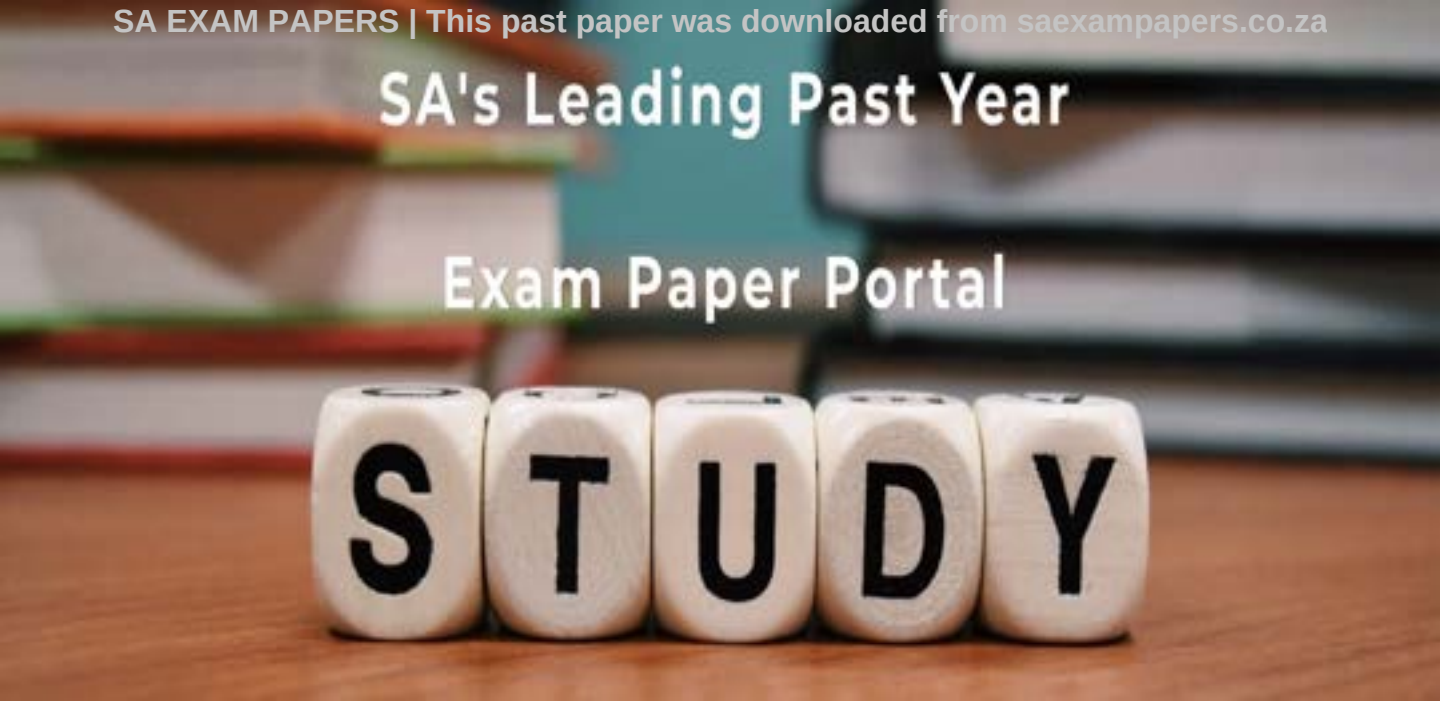


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL

SENIOR CERTIFICATE

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

SEPTEMBER 2024

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

This marking guidelines consist of 15 pages/*Hierdie nasienriglyne bestaan uit 15 bladsye*.

QUESTION/*VRAAG* 1

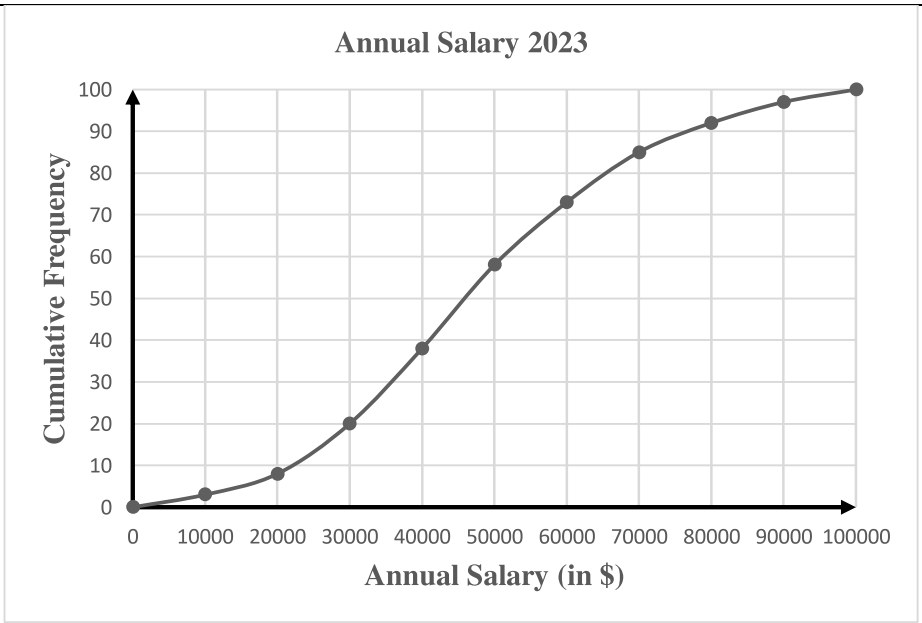
1.1

Annual salary range (in $)/ *Jaarlikse salaris (in $)*	Number of managers/ *Aantal bestuurders*	Cum Freq/ *Kum frekw*
$0 \leq x < 10\,000$	3	3
$10\,000 \leq x < 20\,000$	5	8
$20\,000 \leq x < 30\,000$	12	20
$30\,000 \leq x < 40\,000$	19	39
$40\,000 \leq x < 50\,000$	20	59
$50\,000 \leq x < 60\,000$	14	73
$60\,000 \leq x < 70\,000$	12	85
$70\,000 \leq x < 80\,000$	7	92
$80\,000 \leq x < 90\,000$	5	97
$90\,000 \leq x < 100\,000$	3	100

✓39

✓100 (2)

1.2



✓(0 ; 0)
✓shape/ *vorm*
✓8 other points/ *ander punte* (3)

1.3

IQR/*IKV* = 61000 – 33000
= 28000

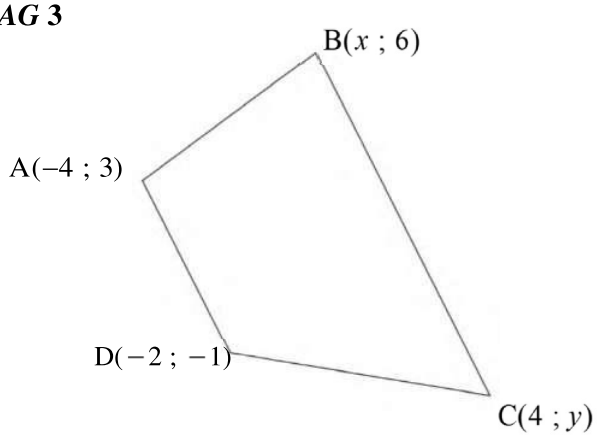
✓accept/ *aanvaar* 61000 - 63000
✓accept/ *aanvaar* 32000 - 34000
✓answ/ *antw* (3)

1.4	Less/*Minder* \$40 000: $\bar{x} = \frac{1055000}{39} = 27051,28$ After 15% increase/*na 15% verhoging*: $\bar{x} = 31108,97$ More than/*Meer as* \$40 000: $\bar{x} = \frac{3685000}{61} = 60409,84$ After 8% increase/*na 8% verhoging*: $\bar{x} = 65242,62$ New/*nuwe*: $\bar{x} = \frac{31108,97 \times 39 + 65242,62 \times 61}{100} = \frac{5034799,92}{100} = 50348$	✓27051,28 ✓31108,97 ✓65242,62 ✓50348	(4)
			[12]

QUESTION/*VRAAG* 2

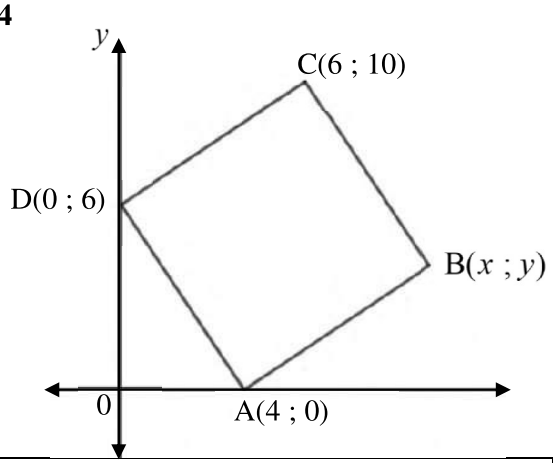
2.1	$a = -26,21740146... = -26,22$ $b = 0,5389468765... = 0,54$ $\hat{y} = -26,22 + 0,54x$	✓$-26,22$ ✓$0,53$ ✓equation/*vergelyking*	(3)
2.2	$\hat{y} = -26,22 + 0,54(150) = 54,78$ OR/OF $\hat{y} = 54,62$ with calculator/ *met sakrekenaar gebruik word*	✓substitution/*vervanging* ✓answer/Antwoord OR/OF ✓✓answer/*antwoord*	(2) (2)
2.3	As the girls get taller, they throw the ball further/ *Namate die meisies langer word, gaan hulle die bal verder gooi*	✓answer/*antwoord*	(1)
2.4	No, the length of the girls cannot get longer indefinitely/ *Nee, die lengte van die meisies kan nie onbepaald langer word nie*	✓No/*nee* ✓reason/*rede*	(2)
			[8]

QUESTION/*VRAAG* 3



3.1	$m(\text{AD}) = m(\text{BC})$ $\frac{3+1}{-4+2} = \frac{6-y}{x-4}$ $\frac{-2}{1} = \frac{6-y}{x-4}$ $-2x+8 = 6-y$ $y = 2x-2$	✓ $m(\text{AD})$ ✓ $m(\text{BC})$ ✓equating/*gelykstel* ✓equation/*vergelyking*	(4)
3.2	BC = 2AD $\sqrt{(x-4)^2+(6-y)^2} = 2\sqrt{(-4+2)^2+(3+1)^2}$ $\sqrt{x^2-8x+16+36-12y+y^2} = 2\left(2\sqrt{5}\right)$ $x^2-8x+y^2-12y+52 = 80$ $x^2-8x+y^2-12y = 28$	✓subst in dist formula/ *vervang in afst formule* ✓simplify/*vereenvoudig* ✓80 ✓equation/*vergelyking*	(4)
3.3	$y = 2x-2$ …….① $x^2-8x+y^2-12y = 28$ …….② Subst/*vervang* ① in ②: $x^2-8x+(2x-2)^2-12(2x-2) = 28$ $x^2-8x+4x^2-8x+4-24x+24-28 = 0$ $5x^2-40x = 0$ $x^2-8x = 0$ $x(x-8) = 0$ $x = 0$ or/*of* $x = 8$ $y = -2$ $y = 14$	✓substitution/*vervanging* ✓simplify/*vereenvoudig* ✓std form/*std vorm* ✓factors/*faktore* ✓values/*waardes x* ✓values/*waaardes y*	(6)
3.4	x translate 2 to left/*transleer 2 links* y translate 4 up/*transleer 4 op*	✓2 left/*links* ✓4 up/*op*	(2)
			[16]

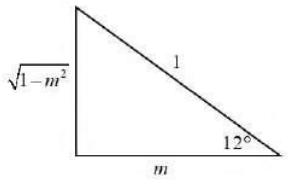
QUESTION/*VRAAG* 4



4.1	4.1.1	B(10 ; 4)	✓ x ✓ y	(2)
	4.1.2	$DB = \sqrt{(0-10)^2 + (6-4)^2} = 2\sqrt{26}$	✓subst in dist formula/ *vervang in afstand formule* ✓answer/*antwoord*	(2)
	4.1.3	$\hat{C} = 90°$ ∴ DB is a diameter (line subtend 90° ∠) E(5 ; 5) Radius = $\frac{1}{2}(2\sqrt{26}) = \sqrt{26}$ $(x-5)^2 + (y-5)^2 = 26$	✓state DB is diameter/ *stel DB as midlyn* ✓ x ✓ y (midpt DB) ✓radius = $\sqrt{26}$ ✓LH✓RH / ✓*LK*✓*RK*	(6)
	4.1.4	$m(\text{radius}) = \frac{6-5}{0-5} = -\frac{1}{5}$ $m(\text{tangent}) = 5$ $y = mx + c$ $y = 5x + c$ $6 = 5(0) + c$ $c = 6$ $y = 5x + 6$	✓ m(radius) ✓ m(tangent/*raaklyn*) ✓subst m and point D/ *vervang m en punt D* ✓equation	(4)

		OR/*OF* $m(\text{radius}) = \frac{6-5}{0-5} = -\frac{1}{5}$ $m(\text{tangent}) = 5$ $y - y_1 = m(x - x_1)$ $y - 6 = 5(x - 0)$ $y - 6 = 5x$ $y = 5x + 6$	✓ m(radius) ✓ m(tangent/*raaklyn*) ✓subst m and point D/ *vervang m en punt D* ✓equation	(4)
	4.1.5	$m(\text{OE}) = \frac{0-5}{0-5} = 1$ $\tan \hat{\text{EOA}} = 1$ $\therefore \hat{\text{EOA}} = 45°$ and/*en* $\hat{\text{DOA}} = 90°$ $\therefore$ EO bisect/*halveer* $\hat{\text{DOA}}$	✓ m(OE) ✓tan def ✓ $\hat{\text{EOA}} = 45°$	(3)
4.2	4.2.1	$(x-1)^2 + (y+1)^2 = 2(x-y)$ $x^2 - 2x + 1 + y^2 + 2y + 1 = 2x - 2y$ $x^2 - 4x + y^2 + 4y = -2$ $x^2 - 4x + 4 + y^2 + 4y + 4 = -2 + 4 + 4$ $(x-2)^2 + (y+2)^2 = 6$ $\therefore$ Centre / *midpunt* $(2\ ;\ -2)$	✓simplify/*vereenvoudig* ✓completing of square/ *vierkantsvoltooiing* ✓std form/*std vorm* ✓ x and/*en* y	(4)
	4.2.2	Radius $= \sqrt{6}$	✓radius = $\sqrt{6}$	(1)
				[22]

QUESTION/*VRAAG* 5

5.1	5.1.1	$\cos(-12°)$ $=\cos 12°$ $=m$  	✓reduction/*reduksie* ✓answer/*antwoord*	(2)
	5.1.2	$\cos 72°$ $=\cos(60°+12°)$ $=\cos 60°\cos 12°-\sin 60°\sin 12°$ $=\left(\frac{1}{2}\right)(m)-\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{1-m^2}\right)$ $=\frac{m-\sqrt{3(1-m^2)}}{2}$	✓$60°+12°$ ✓expansion/*uitbrei* ✓substitution/*vervanging*	(3)
	5.1.3	$\cos 12° = 2\cos^2 6°-1$ $m = 2\cos^2 6°-1$ $m+1 = 2\cos^2 6°$ $\frac{m+1}{2} = \cos^2 6°$ $\cos 6° = \sqrt{\frac{m+1}{2}}$	✓double ∠ formula/ *dubbel ∠ formule* ✓equating to m/*gelykstel m* ✓$\frac{m+1}{2}$ ✓answer/*antwoord*	(4)
5.2		$\frac{\sin 234°}{\cos 36°}-\frac{\sin(x-90°)\cos(90°-2x)}{\sin x}=\cos 2x$ LHS $=\frac{\sin 234°}{\cos 36°}-\frac{\sin(x-90°)\cos(90°-2x)}{\sin x}$ $=\frac{-\sin 54°}{\sin 54°}-\frac{(-\cos x)(\sin 2x)}{\sin x}$ $=-1-\frac{(-\cos x)(2\sin x\cos x)}{\sin x}$ $=-1+2\cos^2 x$ $=2\cos^2 x-1$ $=\cos 2x$	✓$-\sin 54°$ ✓$\sin 54°$ ✓$-\cos x$ ✓$\sin 2x$ ✓$2\sin x\cos x$ ✓$-1+2\cos^2 x$	(6)
5.3	5.3.1	$\sin^2 B-\cos^2 B=1$ $\cos^2 B-\sin^2 B=-1$ $\cos 2B=-1$	✓-1	(1)
	5.3.2	$2B=180°-0°+k.360°$ or $2B=180°+0°+k.360°$ $B=90°+k.180°, k\in Z$ $B=90°+k.180°, k\in Z$ $\therefore \hat{B}=90°$	✓both equations/*beide vergelykings* ✓value/*waarde* $\hat{B}$	(2)

	5.3.3	$\sin(90°-C)-\cos(90°-C)=0$ $\sin(90°-C)=\cos(90°-C)$ $\cos C=\sin C$ $1=\frac{\sin C}{\cos C}$ $\tan C=1$ $\hat{C}=45°+k.180, k\in Z$ $\hat{C}=\{-135°\ ;\ 45°\}$ OR/*OF* $\sin(90°-C)-\cos(90°-C)=0$ $\sin(90°-C)=\cos(90°-C)$ $\cos C=\sin C$ $1=\frac{\sin C}{\cos C}$ $\tan C=1$ $\hat{C}=45°+k.360, k\in Z$ or/*of* $\hat{C}=180°+45°+k.360$ $\hat{C}=225°+k.360, k\in Z$ $\hat{C}=\{-135°\ ;\ 45°\}$	✓substitution/*vervanging* $\hat{B}$ ✓co-functions/*kofunksies* ✓ $\tan C=1$ ✓1 equation with $k.180°$ /1 *vergelyking* $k.180°$ ✓ $-135°$ ✓ $45°$ OR/*OF* ✓substitution/*vervanging* $\hat{B}$ ✓co-functions/*kofunksies* ✓ $\tan C=1$ ✓2 equations with $k.360°$ /*2 vergelykings* $k.360°$ ✓ $-135°$ ✓ $45°$	(6) (6)
5.4		$P=2\cos x-\cos 2x$ $=2\cos x-(2\cos^2 x-1)$ $=-2\cos^2 x+2\cos x+1$ $\cos x=-\frac{b}{2a}$ $=-\frac{2}{2(-2)}$ $=\frac{1}{2}$ $P=-2\cos^2 x+2\cos x+1$ $=-2\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)+1$ $=\frac{3}{2}$	✓double/*dubbel* ∠ ✓std form/*std vorm* ✓substitution/*vervanging* ✓ $\frac{1}{2}$ ✓answer/*antwoord*	(5)
				[29]

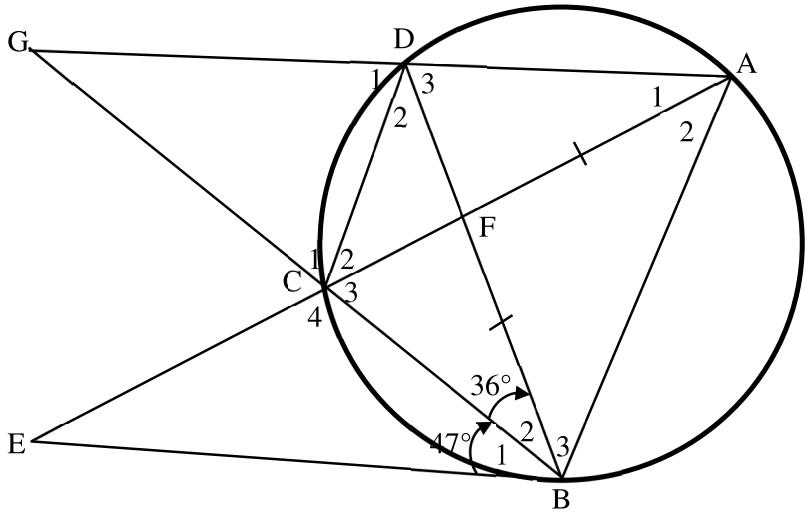
QUESTION/*VRAAG* 6

6.1	$a = 30°$ $b = 1$		✓value/*waarde a* ✓value/*waarde b*	(2)
6.2	$f(x) = 2\cos(0° + 30°)$ $f(x) = 2\cos 30°$ $f(x) = 2\left(\frac{\sqrt{3}}{2}\right)$ $f(x) = \sqrt{3}$		✓substitution/*vervanging* 0° ✓substitution/*vervanging* ✓answer/*antwoord*	(3)
6.3	6.3.1	$x = -135°$ and/*en* $x = 45°$	✓ $-135°$ ✓ $45°$	(2)
	6.3.2	$x \in [-90° ; 0°)$	✓critical values/*kritieke waardes* ✓notation/*notasie*	(2)
6.4	$f(x) = 2\cos(x + 30° - 30°) = 2\cos x$		✓ $-30°$ ✓answer/*antwoord*	(2)
				[11]

QUESTION/*VRAAG* 7

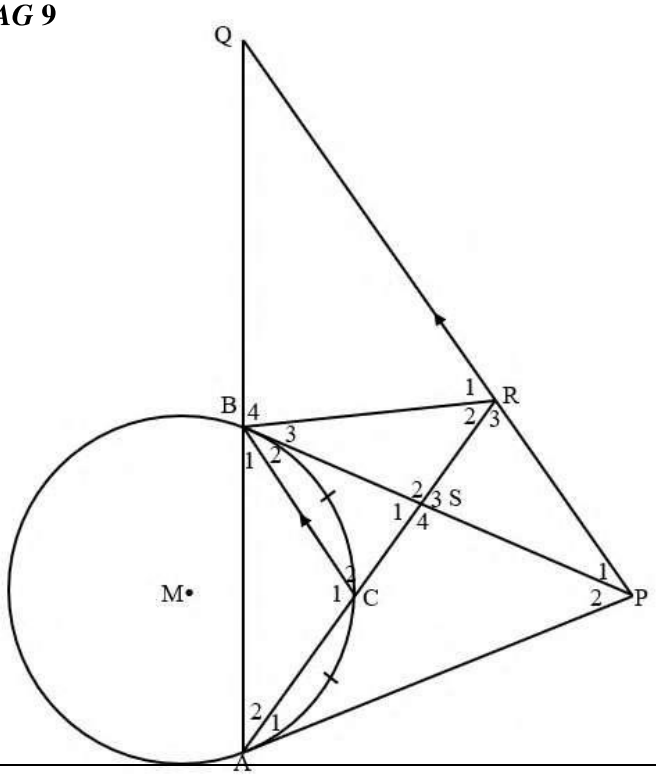
7.1	$B\hat{C}M = 90°$	✓ $90°$	(1)
7.2	$M\hat{A}C = 90° - \alpha$ $A\hat{M}C = 180° - (90° - \alpha) - (90° - \alpha)$ $A\hat{M}C = 180° - 180° + 2\alpha$ $A\hat{M}C = 2\alpha$	✓ $90° - \alpha$ ✓ method/*metode* ✓ 2α	(3)
7.3	In ΔDBC: $\tan\theta = \frac{h}{BC}$ $BC = \frac{h}{\tan\theta}$ In ΔABC: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$ $\frac{AC}{\sin(180° - 2\alpha)} = \frac{\frac{h}{\tan\theta}}{\sin\alpha}$ $AC = \frac{h}{\tan\theta} \times \frac{\sin 2\alpha}{1} \times \frac{1}{\sin\alpha}$ $AC = \frac{h.2\sin\alpha\cos\alpha}{\tan\theta\sin\alpha}$ $AC = \frac{2h\cos\alpha}{\tan\theta}$	✓ tan def ✓ $\frac{h}{\tan\theta}$ ✓ substitute in sine rule/ *vervang in sinreël* ✓ $\sin 2\alpha$ ✓ $2\sin\alpha\cos\alpha$	(5)
7.4	In ΔAMC: $\frac{AM}{\sin C} = \frac{AC}{\sin M}$ $\frac{r}{\sin(90° - \alpha)} = \frac{\frac{2h\cos\alpha}{\tan\theta}}{\sin 2\alpha}$ $\frac{r}{\cos\alpha} = \frac{2h\cos\alpha}{\tan\theta} \times \frac{1}{2\sin\alpha\cos\alpha}$ $r = \frac{2h\cos\alpha}{\tan\theta} \times \frac{1}{2\sin\alpha\cos\alpha} \times \frac{\cos\alpha}{1}$ $r = \frac{h\cos\alpha}{\tan\theta\sin\alpha}$ $r = \frac{h}{\tan\theta\tan\alpha}$	✓ subst is sine rule/ *vervang in sinreël* ✓ $\cos\alpha$ ✓ simplify/*vereenvoudig*	(3)
			[12]

QUESTION/*VRAAG* 8



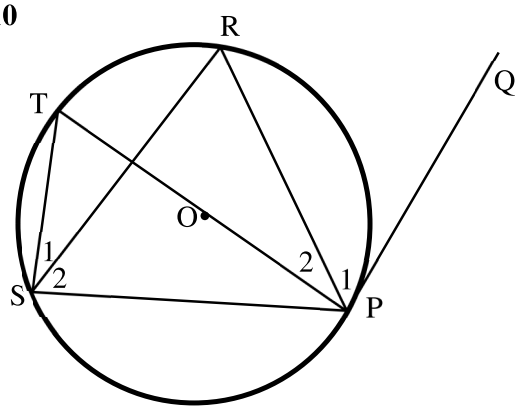
8.1	$\hat{B}_2 = \hat{A}_1 = 36°$	$\angle$'*s* in the same segment / $\angle$'*e in dies segment*	✓S ✓R	(2)
8.2	$\hat{B}_1 = \hat{A}_2 = 47°$	tan-chord theorem/*rklyn krd stelling*	✓S ✓R	(2)
8.3	$\hat{C}_1 = \hat{A}_1 + \hat{A}_2 = 83°$	ext $\angle$ of cyclic quad/*buite* $\angle$ *kvh*	✓S ✓R	(2)
8.4	$\hat{C}_2 = \hat{B}_3 = 47°$ $\hat{C}_1 + \hat{C}_2 = \hat{C}_4$ $\hat{C}_4 = 130°$	$\angle$'*s* in the same segment/ $\angle$'*e in dies segment* vert. opp $\angle$'*s* /*regoorst* $\angle$'*e*	✓S ✓S/R ✓S	(3)
				[9]

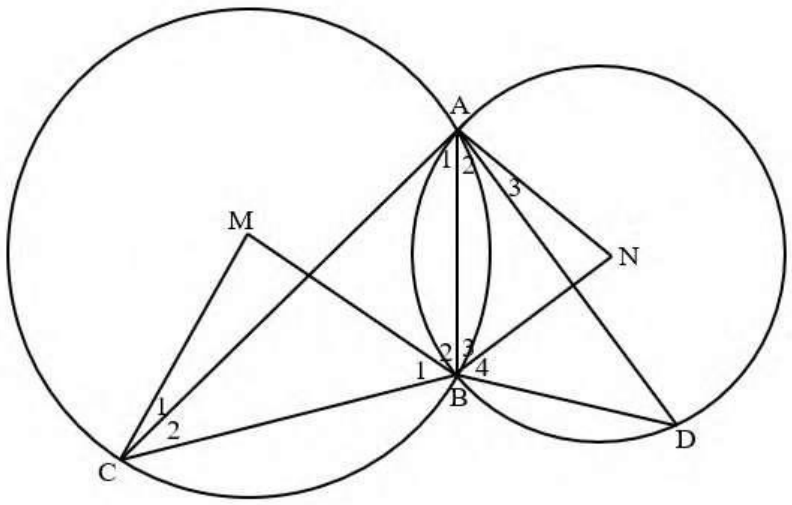
QUESTION/*VRAAG* 9



9.1	$\hat{A}_1 = \hat{B}_1 = x$ tan-chord theorem/*rklyn krd stelling* $\hat{B}_1 = \hat{A}_2 = x$ equal arcs/chords, equal $\angle$'*s /gelyke boë/koorde gelyke* $\angle$'*e* $\hat{A}_2 = \hat{B}_2 = x$ tan-chord theorem/*rklyn krd stelling* $\hat{B}_1 = Q = x$ corresp $\angle$'*s /ooreenk* $\angle$'*e* , BC||QP $\hat{B}_2 = \hat{P}_1 = x$ alt $\angle$'*s / verw* $\angle$'*e* , BC||QP	✓S/R ✓S/R ✓S/R ✓S/R ✓S/R	(5)
9.2	$\hat{A}_2 = \hat{P}_1 = x$ ABRP is a cyclic quadrilateral/*'n kvh* line subtend equal $\angle$'*s /lyn onderspan gelyke* $\angle$'*e* OR/*OF* **converse** $\angle$'*s* in same segment ***/omgekeerde*** $\angle$'*e in dies segment*	✓S ✓R	(2)
9.3	$\hat{A}_1 + \hat{A}_2 = \hat{B}_1 + \hat{B}_2 = 2x$ BP = AP sides opp equal $\angle$'*s / sye teenoor gelyke* $\angle$'*e* $\hat{Q} = \hat{P}_1 = x$ BQ = BP sides opp equal $\angle$'*s /sye teenoor gelyke* $\angle$'*e* $\therefore$ AP = BQ both equal to BP/*albei gelyk aan BP*	✓S ✓R ✓S	(3)
9.4	$\hat{R}_1 = \hat{A}_1 + \hat{A}_2 = 2x$ ext $\angle$ of cyclic quad/ *buite* $\angle$ *kvh* $\hat{C}_2 = \hat{B}_1 + \hat{A}_2 = 2x$ ext $\angle$ of Δ / *buite* $\angle$ Δ $\therefore \hat{R}_1 = \hat{C}_2$ PRQ is a tangent to the circle/ **converse** tan-chord theorem/ r*aaklyn aan die sirkel* ***omgekeerde*** *rklyn krd stelling*	✓S/R ✓S/R ✓R	(3)
			[13]

QUESTION/*VRAAG* 10



10.1	Draw diameter POT and join TS/*trek midlyn POT en verbind TS* $\hat{P}_1 + \hat{P}_2 = 90°$ radius ⊥ tangent/*raaklyn* $\hat{S}_1 + \hat{S}_2 = 90°$ ∠ in semi circle/*sirkel* $\hat{S}_1 = \hat{P}_2$ ∠'*s* in same segment/*dieselfde segment* $\therefore \hat{P}_1 = \hat{S}_2$	✓constr/*konstr* ✓S ✓R ✓S/R ✓S/R	(5)
10.2			
	10.2.1 In ΔABC and/*en* ΔDBA : (i) $\hat{A}_1 = \hat{D}$ tan-chord theorem/*rklyn koord stelling* (ii) $\hat{C}_2 = \hat{A}_2$ tan-chord theorem/*rklyn koord stelling* $\therefore$ ΔABC ||| ΔDBA 3 ∠'*s* **OR/*OF*** In ΔABC and/*en* ΔDBA : (i) $\hat{A}_1 = \hat{D}$ tan-chord theorem/*rklyn koord stelling* (ii) $\hat{C}_2 = \hat{A}_2$ tan-chord theorem/*rklyn koord stelling* (iii) $C\hat{B}A = A\hat{B}D$ 3rd ∠ $\therefore$ ΔABC ||| ΔDBA	✓S/R ✓S ✓R ✓S/R ✓S ✓S	(3) (3)

	10.2.2	$\frac{AB}{DB}=\frac{BC}{AB}$ $\quad\|\|\|\ \Delta's$ $\therefore AB^2 = DB.BC$	✓S ✓R	(2)
	10.2.3	In ΔCBM and/*en* ΔBAN: (i) $\hat{M}=2\hat{A}_1$ midpt $\angle$ = 2×circumf/*omtrek* $\angle$ $\hat{A}_1=\hat{D}$ proven/*bewys* $\therefore 2\hat{A}_1=2\hat{D}$ $\hat{N}=2\hat{D}$ midpt $\angle$ = 2×circumf/*omtrek* $\angle$ $\therefore \hat{M}=\hat{N}$ (ii) $2\hat{B}_1=180°-\hat{M}$ int $\angle's$ of Δ/ *binne* $\angle'e$ Δ $\therefore \hat{B}_1=90°-\frac{\hat{M}}{2}$ $2\hat{B}_3=180°-\hat{N}$ int $\angle's$ of Δ/ *binne* $\angle'e$ Δ $\therefore \hat{B}_3=90°-\frac{\hat{N}}{2}$ But/*maar* $\hat{M}=\hat{N}$ proven/*bewys* $\therefore \hat{B}_1=\hat{B}_3$ $\therefore \Delta CBM \|\|\| \Delta BAN$ 3 $\angle's$ **OR/OF** In ΔCBM and/*en* ΔBAN: (i) $\hat{M}=2\hat{A}_1$ midpt $\angle$ = 2×circumf/*omtrek* $\angle$ $\hat{A}_1=\hat{D}$ proven/*bewys* $\therefore 2\hat{A}_1=2\hat{D}$ $\hat{N}=2\hat{D}$ midpt $\angle$ = 2×circumf/*omtrek* $\angle$ $\therefore \hat{M}=\hat{N}$ (ii) $2\hat{B}_1=180°-\hat{M}$ int $\angle's$ of Δ/ *binne* $\angle'e$ Δ $\therefore \hat{B}_1=90°-\frac{\hat{M}}{2}$ $2\hat{B}_3=180°-\hat{N}$ int $\angle's$ of Δ/ *binne* $\angle'e$ Δ $\therefore \hat{B}_3=90°-\frac{\hat{N}}{2}$ But/*maar* $\hat{M}=\hat{N}$ proven/*bewys* $\therefore \hat{B}_1=\hat{B}_3$ (iii) $M\hat{C}B=B\hat{A}N$ 3rd $\angle$ $\therefore \Delta CBM \|\|\| \Delta BAN$	✓S/R ✓S ✓S/R ✓S ✓R ✓S/R ✓S ✓S/R ✓S ✓S	(5) (5)

	10.2.4	$\frac{CB}{BA}=\frac{BM}{AN}$ $\|\|\| \Delta's$ $\frac{CB}{BA}=\frac{R}{r}$ $\frac{CB^2}{BA^2}=\frac{R^2}{r^2}$ and $AB^2=DB.BC$ proven/*bewys* $\frac{CB^2}{DB.BC}=\frac{R^2}{r^2}$ $\frac{CB}{DB}=\frac{R^2}{r^2}$	✓S/R ✓squaring/*vierkant* ✓substitute/*vervang* $AB^2=DB.BC$	(3)
				[18]

TOTAL/*TOTAAL* : 150