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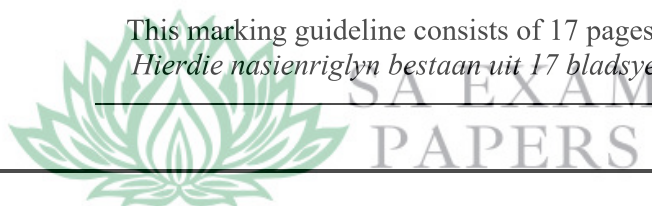
GRADE/*GRAAD* 12

JUNE/JUNIE 2024

**TECHNICAL MATHEMATICS P2/
TEGNIESE WISKUNDE V2
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.



NOTE:

- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

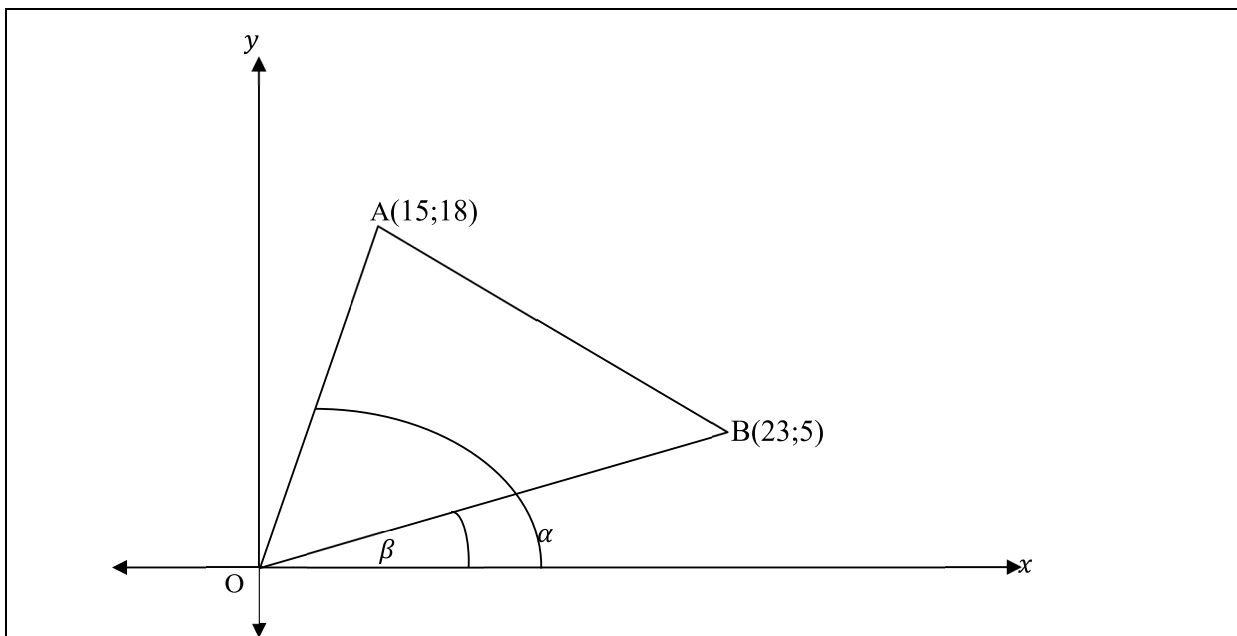
LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method/ <i>Metode</i>
A	Accuracy/ <i>Akkuraatheid</i>
AO	Answer only/ <i>Slegs antwoord</i>
CA	Consistent accuracy/ <i>Deurlopende akkuraatheid</i>
F	Formula/ <i>Formule</i>
I	Identity/ <i>Identiteit</i>
R	Rounding/ <i>Afronding</i>
S	Simplification/ <i>Vereenvoudiging</i>
ST	Statement/ <i>Bewering</i>
RE	Reason/ <i>Rede</i>
ST RE	Statement and correct reason/ <i>Bewering en korrekte rede</i>
SF	Substitution correctly in correct formula/ <i>Korrekte vervanging in die korrekte formule</i>
NPU	No penalty for omitting units/ <i>Geen penalisering vir eenhede uitgelaat</i>



QUESTION/VRAAG 1



1.1	$m_{OA} = \frac{18-0}{15-0}$ $m_{OA} = \frac{6}{5}$ $m_{OB} = \frac{5-0}{23-0}$ $m_{OB} = \frac{5}{23}$	✓ M ✓ S A ✓ M ✓ S A AO: Full marks / Volpunte	(4)
1.2	$\tan \beta = m_{OB}$ $\tan \beta = \frac{5}{23}$ $\therefore \beta = 12,26^\circ$	✓ M ✓ Subst./Vervang A ✓ S CA	(3)
1.3	$\tan \alpha = m_{OB}$ $\tan \alpha = \frac{6}{5}$ $\therefore \beta = 50,19^\circ$ $\therefore \hat{A}OB = 50,19^\circ - 12,26^\circ = 37,93^\circ \approx 38^\circ$	✓ M ✓ Subst. A ✓ S CA ✓ Answer/Antwoord CA	(4)

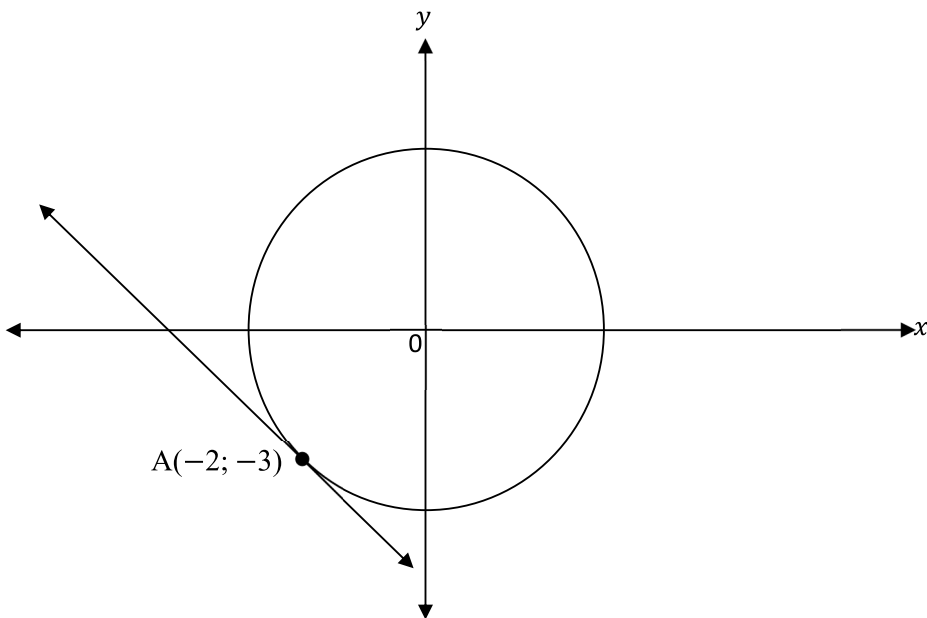


1.4	$M_{OM} = M_{AB}$ $\left(\frac{x_O + x_M}{2}, \frac{y_O + y_M}{2}\right) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$ $\left(\frac{0 + x_M}{2}, \frac{0 + y_M}{2}\right) = \left(\frac{15 + 23}{2}, \frac{18 + 5}{2}\right)$ $\therefore \frac{x_M}{2} = \frac{38}{2} \text{ and/en}$ $\frac{y_M}{2} = \frac{23}{2}$ $\therefore x_M = 38 \text{ and/en}$ $y_M = 23$ $\therefore M(38; 23)$	<p>x-coordinate of /<i>koördinate van</i> M = $23 + 15 = 38$</p> <p>y-coordinate of /<i>koördinate</i> M = $23 + 15 = 38$ $M(38; 23)$</p> <p>(If learner used this method award full marks/Indien leerling hierdie metode gebruik ken volpunte toe.)</p>	<p>✓ M</p> <p>✓ Subst. A</p> <p>✓ M CA</p> <p>✓ S CA</p> <p>✓ Answer/<i>Antwoord</i> CA</p> <p style="text-align: right;">(5)</p>
		[16]	



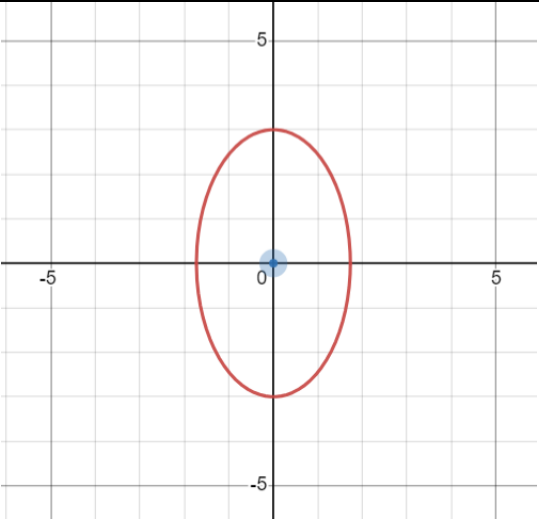
QUESTION/VRAAG 2

2.1



2.1.1	$r = \sqrt{13}$	✓ A (1)
2.1.2	$x \cdot x_1 + y \cdot y_1 = r^2$ $x(-2) + y(-3) = 13$ $-3y = 2x + 13$ $y = -\frac{2}{3}x - \frac{13}{3}$ <p style="text-align: center;">OR/OF</p> $m_{radius} = \frac{3}{2}$ $\therefore m_{tangent/raaklyn} = \frac{-2}{3}$ $y - y_1 = m(x - x_1)$ $y - (-3) = \frac{-2}{3}(x - (-2))$ $y + 3 = \frac{-2}{3}(x + 2)$ $\therefore y = -\frac{2}{3}x - \frac{13}{3}$	✓ F ✓ SF A ✓ S ✓ equation / vergelyking CA <p style="text-align: center;">OR/OF</p> ✓ grad. radius A ✓ grad. tan / raaklyn CA ✓ SF A ✓ equation / vergelyking CA (4)
2.1.3	(3 ; 2)	✓ x-value/waarde A ✓ y-value/waarde A (2)

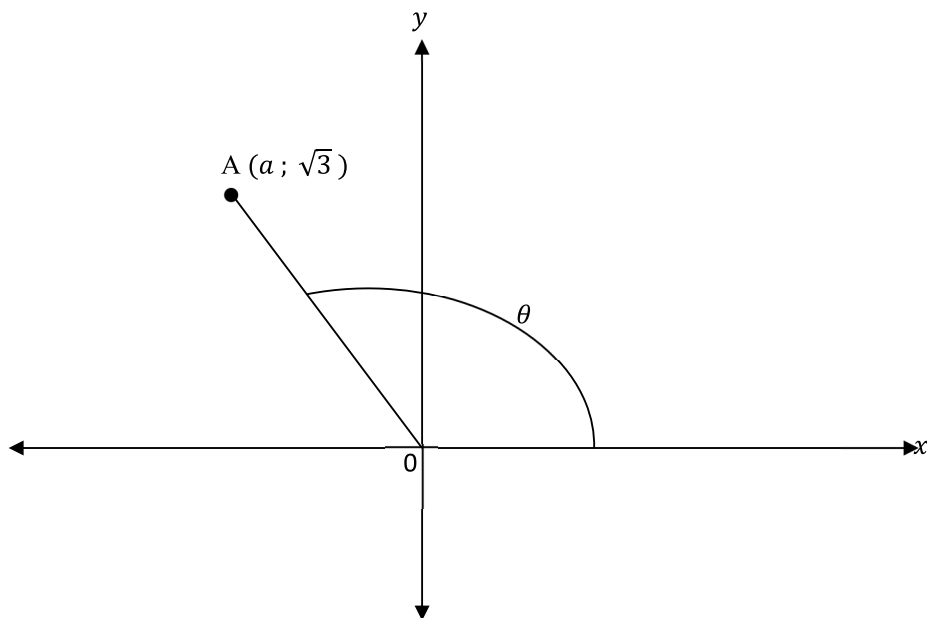


2.2		<ul style="list-style-type: none"> ✓ elliptical shape / elliptiese vorm A ✓ x-intercepts / afsnitte A ✓ y-intercepts / afsnitte A
		(3)
		[10]



QUESTION/VRAAG 3

3.1



3.1.1	$a^2 + (\sqrt{3})^2 = (3)^2$ $a^2 = 9 - 3$ $a^2 = 6$ $a = \pm\sqrt{6}$ $\therefore a = -\sqrt{6}$	✓ M ✓ S ✓ value of / waarde van a (3)
3.1.2	$\frac{-\sqrt{6}}{3}$	✓ Answer/Antwoord (1)
3.1.3	$\text{cosec}(\theta + 360^\circ) = \text{cosec} \theta$ $\therefore \text{cosec} \theta = \frac{3}{\sqrt{3}}$	✓ Reduction/ Reduksie A ✓✓ cosec ratio / verh. CA (3)
3.2	$\tan(x - 30^\circ) = -0,982$ Ref. / Verw. $\leq \tan^{-1}(0,982)$ Ref. / Verw. $\leq 44,48^\circ$ $\therefore \text{I: } x - 30^\circ = 180^\circ - 44,48^\circ$ $\therefore x = 165,52^\circ$ <p style="text-align: center;">AND/EN</p> $\therefore \text{IV: } x - 30^\circ = 360^\circ - 44,48^\circ$ $\therefore x = 345,52^\circ$	✓ Ref. / Verw. \angle CA ✓ Quadrants / kwadrante A ✓✓ values of x / waardes van x CA (4)
		[11]



QUESTION/VRAAG 4

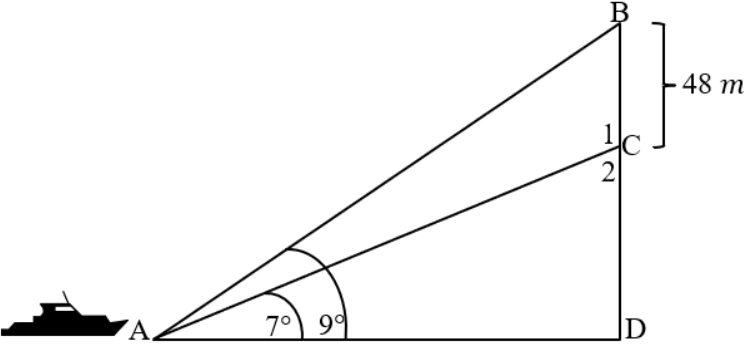
4.1	$\frac{\sin(180^\circ - \theta) \tan(180^\circ + \theta) \sin(270^\circ)}{\cos(360^\circ - \theta) \tan(180^\circ - \theta)}$ $= \frac{\sin(\theta) \cdot \tan(\theta) \cdot (-1)}{\cos(\theta) \cdot -\tan(\theta)}$ $= -\tan\theta$	$\checkmark \sin(\theta)$ A $\checkmark \tan(\theta)$ A $\checkmark -1$ A $\checkmark \cos(\theta)$ A $\checkmark -\tan(\theta)$ A $\checkmark -\tan(\theta)$ A (6)
4.2	$(\operatorname{cosec} B - \cot B)^2 = \frac{1 - \cos B}{1 + \cos B}$ $LHS = (\operatorname{cosec} B - \cot B)^2$ $LHS = \left(\frac{1}{\cos B} - \frac{1}{\tan B} \right)^2$ $LHS = \left(\frac{1}{\sin B} - \frac{\cos B}{\sin B} \right)^2$ $LHS = \left(\frac{1 - \cos B}{\sin B} \right)^2$ $LHS = \frac{(1 - \cos B)^2}{\sin^2 B}$ $LHS = \frac{(1 - \cos B)^2}{1 - \cos^2 B}$ $LHS = \frac{(1 - \cos B)^2}{(1 - \cos B)(1 + \cos B)}$ $LHS = \frac{1 - \cos B}{1 + \cos B} = \text{RHS}$	$\checkmark \frac{1}{\cos B}$ A $\checkmark \frac{1}{\tan B}$ A $\checkmark \frac{\cos B}{\sin B}$ A $\checkmark \frac{1 - \cos B}{\sin B}$ S $\checkmark \frac{(1 - \cos B)^2}{1 - \cos^2 B}$ S $\checkmark (1 - \cos B)(1 + \cos B)$ S (6)
		[12]



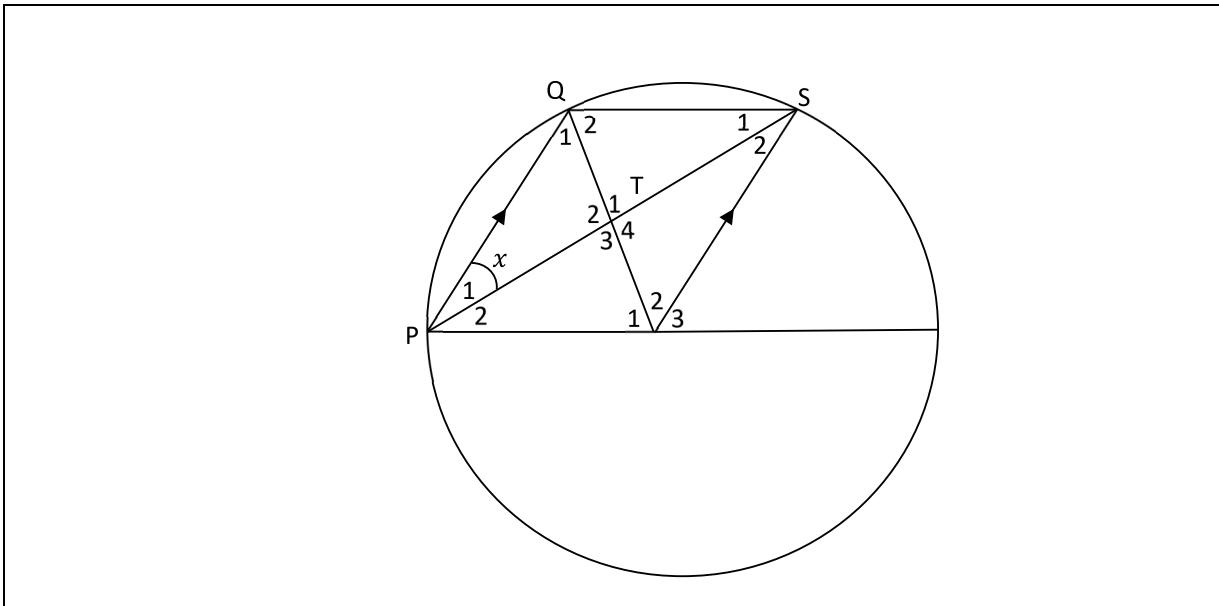
QUESTION/VRAAG 5

	$f(x) = \cos(x - 30)$ and /en $g(x) = 2 \sin x$ for/ vir $x \in (0^\circ; 360^\circ)$	
5.1	$Period_f = 360^\circ$	✓ A (1)
5.2	$Amplitude_g = 2$	✓ A (1)
5.3		<p>f:</p> <ul style="list-style-type: none"> ✓ y-intercept at / y-afsnit by 0,87 ✓ x-intercepts at 120° and 300° x-afsnitte by 120° en 300° ✓✓ turning point at / draaipunt by $(30^\circ; 1)$ and/en $(210^\circ; -1)$ <p>g:</p> <ul style="list-style-type: none"> ✓ y-intercept at / y-afsnit by 0 ✓ x-intercept at 180° and 360° / x-afsnit by 180° en 360° ✓✓ turning point at / draaipunt by $(90^\circ; 1)$ and/en $(270^\circ; -1)$
5.4.1	$0^\circ \leq x \leq 180^\circ$	<ul style="list-style-type: none"> ✓ $0^\circ \leq x$ CA ✓ $x \leq 180^\circ$ CA (2)
5.4.2	$120^\circ < x < 180^\circ$ and/en $300^\circ < x < 360^\circ$	<ul style="list-style-type: none"> ✓ $120^\circ < x < 180^\circ$ CA ✓ $300^\circ < x < 360^\circ$ CA (2)
		[14]

QUESTION/VRAAG 6

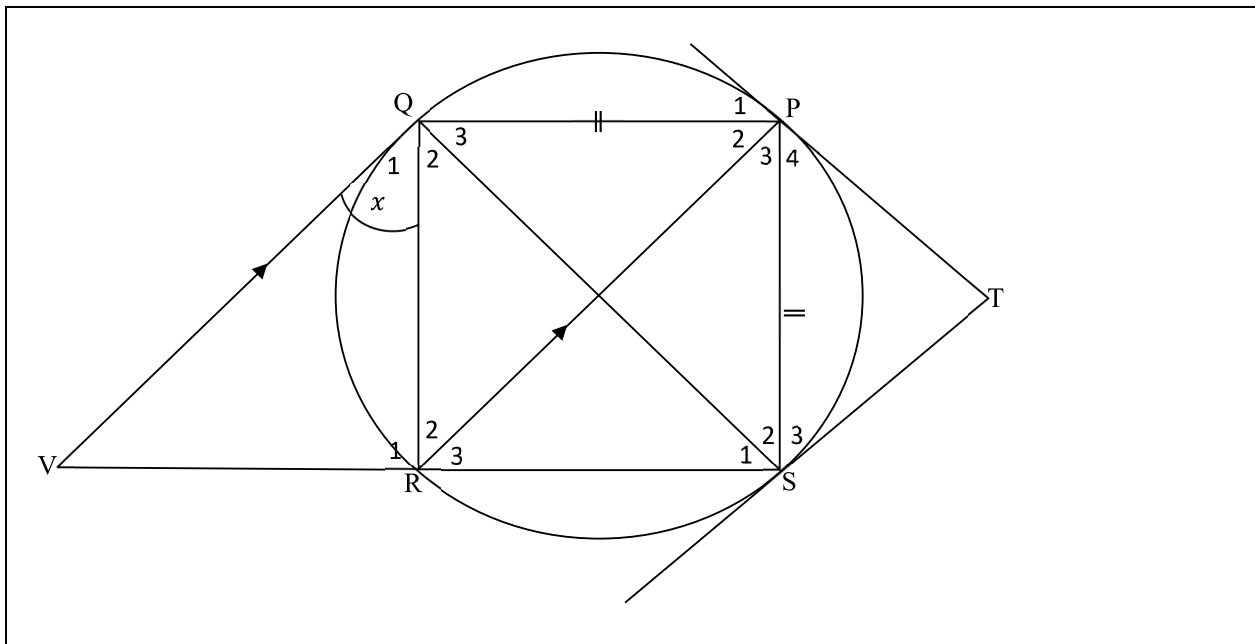
6.1	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{d}{\sin D}$	✓ A (1)
6.2		
6.2.1	$B\hat{A}C = 9^\circ - 7^\circ = 2^\circ$ (given/gegee)	✓ ST ✓ RE A (2)
6.2.2	$A\hat{B}D = 180^\circ - 9^\circ - 90^\circ = 81^\circ$ (int. <'s of Δ / binne <'e van Δ)	✓ ST ✓ RE A (2)
6.2.3	$\frac{AC}{\sin B} = \frac{BC}{\sin B\hat{A}C}$ $\frac{AC}{\sin 81^\circ} = \frac{48}{\sin 2^\circ}$ $AC = \frac{48}{\sin 2^\circ} \times \sin 81^\circ$ $AC = 1358,44 \text{ m}$	✓ F ✓ SF ✓ S ✓ AC value / waarde A CA (4)
6.2.4	$\cos C\hat{A}D = \frac{AD}{AC}$ $\cos 7^\circ = \frac{AD}{1358,44}$ $\cos 7^\circ \times 1358,44 = AD$ $1348,31 \text{ m} = AD$	✓ ratio / verh. ✓ value / waarde A CA (2)
6.2.5	$AC^2 = AD^2 + CD^2$ $(1358,44)^2 = (1348,31)^2 + CD^2$ $27419,3775 = CD^2$ $165,59 \text{ m} = CD$	✓ F ✓ SF ✓ height / hoogte CA (3)
		[14]

QUESTION/VRAAG 7



7.1	$\hat{S}_2 = x$ (alt. \angle 's = / verw. \angle 'e =; $PQ \parallel SO$) $\hat{O}_2 = 2x$ (\angle at centre = $2 \times \angle$ at circumf. / midpts \angle = $2 \times$ omtreks \angle) $\hat{T}_1 = 3x$ (ext. \angle of Δ / buite \angle van Δ)	✓ ST ✓ RE A ✓ ST ✓ RE A ✓ ST ✓ RE A (6)
7.2	$\hat{T}_1 = 3x$ $\hat{T}_1 = 3(30^\circ) = 90^\circ$ $\hat{O}_2 = 2x$ $\hat{O}_2 = 2(30^\circ) = 60^\circ$ $\therefore \hat{Q}_2 = 60^\circ$ ($OQ = OS$; Radii) $\therefore \hat{S}_1 = 30^\circ$	✓ \hat{T}_1 CA ✓ \hat{O}_2 CA ✓ \hat{Q}_2 ✓ RE CA ✓ \hat{S}_1 CA (5)
7.3	In ΔPQS and/en ΔSOP : 1. $\hat{S}_2 = \hat{S}_1$ (proven / bewys) 2. $QS = PO$ (given / gegee) 3. $\hat{P}_1 = \hat{P}_2$ (given / gegee) $\therefore \Delta PQS \equiv \Delta SOP$ ($\ll S$)	✓ ST A ✓ ST A ✓ ST A (3)
		[14]

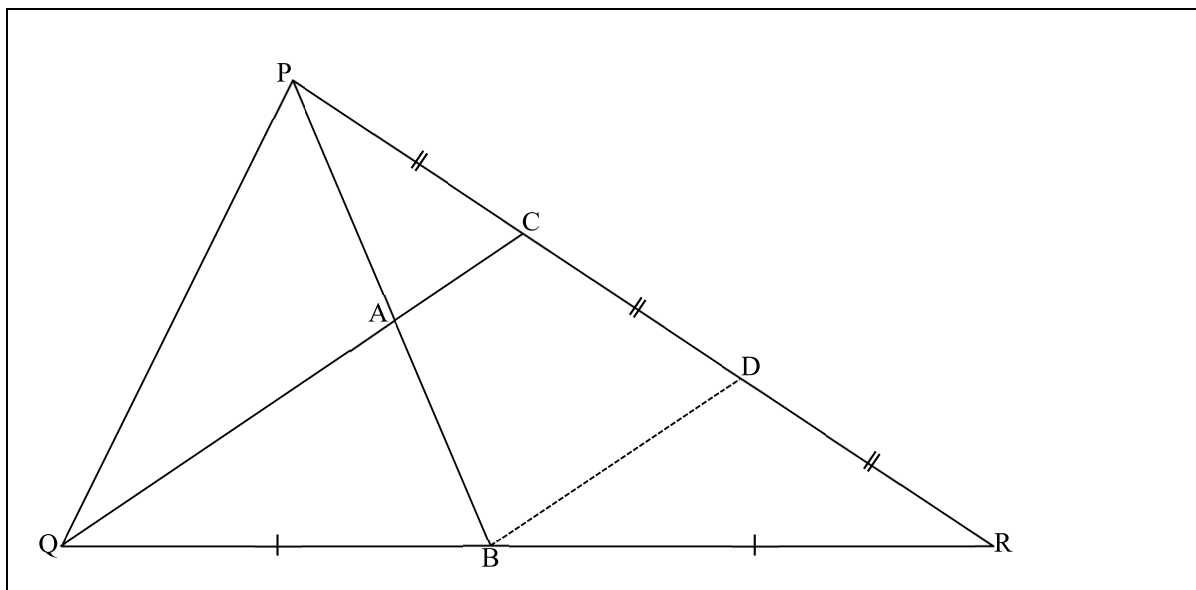
QUESTION/VRAAG 8



8.1	$\hat{S}_1 = x$ (tan-chord thm / <i>stelling</i>) $\hat{P}_2 = x$ (\sphericalangle 's in same segment / \sphericalangle 'e in dieselfde segment) $\hat{R}_2 = x$ (alt. \sphericalangle 's = / <i>verw.</i> \sphericalangle 'e =; $VQ \parallel RP$) $\hat{S}_2 = x$ (\sphericalangle 's in same segment / \sphericalangle 'e in dieselfde segment)	✓ ST ✓ RE A ✓ ST ✓ RE A ✓ ST ✓ RE A ✓ ST ✓ RE A	(8)
8.2	Tangents from same point / <i>Raaklyne vanuit dieselfde punt</i>	✓ RE	A (1)
8.3	$\hat{P}_4 = \hat{R}_3 = x$ (tan-chord thm / <i>stelling</i>) $\therefore \hat{T} = 180^\circ - 2x$ (int. \sphericalangle 's of Δ / <i>binne</i> \sphericalangle 'e van Δ) $\hat{P}_2 + \hat{P}_3 = 180^\circ - 2x$ (int. \sphericalangle 's of Δ / <i>binne</i> \sphericalangle 'e van Δ) $\therefore \hat{T} = Q\hat{P}S$	✓ ST ✓ RE A ✓ ST ✓ RE A ✓ ST A	(5)
			[14]



QUESTION/VRAAG 9



9.1	In $\triangle CRQ$: $RD = DC$ (given / <i>gegee</i>) $RB = BQ$ (given / <i>gegee</i>) $\therefore BD \parallel CQ$ and/en $BD = \frac{1}{2} CQ$ (midpt thm / <i>stelling</i>)	✓ ST ✓ ST ✓ RE	A A A	(3)
9.2	In $\triangle PBD$: $PC = CD$ (given / <i>gegee</i>) $BD \parallel CQ$ (given / <i>gegee</i>) $\therefore PA = AB$ and/en $AC = \frac{1}{2} BD$ (midpt thm / <i>stelling</i>)	✓ ST ✓ ST ✓ RE	A A A	(3)
9.3	$\triangle PAC$ and/en $\triangle PBD$: 1. $\hat{P} = \hat{P}$ (common \angle / <i>gemene \angle</i>) 2. $\hat{PAC} = \hat{PBD}$ (Corresp. \angle 's = / <i>ooreenk. \angle'e =</i> ; $AC \parallel BD$) 3. $\hat{PCA} = \hat{PDB}$ (3^{rd} \angle of \triangle / 3^{de} \angle van \triangle) $\therefore \triangle PAC \parallel \triangle PBD$ (\lll) $\therefore \frac{PA}{PB} = \frac{AC}{BD} = \frac{PC}{PD}$ $\therefore \frac{10}{20} = \frac{AC}{BD} = \frac{5}{10}$ $\therefore \frac{AC}{BD} = \frac{1}{2}$ $\therefore BD : AC = 2 : 1$	✓ ST ✓ ST ✓ ST ✓ ST	A A A A	✓ Subst./ <i>Vervang</i> A ✓ Answer/ <i>Antwoord</i> A (6)
				[12]


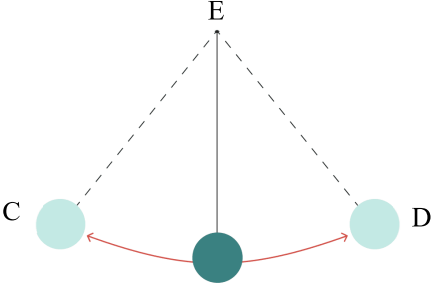


QUESTION/VRAAG 10


10.1	$568 \text{ m/s} = \frac{568 \text{ m}}{1 \text{ sec}} \times \frac{1 \text{ km}}{1\,000 \text{ m}} \times \frac{3600 \text{ sec}}{1 \text{ h}} = 2\,044,8 \text{ km/h}$	<ul style="list-style-type: none"> ✓ conversion factors / herleidingsfaktore ✓ answer / antwoord (2)
10.2	$v = \pi D n$ $2\,044,8 \text{ km/h} = \pi(0,0034 \text{ km})n$ $\therefore n = \frac{2\,044,8}{0,0034\pi}$ $\therefore n \approx 601411,76 \text{ rev/h}$	<ul style="list-style-type: none"> ✓ F ✓ conversion / herleiding ✓ SF A ✓ S ✓ answer / antwoord (5)
10.3	$\omega = 2\pi n$ $= 2\pi \times 38,20$ $\approx 240,02 \text{ rad/s}$	<ul style="list-style-type: none"> ✓ F ✓ SF CA ✓ answer / antwoord (3)
10.4	$s = vt$ OR / OF $D = ST$ $s = 0,568 \times 15$ $s = 8,52 \text{ km}$	<ul style="list-style-type: none"> ✓ F ✓ SF CA ✓ answer / antwoord (3)
10.5	$n = \frac{\text{number of revolutions}}{\text{time}} / \frac{\text{aantal revolusies}}{\text{tyd}}$ $53,18 = \frac{1/2}{t}$ $\therefore t \approx 0,009 \text{ sec/sek}$	<ul style="list-style-type: none"> ✓ SF CA ✓ answer / antwoord (2)
		[15]



QUESTION/VRAAG 11

	<p style="text-align: center;">FIGURE/FIGUUR A</p> 	<p style="text-align: center;">FIGURE /FIGUUR B</p> 
<p>11.1.1</p>	<p>$s = r\theta$</p> <p>$s = (30) \left(60^\circ \times \frac{\pi}{180^\circ} \right)$</p> <p>$\therefore CD = 10\pi \approx 31,42 \text{ cm}$</p>	<p>✓F</p> <p>✓ SF A</p> <p>✓ RT length / lengte</p> <p style="text-align: right;">(3)</p>
<p>11.1.2</p>	<p>$Area = \frac{rs}{2}$</p> <p>$Area = \frac{(30)(10\pi)}{2}$</p> <p>$Area = 150\pi \approx 471,24 \text{ cm}^2$</p> <p style="text-align: center;">OR / OF</p> <p>$Area = \frac{r^2\theta}{2}$</p> <p>$Area = \frac{(30)^2 \left(60^\circ \times \frac{\pi}{180^\circ} \right)}{2}$</p> <p>$Area = 150\pi \approx 471,24 \text{ cm}^2$</p>	<p>✓F</p> <p>✓ SF A</p> <p>✓ Area</p> <p style="text-align: center;">OR / OF</p> <p>✓F</p> <p>✓ SF A</p> <p>✓ Area</p> <p style="text-align: right;">(3)</p>
<p>11.1.3</p>	<p>$D = 2r$</p> <p>$D = 2(30)$</p> <p>$\therefore D = 60 \text{ cm}$</p>	<p>✓M</p> <p>✓Subst./Vervang</p> <p>✓ Answer/Antwoord</p> <p style="text-align: right;">(3)</p>



11.2		
	$4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(30)h + (20)^2 = 0$ $4h^2 - 120h + 400 = 0$ $h^2 - 30h + 100 = 0$ $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $h = \frac{-(-30) \pm \sqrt{(30)^2 - 4(1)(100)}}{2(1)}$ $\therefore h = 26,18 \text{ cm of } h = 3,82 \text{ cm}$ $\therefore h = 3,82 \text{ cm}$ <p>\therefore h hour hand length/lengte van uur aanwyser = radius – height of minor segment/hoogte van kleiner segment</p> $\therefore \text{hour hand length/lengte van uur aanwyser} = \left(\frac{30}{2}\right) - 3,82$ $\therefore \text{hour hand length/lengte van uuraanwyser} = 11,2 \text{ cm}$	<p>✓F ✓SF A</p> <p>✓S</p> <p>✓ value of / waarde van h</p> <p>✓ length/lengte</p> <p>(5)</p>



<p>11.3</p>		
	$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$ $63,525 = \left(\frac{11,55}{7} \right) \left(\frac{7+3}{2} + 6,5 + x + 5 + 7,5 + 6 + 4 \right)$ $63,525 = (1,64)(34 + x)$ $38,734 \dots = 34 + x$ $4,73 = x$ <p style="text-align: center;">OR/ OF</p> $A_T = a(m_1 + m_2 + m_3 + \dots + m_{n-1})$ $63,525 = \left(\frac{11,55}{7} \right) \left(6,75 + \frac{6,5+x}{2} + \frac{x+5}{2} + 6,25 + 6,75 + 5 + 3,5 \right)$ $63,525 = (1,64) \left(28,25 + \frac{11,5+2x}{2} \right)$ $38,734 \dots = 28,25 + \frac{11,5+2x}{2}$ $10,484 \dots = \frac{11,5+2x}{2}$ $20,969 \dots = 11,5 + 2x$ $20,969 \dots = 11,5 + 2x$ $9,469 \dots = 2x$ $4,73 = x$	<p>✓F</p> <p>✓SF A</p> <p>✓S ✓ value of/waarde van a</p> <p style="text-align: center;">OR/ OF</p> <p>✓F</p> <p>✓SF A</p> <p>✓S ✓ value of/ waarde van a</p> <p style="text-align: right;">(4)</p>
		[18]
	TOTAL/TOTAAL:	150

