{AF}{FB} = \dfrac{2}{5}$. BC produced meet FE produced in D. G is a point on FB such that
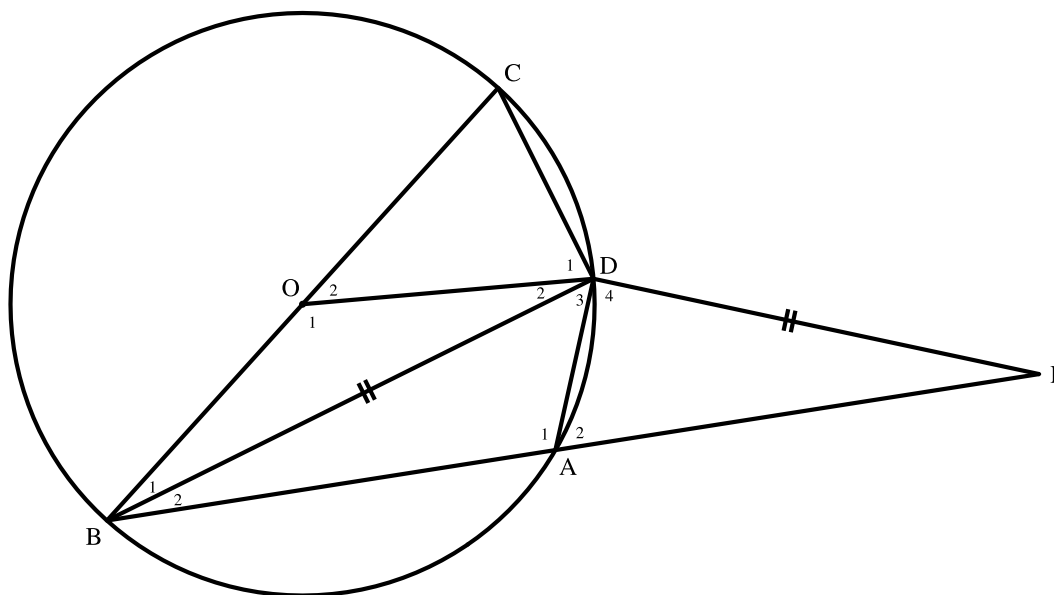FD $\parallel$ GC.
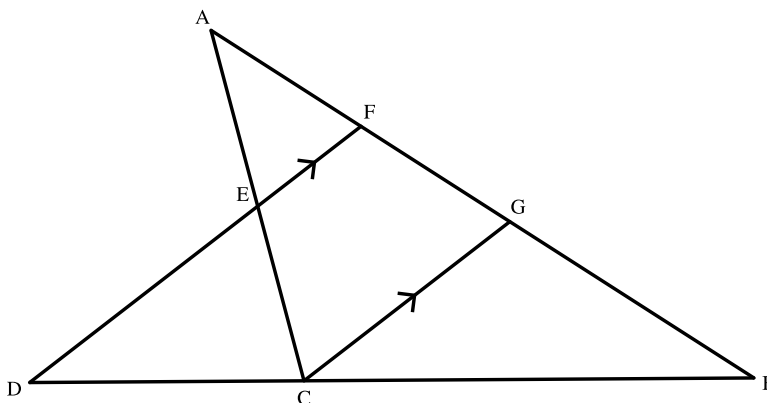
Calculate with reasons, $\dfrac{BC}{CD}$ (5)

[11]

QUESTION 10

In the diagram, O is the centre of circle ABCD. BA produced intersects DE in E. BD bisects $A\hat{B}C$ and
$BD = DE$. Straight lines BOC, OD and AD are drawn. $\hat{B}_1 = x$.

10.1   Determine, with reasons, the size of $C\hat{D}B$ (2)

10.2   Determine the size of $\hat{D}_4$ (5)

10.3   Prove that $\triangle BDO \parallel\parallel \triangle BED$ (3)

10.4   Show that $2DE^2 = BC.BE$ (4)

[14]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \; r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i} \qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\, \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

DIAGRAM SHEET

NAME OF LEARNER:_____
QUESTION 1.3

Time in minutes (t) *Tyd in minute (t)*	Number of children *Getal kinders*	Cumulative frequency *Kumulatiewe frekwensie*
$2 < t \le 6$	2	
$6 < t \le 10$	10	
$10 < t \le 14$	9	
$14 < t \le 18$	7	
$18 < t \le 22$	8	
$22 < t \le 26$	7	
$26 < t \le 30$	2	

QUESTION 1.4

CUMULATIVE FREQUENCY GRAPH (OGIVE)
KUMULATIEWEFREKWENSIEGRAFIEK (OGIEF)

Cumulative frequency
Kumulatiewe frekwensie

Time in minutes • *Tyd in minute*