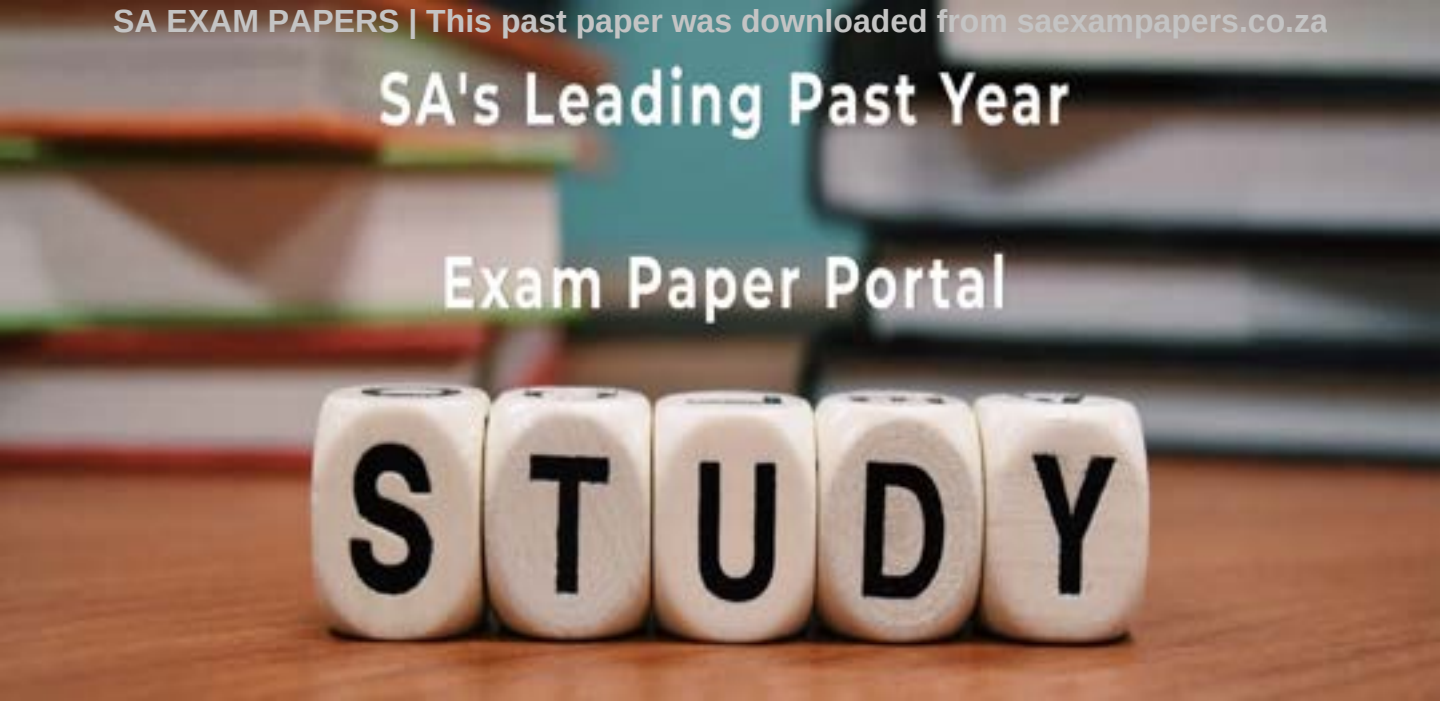


education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

FURTHER EDUCATION AND TRAINING

GRADE 12

MATHEMATICS P2

AUGUST 2024 (PRE -TRIAL)

MARKING GUIDELINE

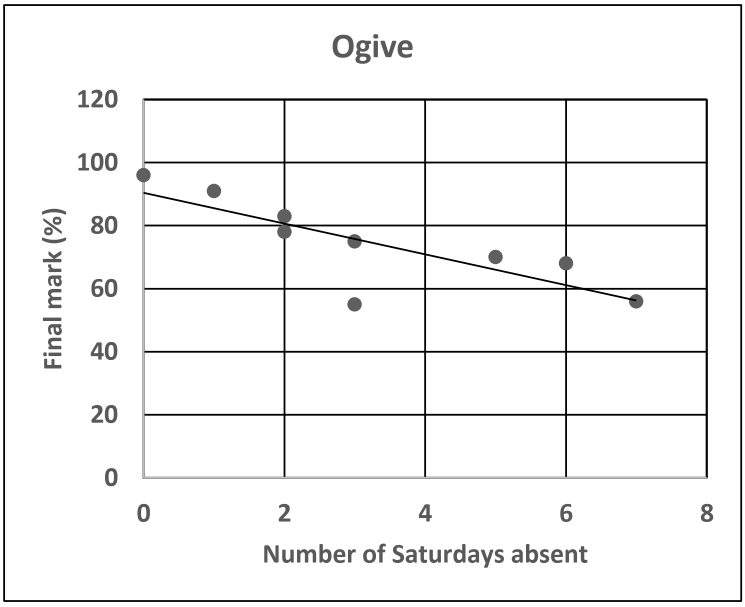
MARKS: 150

This question paper consists of 13 pages.

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/ answers to solve a problem is unacceptable.

QUESTION 1

1.1	$A = 90{,}375$ $B = -4{,}875$ $\hat{y} = 90{,}38 - 4{,}88x$	✓ *value of A* ✓ *value of B* ✓ *equation* (3)
1.2		✓ y-intercept ✓ *passing through mean pt* $(3{,}2; 74{,}67)$) (2)
1.3	$r = -0{,}80$	✓ *answer* (1)
1.4	Negative strong correlation	✓ *negative* ✓ *strong* (2)
1.5	$52 = 90{,}38 - 4{,}88x$ $x = 7{,}86$ It is accurate since $7,86$ is closer to 8. OR	✓ $52 = 90{,}38 - 4{,}88x$ ✓ $x = 7{,}86$ ✓ *conclusion* (3)

	$y = 90{,}38 - 4{,}88(8)$ $= 51{,}34\%$ Which is closest to 52%	✓ $y = 90{,}38 - 4{,}88(8)$ ✓ $y = 51{,}34\%$ ✓ *conclusion* (3)
		[11]

QUESTION 2		
2.1.1	$\bar{x} = \frac{493}{10}$ $= 49{,}3$	✓ $\frac{493}{10}$ ✓ *answer* (2) AO: Full Marks
2.1.2	$\sigma_x = 15{,}67$	✓ *answer* (1)
2.1.3	$(\bar{x} - \sigma_x;\ \bar{x} + \sigma_x)$ $= (49{,}3 - 15{,}67\ ;\ 49{,}3 + 15{,}67)$ $= (33{,}67\ ;\ 64{,}97)$ $7\ players$	✓ *substitution* ✓ *interval* ✓ 3 (3)
2.2	$10 \quad x \quad x \quad 32 \quad 45 \quad y \quad 51 \quad 53 \quad 80$ $x = 20$ $\frac{y + 312}{9} = 40$ $312 + y = 360$ $y = 49$	✓ $x = 20$ ✓ 51 ✓ 53 ✓ 312 ✓ $\frac{y+312}{9} = 40$ ✓ $y = 49$ (6)
		[12]

QUESTION 3		
3.1	$M\left(\frac{3+5}{2}; \frac{8-6}{2}\right)$ $= M(4; 1)$	✓ $x = 4$ ✓ $y = 1$ (2)
3.2	$MB = \sqrt{(17-4)^2 + (6-1)^2}$ $= \sqrt{194} \approx 13{,}93$	✓ *substitution* ✓ *answer* (2)
3.3	$m_{MB} = \frac{6-1}{17-4} = \frac{5}{13}$ $1 = \frac{5}{13}(4) + c$	✓ *subst* ✓ *gradient* ✓ *subst* $M(4; 1)$ *or* $B(17; 6)$ ✓ *equation*

	$c = -\frac{7}{13}$ $y = \frac{5}{13}x - \frac{7}{13}$ **OR** $m_{MB} = \frac{6-1}{17-4} = \frac{5}{13}$ $y - 6 = \frac{5}{13}(x - 17)$ $y = \frac{5}{13}x - \frac{7}{13}$	**OR** ✓ *subst* ✓ *gradient* ✓ *subst* $M(4;1)$ *or* $B(17;6)$ ✓ *equation*	(4) (4)
3.4	$\frac{5+17}{2} = 11$ $11 = \frac{3+x_2}{2}$ $x_2 = 19$ $\therefore x = 19$	✓ *subst* ✓ *answer*	(2)
3.5	$m_{AD} = -1$ $y = -x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = -x + 11$ **OR** $m_{AD} = -1$ $y - 8 = -(x - 3)$ $\therefore y = -x + 11$	✓ $m_{AD} = -1$ ✓ *subst grad and* $(3;8)$ ✓ *equation* **OR** ✓ $m_{AD} = -1$ ✓ *subst grad and* $(3;8)$ ✓ *equation*	(3) (3)
3.6	$\frac{5}{13}x - \frac{7}{13} = -x + 11$ $5x - 7 = -13x + 143$ $18x = 150$ $x = \frac{25}{3}$ $y = \frac{8}{3}$ $\therefore P\left(\frac{25}{3};\frac{8}{3}\right)$	✓ *equating* ✓ $18x = 150$ ✓ *value of* x ✓ *value of* y	(4)
			[17]

QUESTION 4

4.1	$x^2 - 2x + 1 + y^2 + 6y + 9 = 15 + 1 + 9$ $(x-1)^2 + (y+3)^2 = 25$ $A(1; -3)$ $r = 5$	✓ $(x-1)^2 + (y+3)^2$ ✓ 25 ✓ *value of x* ✓ *value of y* ✓ $r = 5$ (5)
4.2	*let* $y = 0$ $x^2 - 2x = 15$ $x^2 - 2x - 15 = 0$ $(x+3)(x-5) = 0$ $x = -3$ *or* $x = 5$ $B(-3; 0)$ *and* $C(5; 0)$ **OR** $(x-1)^2 + (0+3)^2 = 25$ $(x-1)^2 = 16$ $x - 1 = \pm 4$ $x = -3$ *or* $x = 5$ $B(-3; 0)$ and $C(5; 0)$	✓ $y = 0$ ✓ *standard form* ✓ $B(-3; 0)$ ✓ $C(5; 0)$ (4) **OR** ✓ $y = 0$ ✓ $(x-1)^2 = 16$ ✓ $B(-3; 0)$ ✓ $C(5; 0)$ (4)
4.3	$m_{AC} = \dfrac{0-(-3)}{5-1}$ $= \dfrac{3}{4}$ $m_{tan} = -\dfrac{4}{3}$ $y - 0 = -\dfrac{4}{3}(x-5)$ $\therefore y = -\dfrac{4}{3}x + \dfrac{20}{3}$	✓ $m_{AC} = \frac{3}{4}$ ✓ $m_{tan} = -\frac{4}{3}$ ✓ *subst* $C(5; 0)$ ✓ $y = -\frac{4}{3}x + \frac{20}{3}$ (4)
4.4	$r_1 + r_2 = 6$ $-3 - b = 6$ $b = -9$	✓ 6 ✓ $b = -9$ (2)
4.5	$BC = 8$ $(8)^2 = (5)^2 + (5)^2 - 2(5)(5)\cos\theta$ $\cos\theta = -\dfrac{7}{25}$ $\theta = 106{,}3°$ ***OR***	✓ $BC = 8$ ✓ *substitution* ✓ $\cos\theta = -\frac{7}{25}$ ✓ *answer* (4)

	$m_{AB} = \frac{-3-0}{1-(-3)}$ $= -\frac{3}{4}$ $\tan\alpha = -\frac{3}{4}$ $\alpha = 143{,}13°$ $\tan\beta = \frac{3}{4}$ $\beta = 36{,}87°$ $\theta = 106{,}3°$ **OR** $\tan\beta = \frac{3}{4}$ $\beta = 36{,}87°$ $\therefore B\hat{C}A = 36{,}87°$ $A\hat{B}C = 36{,}87°$ $\mid < s\ opp = sides$ $\therefore \theta = 106{,}3°$ $\mid sum\ of < s\ of\ \Delta ABC$	**OR** ✓ $\tan\alpha = -\frac{3}{4}$ ✓ $143{,}13°$ ✓ $\tan\beta = \frac{3}{4}$ ✓ *answer* (4) **OR** ✓ $\tan\beta = \frac{3}{4}$ ✓ $\therefore B\hat{C}A = 36{,}87°$ ✓ $A\hat{B}C = 36{,}87°$ ✓ *answer* (4)
4.6	$\text{area}\Delta\text{ABC} = \frac{1}{2}(8)(3)$ $= 12$ $\text{area of circle} = \pi(5)^2 = 25\pi$ $\text{area unshaded} = 25\pi - 12$ $= 66{,}54\ \text{units}^2$ **OR** $\text{area}\Delta\text{ABC} = \frac{1}{2}(5)(5)\sin 106{,}3°$ $= 12{,}00$ $\text{area of circle} = \pi(5)^2 = 25\pi$ $\text{area unshaded area} = 25\pi - 12$ $= 66{,}54\ \text{units}^2$	✓ *substitution* ✓ 12 ✓ 25π ✓ *answer* (4) **OR** ✓ *substitution* ✓ 12 ✓ 25π ✓ *answer* (4)
		[23]

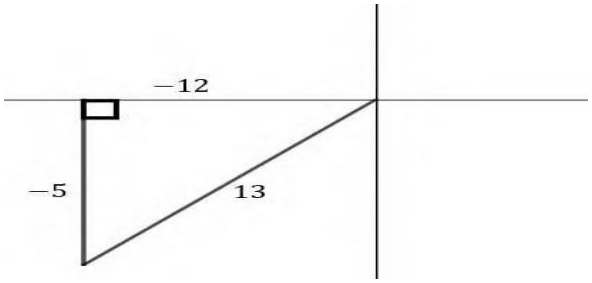
QUESTION 5

5.1.1

$x^2 + (-5)^2 = (13)^2$

$x^2 = 144$

$x = -12$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$

$= \frac{120}{169}$

✓ $3rd\ quad$
✓ $x = -12$
✓ $2\sin\theta\cos\theta$
✓ Substution
✓ Answer

(5)

5.1.2

$\cos(\theta + 30°) = \cos\theta\cos 30° - \sin\theta\sin 30°$

$= \left(-\frac{12}{13}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{5}{13}\right)\left(\frac{1}{2}\right)$

$= \frac{-12\sqrt{3}+5}{26}$

✓ $expansion$
✓ $subst\ -\frac{12}{13}\ and -\frac{5}{13}$
✓ $subst\ \frac{\sqrt{3}}{2}\ and \frac{1}{2}$
✓ $answer$ (4)

5.2

$\frac{\sin 35°\cos 35°}{\tan 225°\cos 200°}$

$= \frac{\sin 35°\cos 35°}{\tan 45°(-\sin 70°)}$

$= \frac{\sin 35°\cos 35°}{(-1)\sin 70°)}$

$= \frac{\sin 35°\cos 35°}{(-1)2\sin 35°\cos 35°}$

$= -\frac{1}{2}$

✓ $\tan 45°$
✓ $-\sin 70°$
✓ -1
✓ $2\sin 35°\cos 35°$
✓ $-\frac{1}{2}$

(5)

OR

$\frac{\sin 35°\cos 35°}{\tan 45°(-\cos 20°)} \times \frac{2}{2}$

$= \frac{\sin 70°}{2(-1)\cos 20°}$

$= \frac{\sin 70°}{-2\sin 70°}$

OR

✓ $\tan 45°$
✓ $-\cos 20°$
✓ -1
✓ $\sin 70°\ in\ numerator$
✓ $\sin 70°\ in\ denominator$

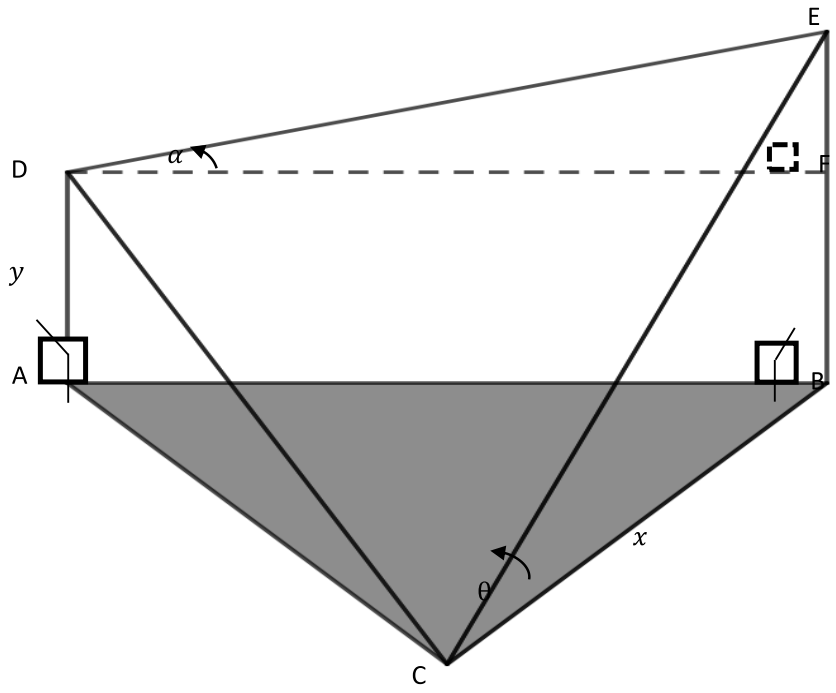
(5)

	$= -\frac{1}{2}$	
5.3.1	$LHS: \frac{1-\tan A}{1+\tan A}$ $= \frac{1-\frac{\sin A}{\cos A}}{1+\frac{\sin A}{\cos A}}$ $= \frac{\cos A-\sin A}{\cos A+\sin A} \times \frac{\cos A+\sin A}{\cos A-\sin A}$ $= \frac{\cos^2 A-\sin^2 A}{\cos^2 A+2\cos A\sin A+\sin^2 A}$ $= \frac{\cos 2A}{1+\sin 2A} = \text{RHS}$ **OR** $\frac{1-\tan A}{1+\tan A}$ $= \frac{1-\frac{\sin A}{\cos A}}{1+\frac{\sin A}{\cos A}}$ $= \frac{\cos A-\sin A}{\cos A+\sin A}$ $RHS: \frac{\cos 2A}{1+\sin 2A}$ $= \frac{\cos^2 A-\sin^2 A}{1+2\sin A\cos A}$ $= \frac{\cos^2 A-\sin^2 A}{\sin^2 A+\cos^2 A+2\sin A\cos A}$ $= \frac{(\cos A-\sin A)(\cos A+\sin A)}{(\sin A+\cos A)(\sin A+\cos A)}$ $= \frac{\cos A-\sin A}{\cos A+\sin A}$ $\text{LHS} = \text{RHS}$	✓ $\frac{\sin A}{\cos A}$ ✓ $\frac{\cos A-\sin A}{\cos A+\sin A}$ ✓ $\cos^2 A - \sin^2 A$ ✓ $\cos^2 A + 2\cos A\sin A + \sin^2 A$ (4) OR ✓ $\frac{\sin A}{\cos A}$ ✓ $\frac{\cos A-\sin A}{\cos A+\sin A}$ ✓ $\frac{\cos^2 A-\sin^2 A}{1+2\sin A\cos A}$ ✓ $\frac{(\cos A-\sin A)(\cos A+\sin A)}{(\sin A+\cos A)(\sin A+\cos A)}$ (4)
5.3.2	$\frac{1-\tan 22{,}5°}{1+\tan 22{,}5°}$ $= \frac{\cos 45°}{1+\sin 45°}$ $= \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$ $= \frac{\sqrt{2}}{2+\sqrt{2}}$ or $-1+\sqrt{2}$	✓ $\frac{\cos 45°}{1+\sin 45°}$ ✓ $\frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$ ✓ *answer* (3)
5.4.1	$P = \sqrt{\frac{2\sin x\cos x}{2}}$ $= \sqrt{\frac{1}{2}\sin 2x}$	✓ $P = \sqrt{\frac{2\sin x\cos x}{2}}$ ✓ Answer (2)

	$\therefore maximum\left\{\sqrt{\sin x\,.\cos x}\right\} = \frac{1}{\sqrt{2}}$	
5.4.2	$\sin x \cos x = -0{,}24$ $\sin 2x = -0{,}48$ Ref <: $2x = 28{,}69°$ $2x = 208{,}69° + k360°; k \in Z$ $x = 104{,}35° + k180°$ Or $2x = 331{,}31° + k360°$ $x = 165{,}66° + k180°$ **OR** $\sin 2x = -0{,}48$ $2x = -28{,}69° + k360°; k \in Z$ $x = -14{,}35° + k180°$ or $2x = -151{,}31° + k360°$ $x = -75{,}66° + k180°$	✓ $\sin 2x = -0{,}48$ ✓ $2x = 208{,}69° + k360°$ ✓ $x = 104{,}35° + k180°$ ✓ $x = 104{,}35° + k180°$ ✓ $k \in Z$ (5) **OR** ✓ $\sin 2x = -0{,}48$ ✓ $2x = -28{,}69° + k360°$ ✓ $x = -14{,}35° + k180°$ ✓ $x = -75{,}66° + k180°$ ✓ $k \in Z$ (5)
		[28]

QUESTION 6			
6.1.1	$y = -1$	✓ *answer*	(1)
6.1.2	$amplitude = 1$	✓ *answer*	(1)
6.1.3	$b = 2$	✓ *answer*	(1)
6.1.4	$(-45°; 0)\ and\ (45°; 0)$	✓ $(-45°; 0)$ ✓ $(45°; 0)$	(2)
6.2	$-90° \leq x < 0\ \ or\ 0° < x \leq 90°$	✓✓ $-90° \leq x < 0$ ✓ $0° < x \leq 90°$	(3)
			[8]

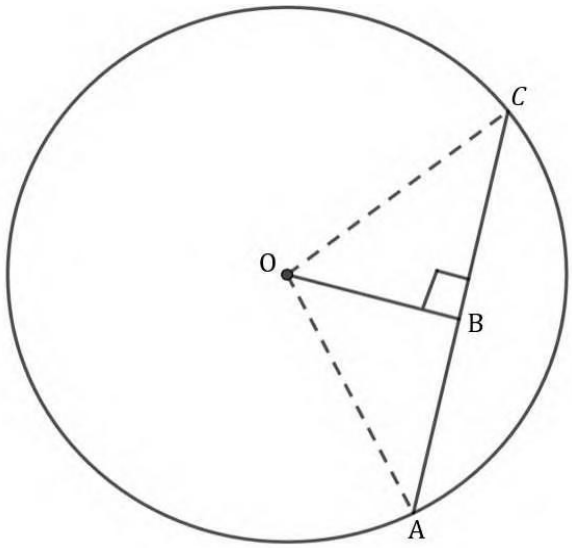
QUESTION 7



7.1	$In\ \Delta BCE$: $\tan\theta = \frac{EB}{x}$ $EB = x\tan\theta$ $In\ \Delta DEF$: $\sin\alpha = \frac{EF}{DE}$ $DE = \frac{EF}{\sin\alpha}$ $BF = AD = y$ ∣ *sides of rectangles* $EF = EB - y$ $= x\tan\theta - y$ $DE = \frac{x\tan\theta - y}{\sin\alpha}$ Q. E. D	✓ $\tan\theta = \frac{EB}{x}$ ✓ $EB = x\tan\theta$ ✓ $DE = \frac{EF}{\sin\alpha}$ ✓ $BF = y$ ✓ $EF = EB - y$ (5)
7.2	$DF^2 = DE^2 - EF^2$ ∣ *Pythagoras* $DF^2 = \left(\frac{x\tan\theta - y}{\sin\alpha}\right)^2 - (x\tan\theta - y)^2$ $= \left(\frac{1000\tan 45° - 250}{\sin 45°}\right)^2 - (1000\tan 45° - 250)^2$	✓ *subst given info* ✓ *subst into Pythagoras* ✓ *answer* (3)

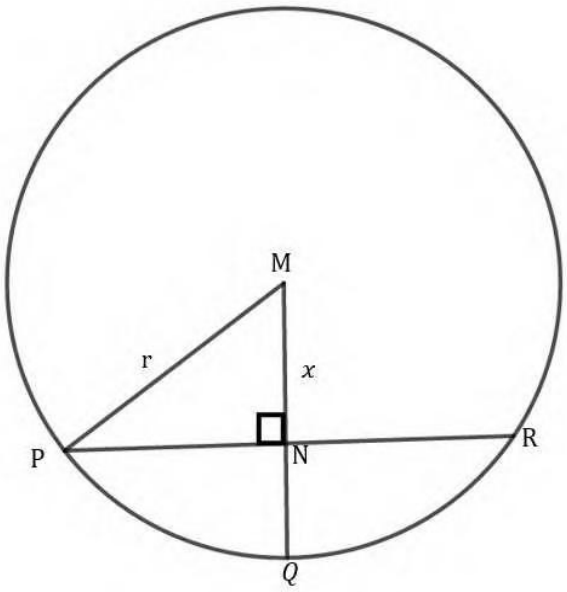
	$\therefore DF = 750m$ **OR** $\tan\alpha = \frac{EF}{DF}$ $DF = \frac{EF}{\tan\alpha}$ $DF = \frac{x\tan\alpha - y}{\tan\alpha}$ or $\cos\alpha = \frac{DF}{DE}$ $DF = DE\cos\alpha$ $= \frac{1000\tan 45° - 250}{\tan 45°}$ $\therefore DF = 750m$	**OR** ✓ $\tan\alpha = \frac{EF}{DF}$ ✓ *substitution* ✓ *answer* (3)
		[8]

QUESTION 8



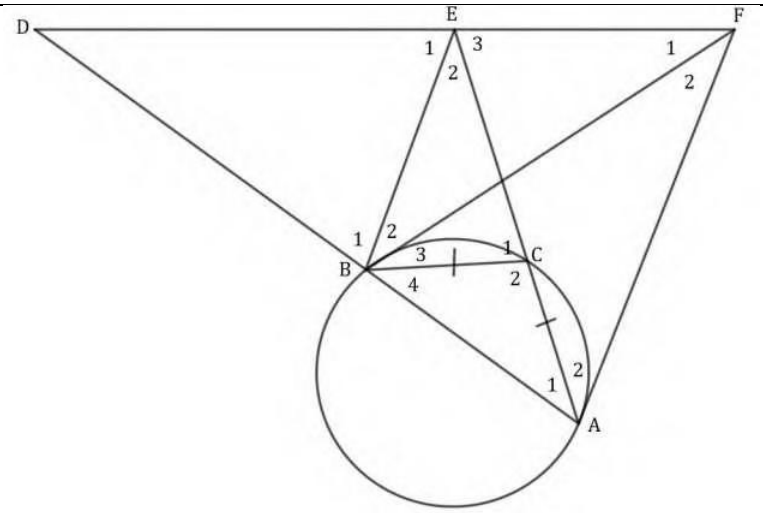
8.1	*In* ΔOBC *and* ΔOBA: $O\hat{B}C = O\hat{B}A$ $[both = 90°]$ $OA = OC$ $[radii]$ $OB = OB$ $[common]$ $\therefore \Delta OBC \equiv \Delta OBA$ $[RHS]$ $AB = BC$ $[\equiv \Delta s]$	✓ *construction* ✓ S/R ✓ S/R ✓ $\therefore \Delta OBC \equiv \Delta OBA$ $[RHS]$ ✓ $AB = BC$ $[\equiv \Delta s]$ (5)

8.2



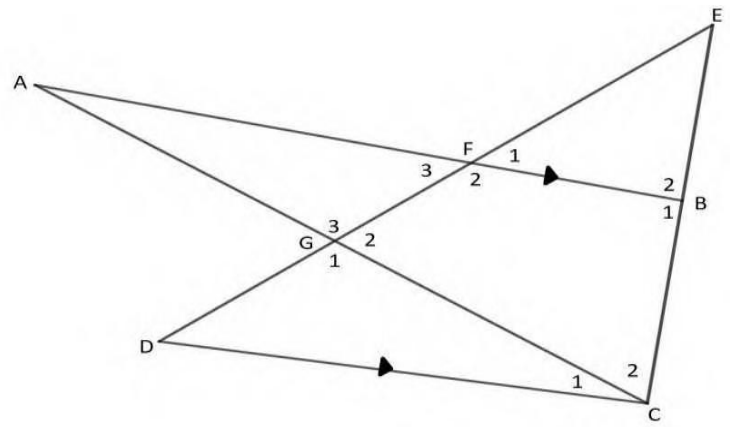
8.2.1	$r = x + 20$	✓ *answer* (1)
8.2.2	$PN = NR$ $= 60$ ∣ *line from centre* $\perp$ *to chord* $PM^2 = PN^2 + NM^2$ ∣ *Pythagoras* $(x + 20)^2 = (60)^2 + x^2$ $x^2 + 40x + 400 = 3600 + x^2$ $40x = 3200$ $x = 80$ $r = 100mm$	✓ S ✓ R ✓ *subst into Pythagoras* ✓ $x^2 + 40x + 400$ ✓ $x = 80$ ✓ 100 (6)
		[12]

QUESTION 9



9.1	$\hat{B}_4 = \hat{A}_2 = x$ [tan *chord theorem*] $\hat{B}_4 = \hat{A}_1 = x$ [$\angle$ *s opp = sides*] $\hat{B}_3 = \hat{A}_1 = x$ [tan *chord theorem*] $\hat{D} = \hat{B}_4 = x$ [*corresp* $\angle$ *s*, $BC \| DF$] $\hat{\hat{F}}_1 = \hat{B}_3 = x$ [*alt* $\angle$ *s*, $BC \| DF$]	✓ *S* ✓ *R* ✓ *S* ✓ *R* ✓ *S* ✓*R* ✓ *S*|*R* ✓ *S*|*R* (8)
9.2.1	$\hat{\hat{F}}_1 = \hat{A}_1 = x$ [*proved*] *ABEF is a cyclic quad* [*converse* $\angle$ *s in the same seg*]	✓ *S* ✓ *R* (2)
9.2.2	$AF = BF$ [*tans from common pt*] $BD = BF$ [$\angle$ *s opp = sides*] $AF = BD$ [$= BF$]	✓ *S*|*R* ✓ *S* ✓*R* (3)
9.3	*In* ΔBDE *and* ΔFAE: $\hat{D} = \hat{A}_2 = x$ | *proved* $\hat{B}_1 = \hat{F}_1 + \hat{F}_2$ | *ext* $<$ *of cyclic quad* $\hat{E}_1 = \hat{E}_3$ | *int* $<$ *s of* Δ*s* $\therefore \Delta BDE \Vert\Vert\Vert \Delta FAE$ | *AAA* $\frac{BD}{AF} = \frac{DE}{AE}$ | $\Vert\Vert\Vert\Delta s$ *but* $BD = AF$ *and* $AF = BF$ $\therefore \frac{AF}{FB} = \frac{AF}{AE}$	✓ ΔBDE *and* ΔAEF ✓ *S* ✓ *S*|*R* ✓ *AAA* ✓ *S* ✓ *R* (6)
		[19]

QUESTION 10



10.1	$\frac{\text{EF}}{\text{ED}} = \frac{EB}{EC}$ $[prop\ theorem, FB \| DC]$ $= \frac{10}{30} = \frac{1}{3}$	✓ $S\|R$ ✓ $\frac{1}{3}$ (2)
10.2	$\frac{\text{EF}}{\text{ED}} = \frac{EB}{EC}$ $[proved]$ $\frac{16}{ED} = \frac{1}{3}$ $\therefore ED = 48cm$	✓ $\frac{16}{ED} = \frac{1}{3}$ ✓ $answer$ (2)
10.3	$\Delta\text{EFB} \| \| \| \Delta\text{EDC}$	✓ $answer$ (1)
10.4	$\frac{DC}{12} = \frac{30}{10}$ $[\| \| \| \Delta s]$ $\therefore DC = 36cm$	✓ $S\|R$ ✓ $answer$ (2)
10.5	$\frac{area\Delta EFB}{area\Delta EDC} = \frac{\frac{1}{2}(16)(10)\sin E}{\frac{1}{2}(48)(30)\sin E}$ $= \frac{1}{9}$ $\frac{area\Delta EDC}{area\Delta DGC} = \frac{48}{DG}$ $[same\ height, common\ vertex]$ $\frac{area\Delta EFB}{area\Delta EDC} \times \frac{area\Delta EDC}{area\Delta DGC} = \frac{1}{9} \times \frac{48}{DG} = \frac{16}{3DG}$ $\frac{area\Delta EFB}{area\Delta DGC} = \frac{16}{3DG}$ $\therefore \frac{3area\Delta EFB}{16area\Delta DGC} = \frac{1}{DG}$	✓ $\frac{area\Delta EFB}{area\Delta EDC} = \frac{\frac{1}{2}(16)(10)\sin E}{\frac{1}{2}(48)(30)\sin E}$ ✓ $\frac{1}{9}$ ✓ $\frac{area\Delta EDC}{area\Delta DGC} = \frac{48}{DG}$ ✓ $multiplying$ ✓ $\frac{area\Delta EFB}{area\Delta DGC} = \frac{16}{3DG}$ (5)
		[12]
		TOTAL: 150