Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2024

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper has 15 pages, a 2-page information sheet, and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the **instructions**. **Answer** the **questions**.

1. This question paper has **11 (ELEVEN) questions**.

2. **Answer ALL** the **questions**.
   Write in the **SPECIAL ANSWER BOOK**.

3. **Show ALL calculations**, **diagrams**, **graphs**, etc. that you used in your calculations.

4. **Answers only** will **NOT always** get **full marks**.

5. You **may use** a prescribed **calculator**.
   Some questions will **tell** you **NOT** to use a **calculator**.

6. **Round off** answers to **TWO decimal places**.
   Some questions will **tell** you **how** to **round off**.

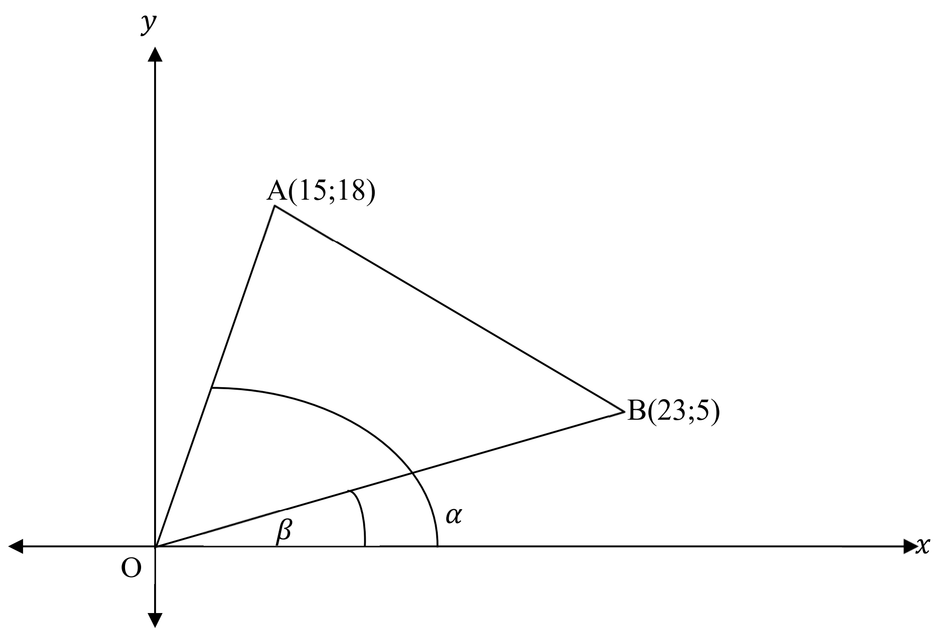
7. **Diagrams** are **NOT** always drawn to **scale**.

8. An **information sheet** with formulae is at the **end** of the **question paper**.

9. Write **neatly**.
   Your **answers** must be **easy** to **read**.

QUESTION 1

Diagram:
AOB is a **triangle** with **vertices** A(15 ; 18); O(0 ; 0) and B(23 ; 5).
β is the **angle of inclination** of **line OB**.
α is the **angle of inclination** of **line OA**.



1.1 **Determine** the **gradients** of **OA** and **OB.** (4)

1.2 **Determine** the **angle of inclination** of **line OB**. (3)

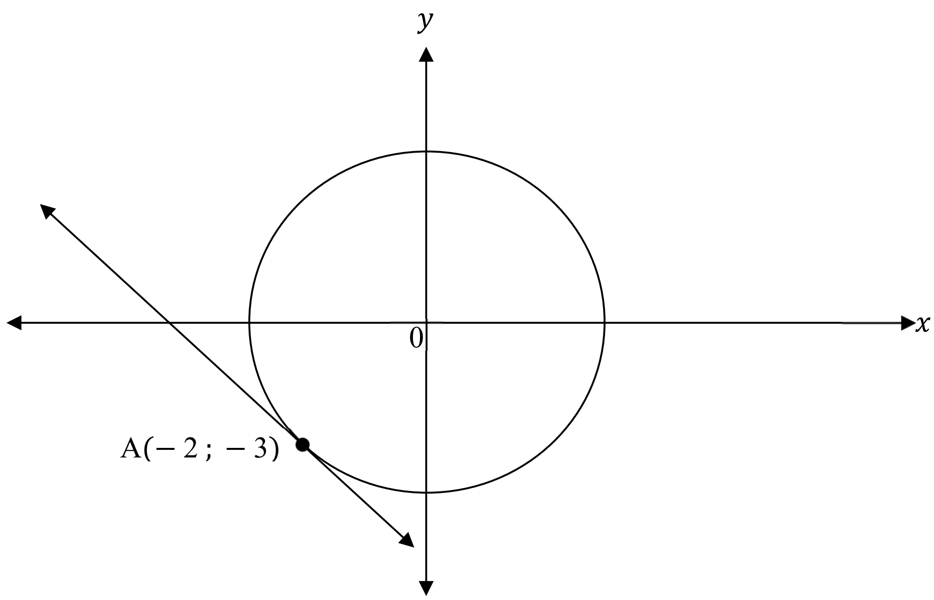
1.3 **Find** the **size** of $A\hat{O}B$, **correct** to the **nearest whole number**. (4)

1.4 AOBM is a **parallelogram**.
Find the **coordinates of M**. (5)

[16]

QUESTION 2

2.1 **Diagram:**
It shows the **circle with equation** $x^2 + y^2 = 13$.
The **contact point** of a **tangent** to the **circle** is at A$(-2\,;\,-3)$.



2.1.1 **Write** down the **radius** of the circle in **simplified surd form**. (1)

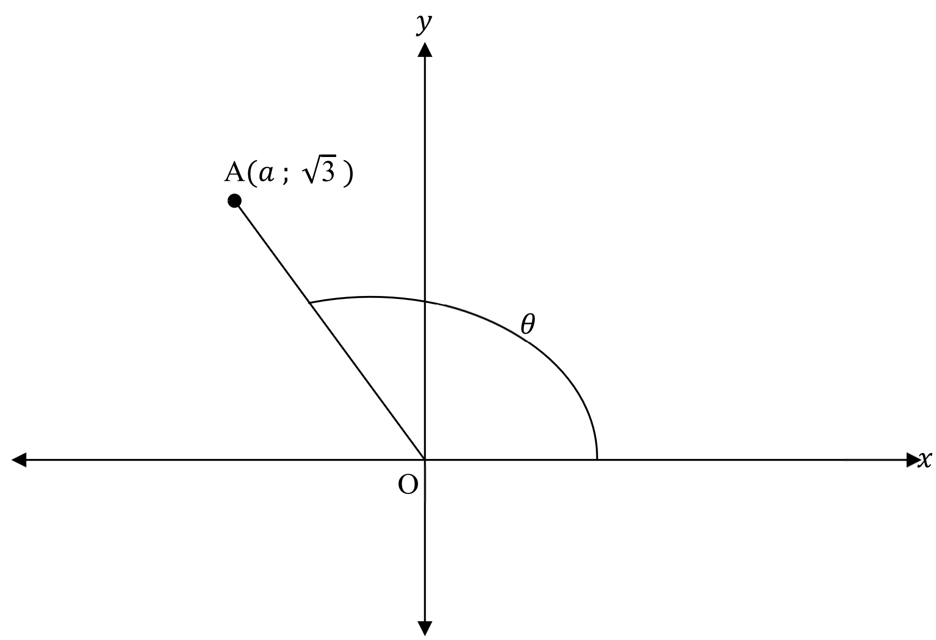
2.1.2 **Determine** the **equation** of the **tangent** to the **circle at point** A in the **form** $y = \ldots$ (4)

2.1.3 **Write** the **coordinates** of **another point where** the **line AO intersects** with the **circle**. (2)

2.2 **Draw** the **graph** of $\frac{x^2}{3} + \frac{y^2}{9} = 1$.
Show ALL the **intercepts.** (3)
[10]

QUESTION 3

3.1 **Diagram**:
A$(a\ ;\ \sqrt{3}\ )$ and OA = 3.



Do **NOT** use a **calculator**.
Determine the **value** of:

3.1.1 a (3)

3.1.2 $\sec\theta$ (1)

3.1.3 *cosec* $(\theta + 360°)$ (3)

3.2 **Determine** the **values** of x, if $\tan(x - 30°) = -0{,}982$ and $0° \leq x - 30° \leq 360°$. (4)

[11]

QUESTION 4

4.1 **Simplify**: $\dfrac{\sin(180°-\theta)\tan(180°+\theta)\sin(270°)}{\cos(360°-\theta)\tan(180°-\theta)}$ (6)

4.2 **Prove that**: $(cosec\ B-\cot B)^2=\dfrac{1+cos\ B}{1-\cos B}$ (6)

[12]

QUESTION 5

Given the **functions defined** by $f(x)=\cos(x-30)$ and $g(x)=2\sin x$ for $x\in(0°;360°)$.

5.1 **Write** down the **period** of f. (1)

5.2 **Write** down the **amplitude** of g. (1)

5.3 **On** the **same axes** given in your **SPECIAL ANSWER BOOK** draw the **graphs** of f and g.
Show the **turning points**, **endpoints**, and the **intercepts** with the **axes**. (8)

5.4 Use **graphs** to **determine** for which **values** of x is:

5.4.1 $g(x)\geq 0$ (2)

5.4.2 $f(x).g(x)<0$ in the **second quadrant** (2)

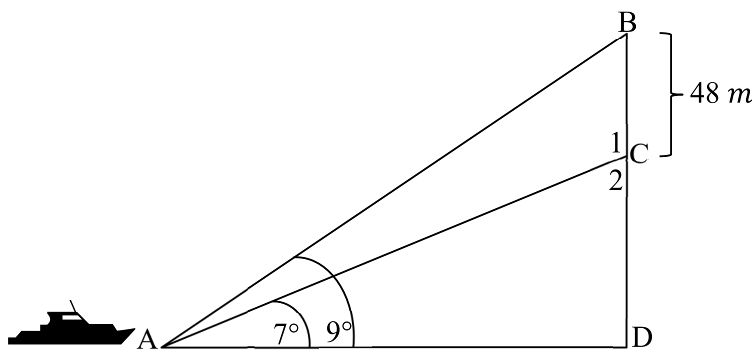
[14]

QUESTION 6

6.1 **Write** the **sine rule** for ΔABD. (1)

6.2 **Diagram:**

A s**hip** at sea, **observes** that the **angles of elevation** to the **top** and **bottom** of a **lighthouse** on a **cliff** are 7° and 9° respectively.
It is **known** that the **heigh**t of the **lighthouse** is 48 m.



Determine:

6.2.1 The **size** of $B\hat{A}C$. **Give** a **reason** (2)

6.2.2 The **size** of $A\hat{B}D$.**Give** a **reason** (2)

6.2.3 The **length** of AC (4)

6.2.4 The **distance** between the **ship** and the **bottom** of the **cliff** (2)

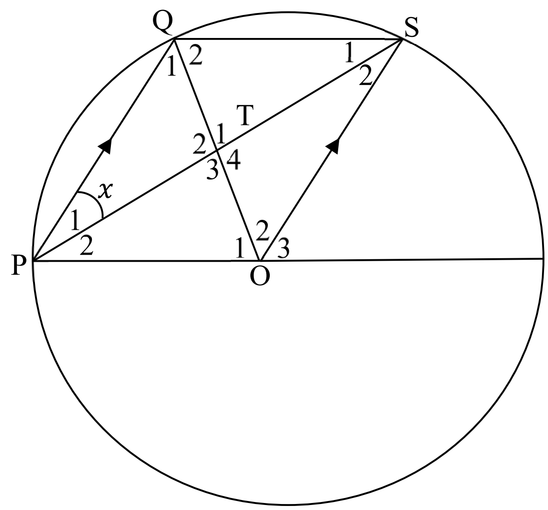
6.2.5 The **height** of the **cliff** (3)

[14]

QUESTION 7

Diagram:

O is the c**entre** of the **circle.**
OS ∥ PQ and **PS** meet **OQ** at **T**.



7.1 If $P_1 = x$, **express** T_1 in **terms of** $\boldsymbol{x}$.
Give reasons. (6)

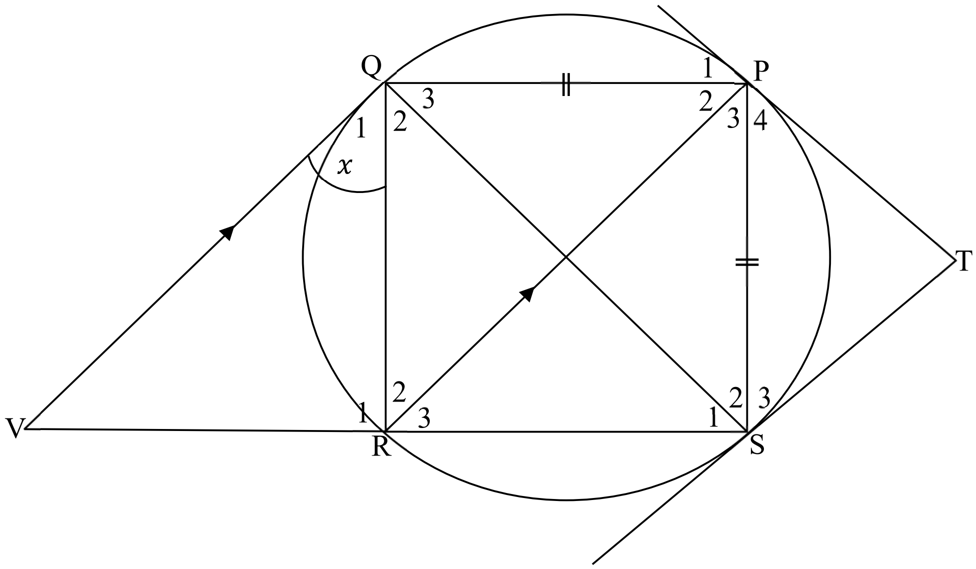
7.2 If $x = 30°$, **calculate** the **sizes** of the **angles** in ΔQST.
Give reasons where necessary. (5)

7.3 **Show** that ΔPQS ≡ ΔSOP. (3)
[14]

QUESTION 8

Diagram:

PQRS is a **cyclic quadrilateral** with **PS = PQ**.
SR is **produced** to meet **V** such that PR ∥ QV.
TP and **TS** are **tangents** to the **circle**. $Q_1 = x$.



8.1 **Name**, with **reasons**, **four other angles equal** to x. (8)

8.2 **Give** a **reason** for $P_4 = S_3$. (1)

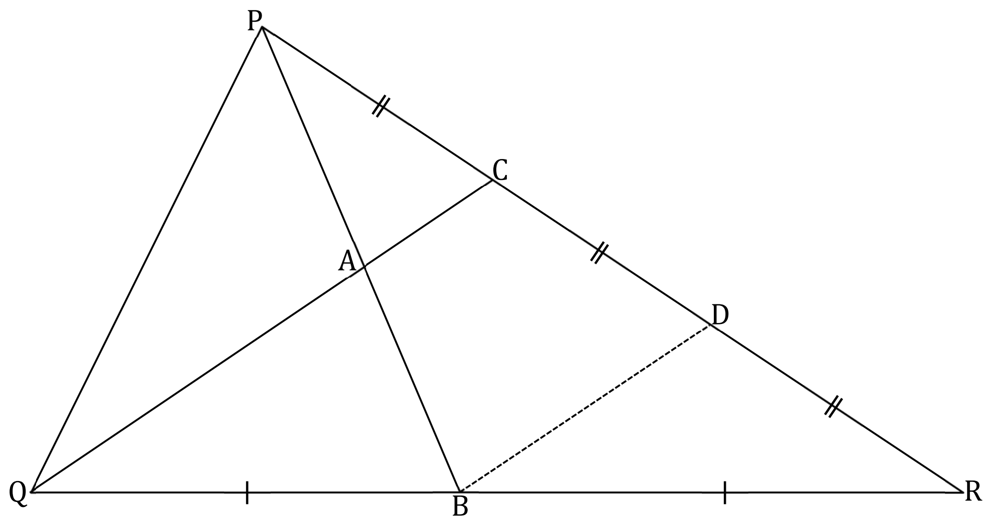
8.3 **Prove, with reasons**, that $\hat{T} = Q\hat{P}S$. (5)

[14]

QUESTION 9

Diagram:

B is the **midpoin**t of **side QR**.
C and D are **points** on **PR** such **that PC = CD = DR**.
PR = 15 cm.



9.1 **Show** that BD ∥ QC. (3)

9.2 **Prove** that PA = AB. (3)

9.3 **Determine** the **length** of **QR**, if PD : DR = 2 : 1. (6)

[12]

QUESTION 10

A **fan** in a **jet engine** has a **diameter** of **340 cm** and a **circumferential velocity** of **568 metres per second**.

10.1	**Convert** 568 m/s to km/h.	(2)
10.2	**Determine** the **rotational frequency** of the **wheel** in **hours**.	(5)
10.3	**Determine** the **angular velocity$_{\text{(speed)}}$** of the **wheel** in **seconds**.	(3)
10.4	**Determine** the **distance**, **in km**, a **point on** the **fan** will **cover** in **15 seconds**.	(3)
10.5	**Determine how long it** will take the **fan** to **make half a revolution**.	(2)
		[15]

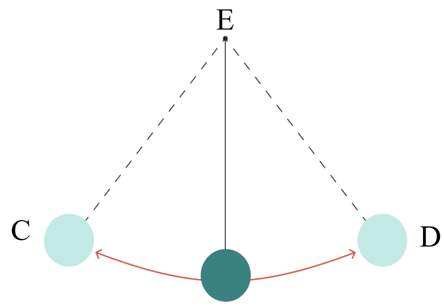
QUESTION 11

11.1 **Diagram:**

A **pendulum** in a **clock**, **FIGURE A**, **follows** the **path** as **depicted(shown)** in the **diagram**, **FIGURE B**.
There is a **radius** of **30 cm** and the **angle formed** is **60°.**

FIGURE A

FIGURE B



11.1.1 **Determine** the **length** of arc **CD**, that the **pendulum follows**. (3)

11.1.2 **Determine** the **area** of **sector ECD**. (3)

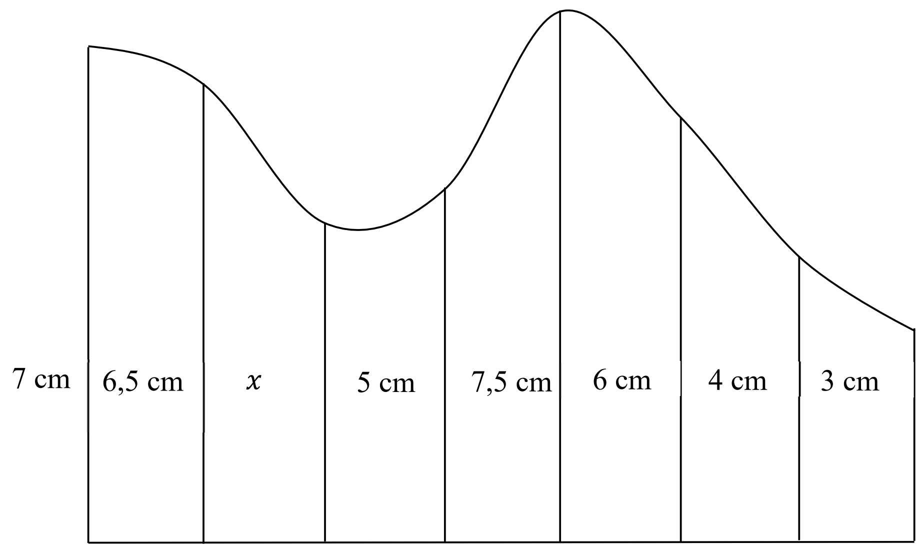
11.1.3 **Calculate** the **length** of the **pendulum**. (3)

11.2 An **analogue clock** has a **diameter** of **30 cm**, and a **chord length** of **20 cm**.

Determine the **length** of the **hour hand**. (5)

11.3 **Diagram:**

The **ordinates** in the **irregular figure** are: 7 cm, 6,5 cm, x , 5 cm; 7,5 cm, 6 cm, 4 cm and 3 cm respectively as i**ndicated(shown).**
The **width** of the **irregular figure** is 11,55 cm and the **area** is 63,525 cm^2.



Determine the **length** of the **unknown ordinate** x. (4)
[18]

TOTAL: 150

INFORMATION SHEET

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\frac{b}{2a}$ $\qquad y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1 \text{ and } b > 0$

$A = P(1+ni)$ $\qquad A = P(1-ni)$ $\qquad A = P(1+i)^n$ $\qquad A = P(1-i)^n$

$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \ n \neq -1$ $\qquad \int kx^n dx = k \cdot \frac{x^{n+1}}{n+1} + C \quad , \ n \neq -1$

$\int \frac{1}{x} dx = \ln(x) + C, \quad x > 0$ $\qquad \int \frac{k}{x} dx = k \cdot \ln(x) + C, \quad x > 0$

$\int a^x dx = \frac{a^x}{\ln a} + C \quad , \ a > 0$ $\qquad \int ka^{nx} dx = k \cdot \frac{a^{nx}}{n \ln a} + C \quad , \ a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad m = \tan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In ΔABC:

$\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$ $\qquad a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$ $\qquad \text{Area} = \frac{1}{2} ab.\sin \mathrm{C}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad \tan^2\theta + 1 = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360°n$ where n = rotation frequency

Circumferential velocity = $v = \pi Dn$ where D = diameter and n = rotation frequency

Circumferential velocity = $v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius and s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of the circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $o_i = i^{th}$ ordinate and
n = number of ordinates