LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL

SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2024

MARKS: 150

TIME: 3 HOURS

EMATHP2

This paper consists of 12 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Please read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the ANSWER BOOK.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you used to determine your answers.

4. ANSWERS ONLY will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphically) unless otherwise stated.

6, If necessary, round off answers to TWO decimal places unless otherwise noted.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this paper.

9. Write legibly and present your work neatly.

QUESTION 1

A statistician at a large corporation, with branches around the world, has been assigned to analyse the annual salaries (in \$) of 100 global customer service managers across all branches during 2023. The following data were collected:

Annual salary range (in \$)	Number of managers
$0 \leq x < 10\ 000$	3
$10\ 000 \leq x < 20\ 000$	5
$20\ 000 \leq x < 30\ 000$	12
$30\ 000 \leq x < 40\ 000$	19
$40\ 000 \leq x < 50\ 000$	20
$50\ 000 \leq x < 60\ 000$	14
$60\ 000 \leq x < 70\ 000$	12
$70\ 000 \leq x < 80\ 000$	7
$80\ 000 \leq x < 90\ 000$	5
$90\ 000 \leq x < 100\ 000$	3

1.1 Complete the cumulative frequency table in the ANSWER BOOK provided. (2)

1.2 Draw the cumulative frequency graph (ogive) on the grid provided. (3)

1.3 Use the graph to determine the interquartile range of the data. (3)

1.4 The CEO of the company wants to motivate the managers whose salaries are less than \$40 000 and promise them a raise of 15% in their annual salary. The managers who earn more than \$40 000 will get a raise of 8%. What will the new mean annual salary be at the end of 2024? (4)

[12]

QUESTION 2

10 girls compete to see who can throw a ball the furthest. They want to see whether there is a relationship between their height and the distance which they can throw. The following table shows the height of each girl (x cm) to the nearest cm and the distance (y m) she can throw the ball.

	A	B	C	D	E	F	G	H	I	J
x	122	124	133	138	144	156	158	161	164	168
y	41	38	56	56	29	54	59	61	68	67

2.1 Determine the equation for the least squares regression line. (3)

2.2 Estimate the distance (to the nearest metre) which a ball can be thrown by a girl who is 150 cm tall. (2)

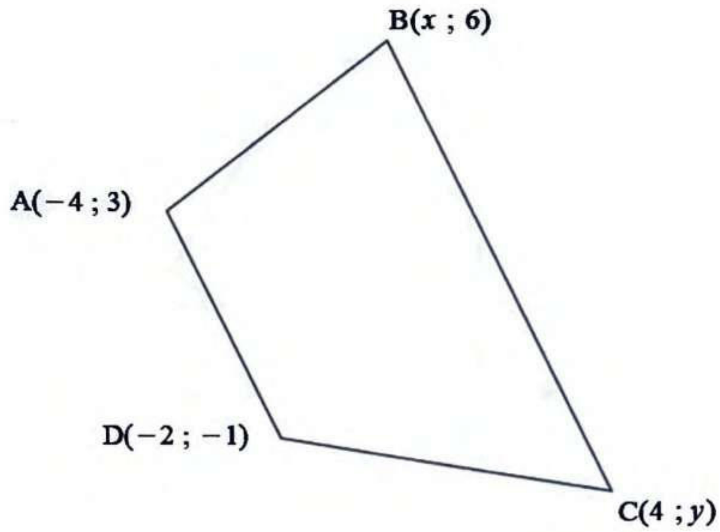
2.3 Describe the trend of the data above. (1)

2.4 Do you think this trend can continue indefinitely? Give a valid reason for your answer. (2)

[8]

QUESTION 3

The following diagram shows a trapezium ABCD with $A(-4\,;3)$, $B(x\,;6)$, $C(4\,;y)$ and $D(-2\,;\,-1)$.

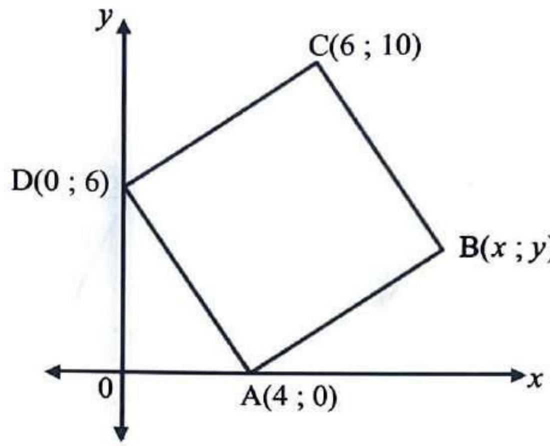


3.1	If AD ∥ BC, determine an equation in the form $y = \ldots$.	(4)
3.2	If $BC = 2AD$, determine another equation in terms of x and y.	(4)
3.3	Determine the values of x and y.	(6)
3.4	If $y < 0$, describe the translation of point C so that ABCD is a parallelogram.	(2)

[16]

QUESTION 4

4.1 In the figure $A(4\ ;\ 0)$, $B(x\ ;y)$, $C(6\ ;\ 10)$ and $D(0\ ;\ 6)$ are the vertices of a square. O is the origin.



4.1.1 Determine the coordinates of B. (2)

4.1.2 Determine the length of DB. (2)

4.1.3 Determine the equation of the circle passing through B, C and D in the form $(x-a)^2+(y-b)^2=r^2$. (6)

4.1.4 Determine the equation of the tangent to the circle at $D(0\ ;\ 6)$ in the form $y = ...$ (4)

4.1.5 E is the point intersection of the diagonals of ABCD. Show, by using analytical methods, that OE bisects $D\hat{O}A$. (3)

4.2 $(x-1)^2+(y+1)^2=2(x-y)$ is the equation of a circle. Determine the coordinates of :

4.2.1 the centre of the circle. (4)

4.2.2 the length of the radius of the circle. (1)

[22]

QUESTION 5

5.1 If $\cos 12° = m$, determine the value of the following in terms of *m*, without using a calculator:

5.1.1 $\cos(-12°)$ (2)

5.1.2 $\cos 72°$ (3)

5.1.3 $\cos 6°$ (4)

5.2 Show that: $\dfrac{\sin 234°}{\cos 36°} - \dfrac{\sin(x - 90°)\cos(90° - 2x)}{\sin x} = \cos 2x$ (6)

5.3 If $\sin^2 B - \cos^2 B = 1$ and $B \in [0° ; 90°]$, determine without using a calculator:

5.3.1 $\cos 2B$. (1)

5.3.2 the size of $\hat{B}$. (2)

5.3.3 hence the size of $\hat{C}$, when $\sin(B - C) - \cos(B - C) = 0$ and $-180° \le C \le 90°$. (6)

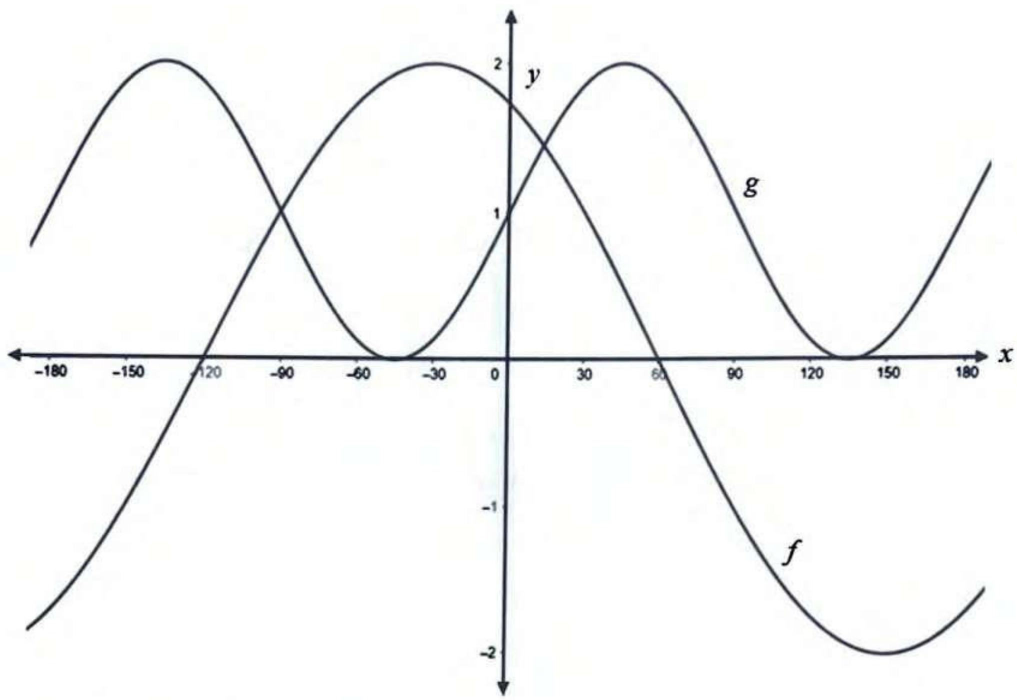
5.4 Consider $P = 2\cos x - \cos 2x$.

Use algebraic methods to determine the maximum value of P. (5)

[29]

QUESTION 6

In the diagram below the graphs of $f(x)=2\cos(x+a)$ and $g(x)=b+\sin 2x$, for $x\in[-180°\ ;\ 180°]$ are drawn.



6.1 Determine the values of a and b. (2)

6.2 Determine the value of $f(x)$ if $x=0°$, without using a calculator. (3)

6.3 Determine the value(s) of x, by using the graphs if:

6.3.1 $g(x)=2$ (2)

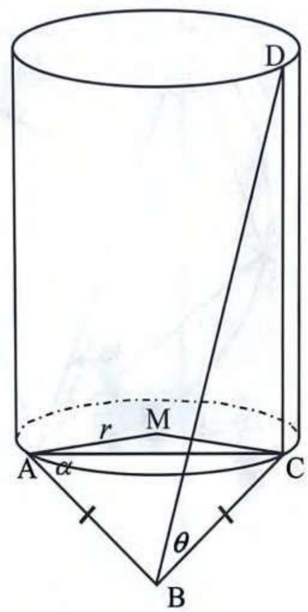
6.3.2 $f(x)\geq g(x)$; for $x\in[-180°\ ;\ 0°)$ (2)

6.4 The $y-$axis is shifted to the left to pass through the maximum turning point of f. Determine the new equation of f in the form $y=.....$ (2)

[11]

QUESTION 7

BA and BC are tangents drawn to the base of a right cylindrical silo with A and C the points of contact. D is a point h metres vertically above C and the angle of elevation of D from B is θ. M is the centre of the base and $B\hat{A}C = \alpha$. B, A, C and M are in the same horizontal plane with BA = BC. r is the radius of the circle.

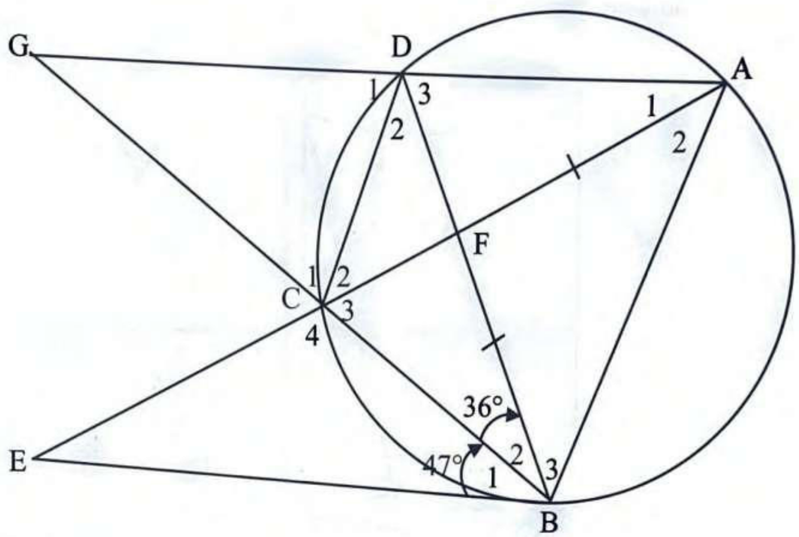


7.1	Write down the size of $B\hat{C}M$.	(1)
7.2	Find $M\hat{A}C$ and $A\hat{M}C$ in terms of α.	(3)
7.3	Hence, prove that: $AC = \dfrac{2h\cos\alpha}{\tan\theta}$	(5)
7.4	Use ΔAMC to prove that: $r = \dfrac{h}{\tan\alpha\tan\theta}$	(3)

[12]

QUESTION 8

In the diagram, ABCD is a cyclic quadrilateral. Chords AC and BD intersect at F. AD produced meets BC produced at G. EB is a tangent to the circle at B. AC produced cuts the tangent EB at E. AF = BF, $E\hat{B}F = 47°$ and $G\hat{B}D = 36°$.



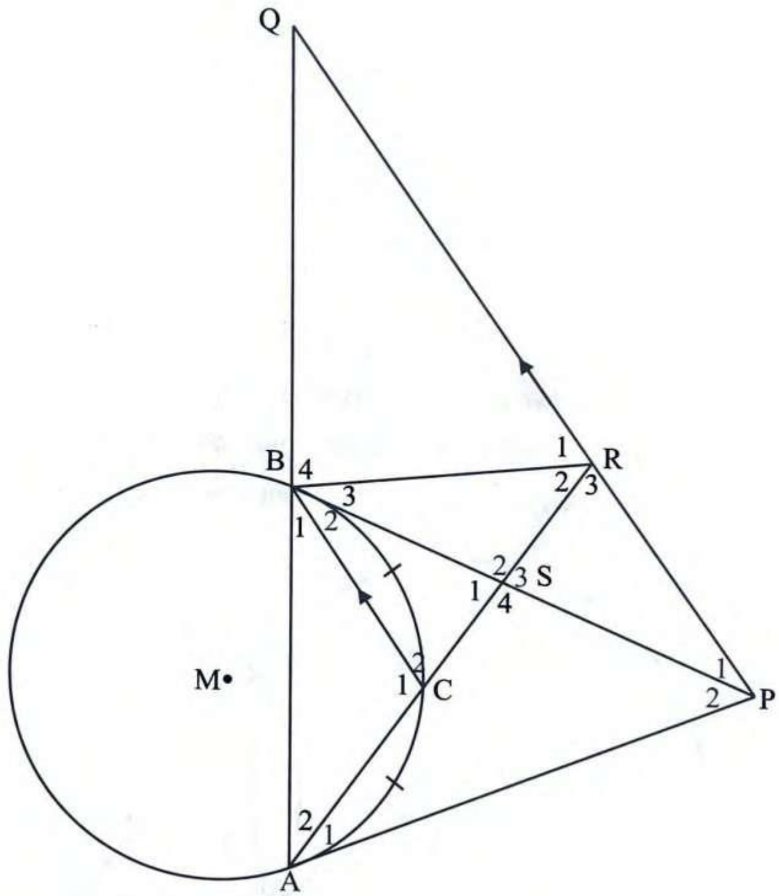
Determine with reasons:

8.1	$\hat{A}_1$	(2)
8.2	$\hat{A}_2$	(2)
8.3	$\hat{C}_1$	(2)
8.4	$\hat{C}_4$	(3)

[9]

QUESTION 9

In the diagram, PA and PB are tangents to the circle with centre M. C is the midpoint of arc ACB and PQ||CB. It is also given that $P\hat{A}C = x$.

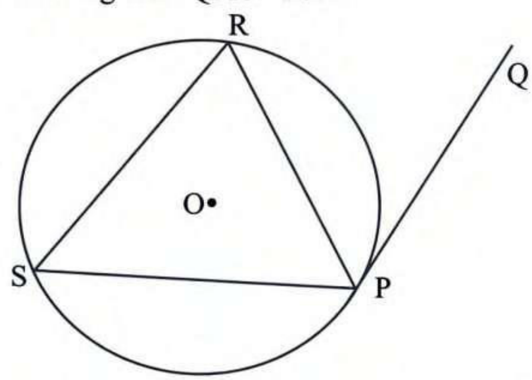


9.1	Write down, with reasons, 5 more angles each equal to x.	(5)
9.2	Prove that ABRP is a cyclic quadrilateral.	(2)
9.3	Prove that AP = BQ.	(3)
9.4	Prove that PRQ is a tangent to the circle passing through BCR at R.	(3)
		[13]

QUESTION 10

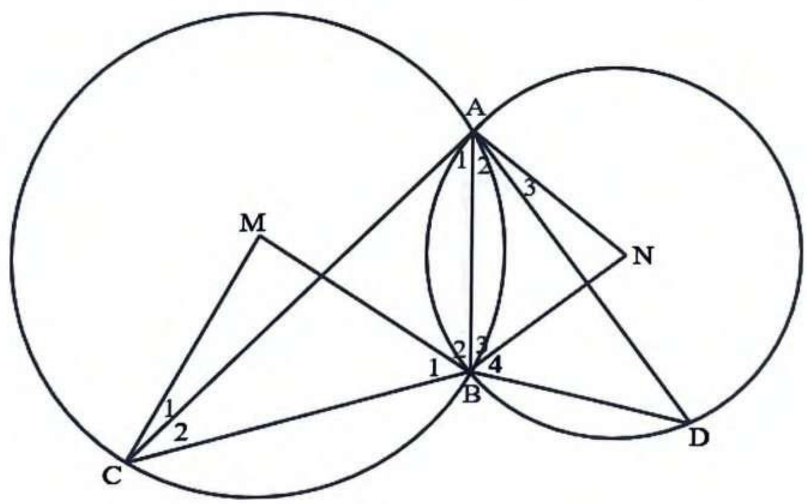
10.1 In the diagram PQ is a tangent to the circle with centre O. PR is a chord drawn from the point of contact and S is a point on the circumference.

Proof the theorem stating that $Q\hat{P}R = P\hat{S}R$.



(5)

10.2 In the diagram, two circles with centres M and N intersect at A and B. The radius of circle M is R, and the radius of circle N is r. AC is a tangent to circle N at A, and AD is a tangent to circle M at A.



Proof the following:

10.2.1	$\Delta ABC \,|||\, \Delta DBA$	(3)
10.2.2	$AB^2 = DB.BC$	(2)
10.2.3	$\Delta CBM \,|||\, \Delta BAN$	(5)
10.2.4	$R^2 : r^2 = BC : DB$	(3)

[18]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$