education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

SEPTEMBER 2024

MARKS: 150

TIME: 3 HOURS

This question paper consists of 13 pages and 1 information sheet and an answer book is provided.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

QUESTION 1

A Mathematics teacher wants to create a model by which she can predict a learner's final marks. She decided to use her 2015 results to create the model.

Preparatory exam(x)	55	35	67	85	91	48	78	72	15	75	69	37
Final exam(y)	57	50	74	80	92	50	80	81	23	80	75	42

1.1 Determine the equation of the least squares regression line in the form $y = a + bx$. (3)

1.2 Draw a scatter plot and show the regression line (3)

1.3 Predict the final mark for a learner who attained 46% in the preparatory examination. (2)

1.4 Determine the correlation coefficient of the data. (1)

1.5 Describe the relationship between preparatory and final exam results. (1)

1.6 Could you use this equation to estimate the preparatory exam mark for a learner who attained 73% in the final exam? Give a reason for your answer. (2)

[12]

QUESTION 2

The table below shows the results from a survey of cell phone expenditure for 100 learners from a secondary school in Rustenburg.

Expenditure(in rand)	**Frequency**	**Cumulative frequency**
$50 \leq x < 100$	24	
$100 \leq x < 150$	52	
$150 \leq x < 200$	14	
$200 \leq x < 250$	6	
$250 \leq x < 300$	4	

2.1 Complete the cumulative frequency table in the ANSWER BOOK. (2)

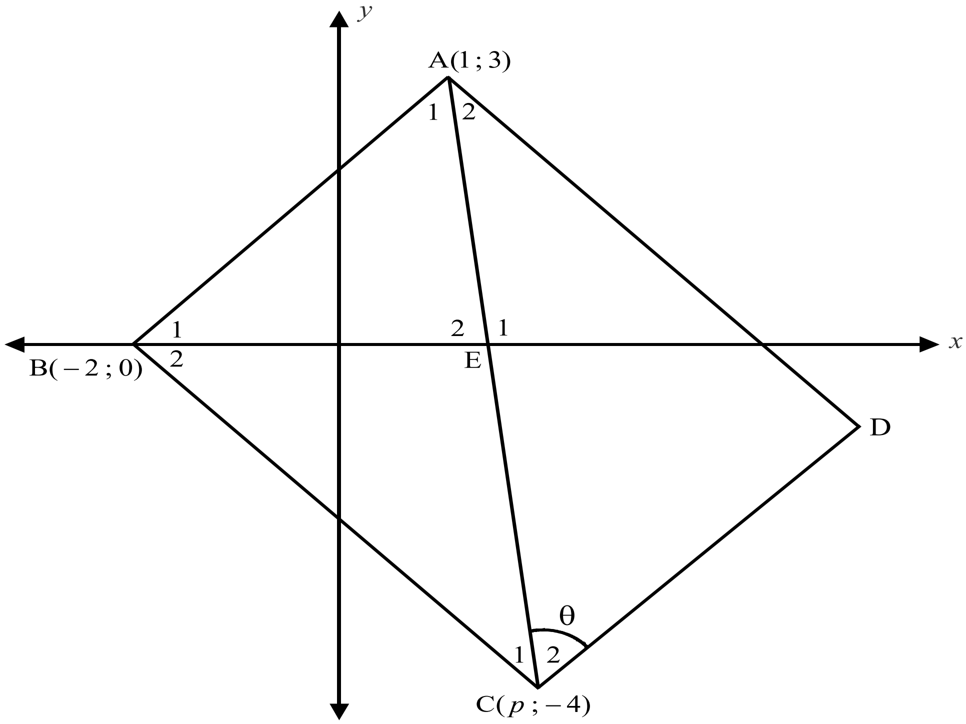
2.2 Draw an Ogive (cumulative frequency graph) for the data. (3)

2.3 Calculate the estimated mean of cell phone expenditure. (3)

[8]

QUESTION 3

3.1 ABC is a triangle with vertices $A(1\,;3)$, $B(-2\,;0)$ and $C(p\,;-4)$ where $p>0$.
the length of AC is $\sqrt{50}$ units.

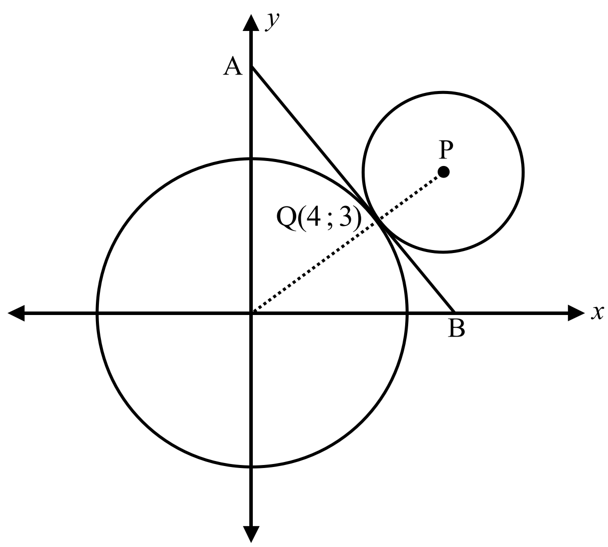


3.1.1	Determine the gradient of AB.	(2)
3.1.2	Show, by calculation, that $p=2$.	(4)
3.1.3.	Determine the equation of the perpendicular bisector of AB.	(4)
3.1.4	Write down the coordinates of D such that ABCD is a rectangle	(2)
3.1.5	Determine the equation of the circle passing through A, B and C.	(4)
3.1.6	Calculate the size of θ rounded off to the nearest whole number	(5)
3.2	Three straight lines AB, RS and $x=-3$ intersect each other. The equation of AB is $3x+by=-2$ with $b\neq 0$, and the equation of RS is $y=-\frac{2}{3}x+2$. Calculate the value of *b*.	(4)

[25]

QUESTION 4

Two circles in the diagram below represent two interlocking gears, which touch at the point Q(4 ; 3). The circles have the following equations: $x^2 + y^2 = 25$ and $x^2 - 12x + y^2 - 9y + 50 = 0$



4.1 Show that the coordinates of P are $(6 ; 4\frac{1}{2})$. (3)

4.2 Determine the equation of the common tangent AB. (4)

4.3 If the larger gear makes one full revolution, how many times will the smaller gear turn completely? (4)

4.4 Determine the area of ΔAOB (3)

4.5 Another tangent to the circle with centre O, drawn from A, touches the circle at C. And C is the reflection of Q by the y axis.
Determine the length of CQ. (2)

[16]

QUESTION 5

5.1 If $\cos 21° = p$ determine the following in terms of p.

5.1.1 $\tan 201°$ (3)

5.1.2 $\sin 42°$ (3)

5.1.3 $\cos 51°$ (3)

5.2 Simplify : $\dfrac{\sin 210°.\cos 510°}{\cos 315°.\sin(-135°)}$ (7)

5.3 Prove the identity:

$$\frac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta} = \frac{1+\cos\theta}{\sin\theta}$$ (5)

5.4 Determine the general solution of $\sin\theta\sin\dfrac{3\theta}{2} + \cos\dfrac{3\theta}{2}\cos\theta = -\dfrac{\sqrt{3}}{2}$. (4)

5.5 Given: $\sin\theta.\cos\beta = -1$

5.5.1 Write down the maximum and minimum value of $\cos\beta$ (1)

5.5.2 Solve for $\theta \in [0^\circ;270^\circ]$ and $\beta \in [-180^\circ;90^\circ]$. (4)

[30]

QUESTION 6

Given that $y = f(x) = 2\cos x$ and $y = g(x) = \sin(x + 30^\circ)$:

6.1 Sketch the graphs of f and g on the ANSWER BOOK on the same set of axes for $x \in [-180^\circ; 180^\circ]$. (6)

6.2 Read the following answers from your graph and show where these answers have been read:

6.2.1 Write down the period of f. (1)

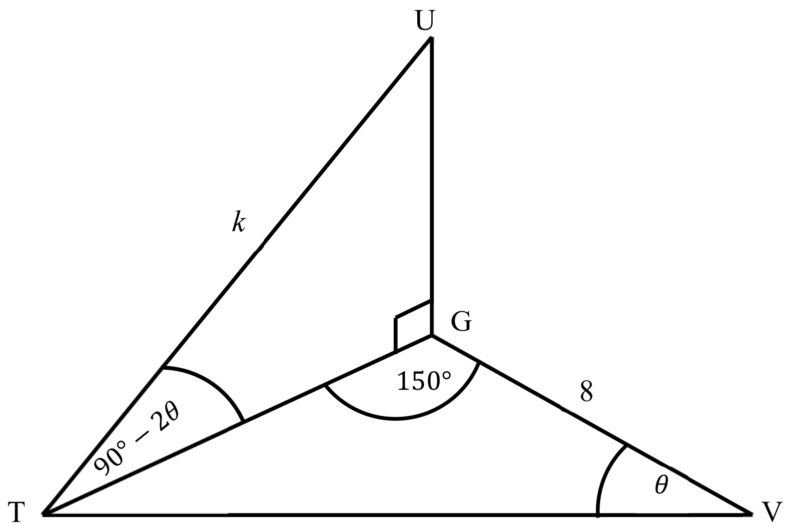
6.2.2 Determine one value of x for which $f(x) - g(x) = 1{,}5$. (1)

6.2.3 Determine the positive values of x for which $2\sin(x + 30^\circ).\cos x < 0$ (2)

[10]

QUESTION 7

A mouse on the ground at point T is looking up to an owl in a tree at point U and a cat to his right on the ground at point V. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$.
$\text{TU} = k\ \ units$, $\text{GV} = 8\, units$, $\text{T}\hat{\text{G}}\text{V} = 150°$ and $\text{T}\hat{\text{V}}\text{G} = \theta$.



7.1 Write down the size of $\text{T}\hat{\text{U}}\text{V}$ in terms of θ (1)

7.2 Show that $\text{TG} = k \sin 2\theta$. (2)

7.3 Show that $\text{TV} = k \cos \theta$ (4)

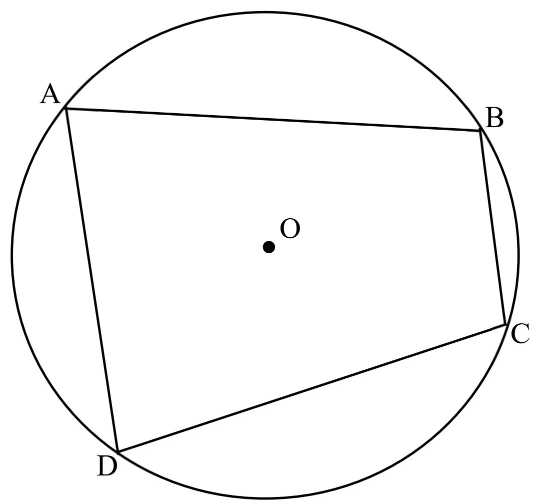
7.4 Show that the area of $\Delta\text{TGV} = 2k \sin 2\theta$. (2)

[9]

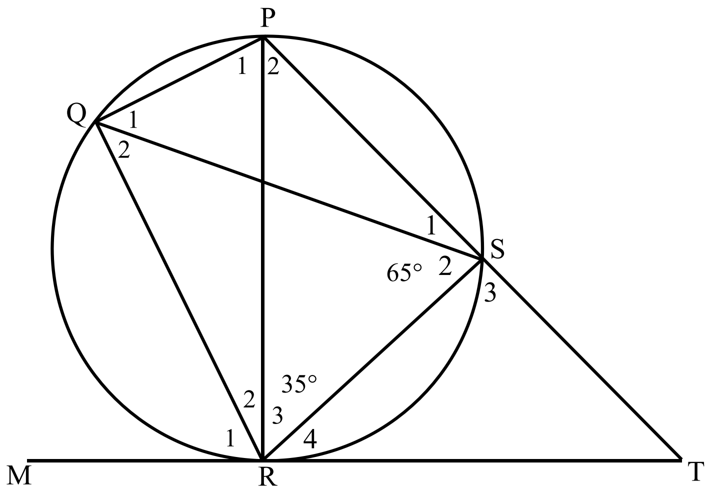
Give reasons for your statements and calculations in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 A, B, C and D are points on the circumference of the circle centre O.
Prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)



8.2 In the diagram below, PQRS is a cyclic quadrilateral and PR is a diameter of the circle. The line MRT is a tangent to the circle at R. $\hat{S}_2 = 65°$ and $\hat{R}_3 = 35°$.



Determine the sizes of each of the following angles, with reasons:

8.2.1 $\hat{R}_1$ (1)

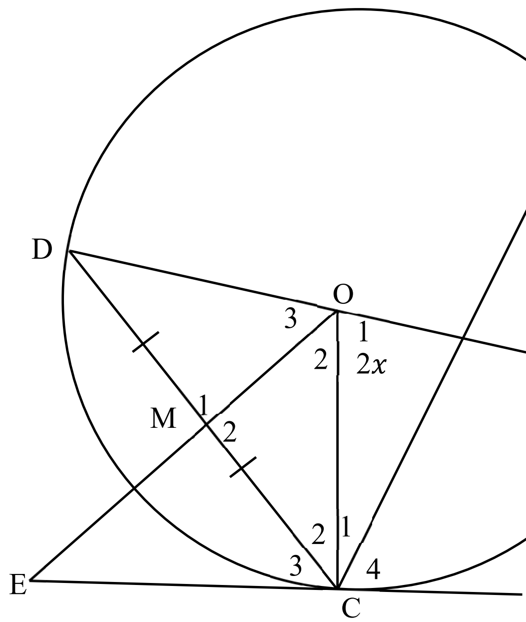
8.2.2 $\hat{R}_4$ (2)

8.2.3 $\hat{T}$ (3)

[11]

QUESTION 9

In the diagram, O is the centre of the circle and EC is a tangent to the circle at C. $DM = MC$ and OME is a straight line. Let $\widehat{O}_1 = 2x$.



9.1 Write, with reasons, THREE angles equal to x. (6)

9.2 Prove that $\widehat{O}_2 = 90° - x$ (3)

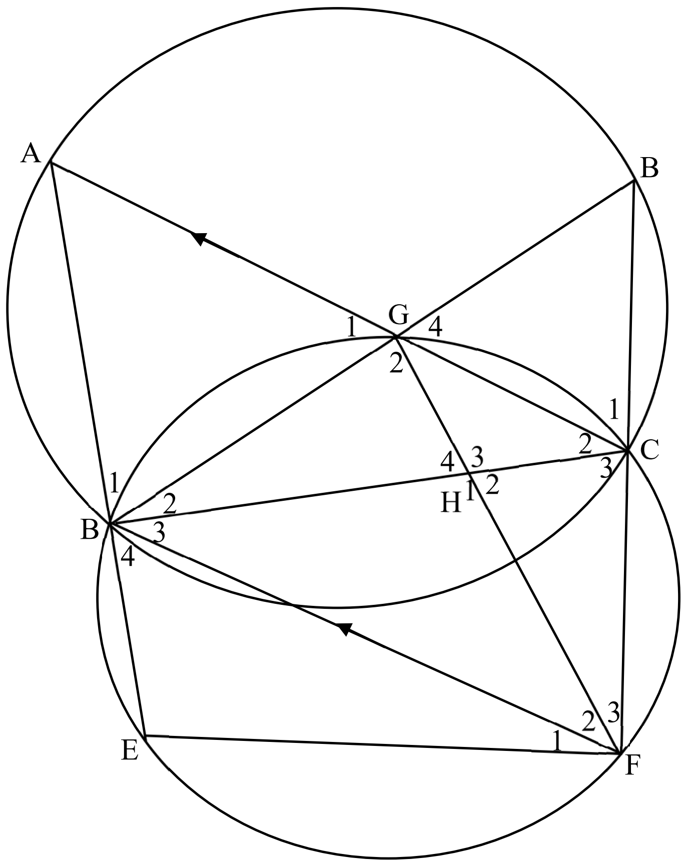
9.3 Prove that EC is a tangent to the circle passing through points, M,C and O. (4)

9.4 Prove that DOCE is a cyclic quadrilateral. (3)

[16]

QUESTION 10

In the diagram below, two circles ABCD and BEFCG intersect at B, C and G. AC || BF. AC and BD intersect at G. BC and FG intersect at H.



10.1 Complete the following reasons

10.1.1 $\hat{G}_1 = \hat{F}_2 + \hat{F}_3$ …………………………………….. (1)

10.1.2 $B\hat{F}D = \hat{C}_1$ …………………………………….. (1)

10.2 Prove that

10.2.1 BH = FH (4)

10.2.2 $\Delta BEF \,|||\, \Delta DGF$ (3)

10.2.3 $FH.\,BG = BH..FC$ (4)

[13]

TOTAL: 150

FORMULA SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$

$b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$ $S_\infty = \frac{a}{1-r}\ ; -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\hat{y} = a + bx$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

$In\ \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \triangle ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = cos^2\alpha - sin^2\alpha$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 1 - 2sin^2\alpha$

$= 2cos^2\alpha - 1$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_1 - x)^2}{\text{n}}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A\ of\ B) = P(A) + P(B) - P(A\ and\ B)$