**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

JUNE 2024

TECHNICAL MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages, including a
2-page formula sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of NINE questions. Answer ALL the questions.

2.   Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

3.   Answer QUESTION 4.1.4 and QUESTION 7.4 on the ANSWER SHEETS provided. Write your name in the spaces provided and then hand in the ANSWER SHEETS with your ANSWER BOOK.

4.   You may an use approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

5.   If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

6.   Number the answers correctly according to the numbering system used in this question paper.

7.   Diagrams are NOT necessarily drawn to scale.

8.   An information sheet with formulae is included at the end of the question paper.

9.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x:

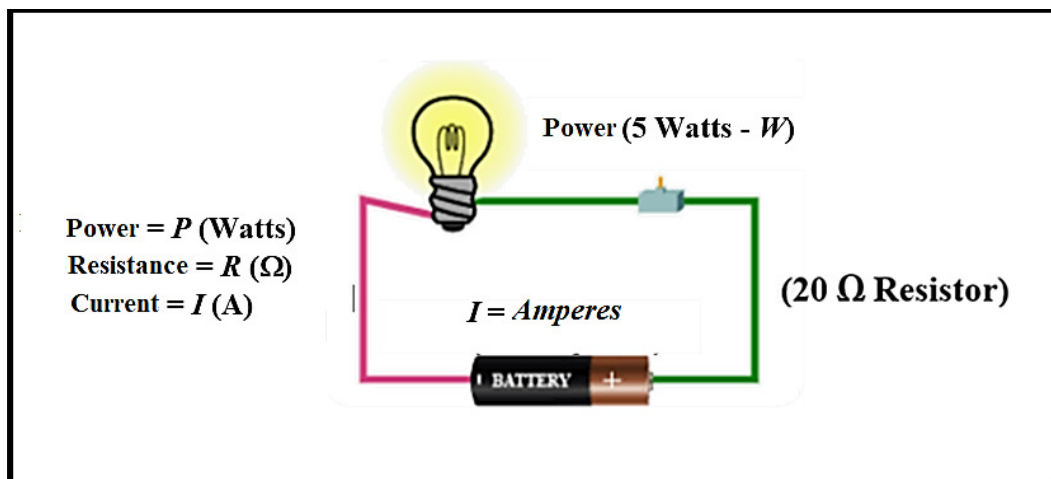   1.1.1 $3x - \dfrac{x^2}{4} = 0$ (3)

     1.1.2 $x(x - 3) = 13$ (Correct to TWO decimal places) (4)

     1.1.3 $-2x^2 - x + 10 \le 0$ (Represent the solution set on a **NUMBER LINE**) (4)

1.2   Solve for x and y if:

     $y - x = 2$ and $\dfrac{x^2}{y} = 1$ (6)

1.3   The diagram below shows a simple electric circuit with a light bulb, connected by a conducting wire to a small battery.

     If $P = I^2 R$:

     1.3.1   Make I the subject of the formula. (2)

     1.3.2   Determine the amount of current flowing through the circuit if the power of the light bulb is 5 Watts and the resistance of the conducting wire = 20 Ω. (2)

     1.3.3   Write the value of the current obtained in QUESTION 1.3.2 in **Scientific Notation.** (1)

1.4   Determine the value of $A = 1010100_2 - 111011_2$ in decimal form. (3)

                                                                               [25]

QUESTION 2

2.1   Given: $f(x) = 1 \pm \sqrt{\dfrac{x}{1-x^2}}$

   2.1.1   Write down the value of x for which $f(x) = 1$.   (1)

   2.1.2   Determine the values of x for which the roots of $f(x)$ will be undefined.   (3)

2.2   Show that the roots of $g(x) = 9x^2 - 12kx + 4k^2$ will always be equal for all values of k.   (4)

                                                                                                     [8]

QUESTION 3

3.1   Simplify the following WITHOUT using a calculator:

   3.1.1 $\log_x x + \log_y 1$ (2)
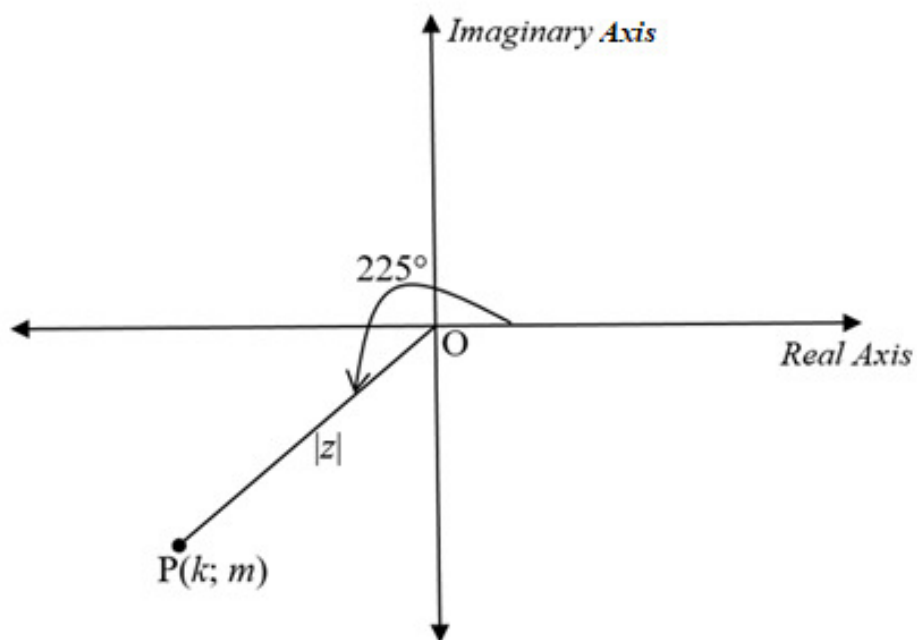
   3.1.2 $\dfrac{\sqrt{18} - \sqrt{128}}{5\sqrt{2}}$ (3)

3.2   Solve for x WITHOUT using a calculator:

   $7^{x-1} - 7^x = -\dfrac{2}{9}$ **HINT: $\log_7 27 = 1{,}69$**   (7)

3.3   The Argand diagram represents the complex: $z = \sqrt{2}\,cis\,225°$. P $(k\,;\,m)$ is a point on z.

3.3.1   Write down the length of the modulus of z.   (1)

3.3.2   Determine the length of k and m.   (2)

3.3.3   Hence or otherwise, write z in rectangular form.   (1)

3.4   Solve for x and y if $x - iy = 0$.   (2)

                                                                                          [18]

QUESTION 4

4.1   Given the functions f and g defined by $f(x) = 3^x + 1$ and $g(x) = 3x + 2$.

    4.1.1   Write down the asymptote of f.   (1)

    4.1.2   Determine the coordinates of the y-intercept of f.   (2)

    4.1.3   Determine the x and y-intercepts of g.   (3)

    4.1.4   Sketch the graph of f and g on the ANSWER SHEET provided.
   Clearly show all the intercepts with the axes and the asymptote of the graph.   (6)

    4.1.5   Write the coordinates of a point where the graph of g cuts the asymptote of f.   (2)

    4.1.6   Hence, determine the values of x for which $g(x) <$ asymptote of f.   (2)

4.2   The diagram below represents the graphs of the functions defined by:
       $$h(x) = -\sqrt{4 - x^2} \text{ and } k(x) = \frac{1}{x} + q$$

    •   The asymptotes of k cut the graph of h at point A, its y-intercept.

    •   Point B is the x-intercept of k.
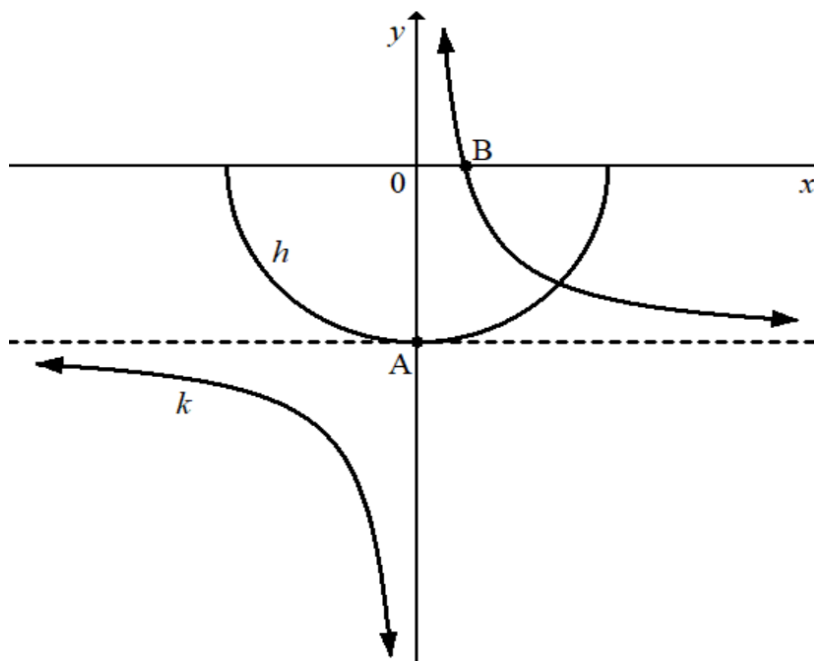
    4.2.1   Write down the y-intercept of h.   (1)

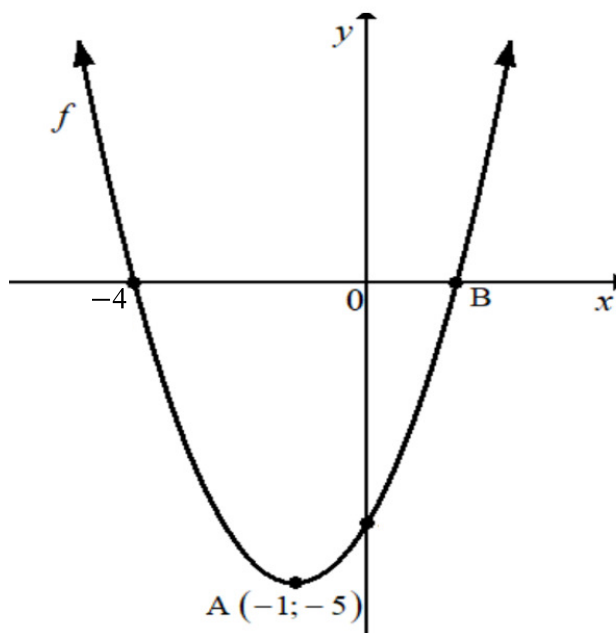   4.2.2   Hence, write down the value of q.   (1)

    4.2.3   Determine the coordinates of B.   (2)

    4.2.4   Write down the domain of k.   (1)

    4.2.5   Determine the coordinates of two points on k that are closest to point A.   (4)

4.3 The graph of the function defined by: $y = a(x - p)^2 + q$ is drawn below.

$x = -4$ and B are the $x-$intercepts of f and A $(-1 ; -5)$ is a turning point of f.

4.3.1 Write down the equation of the axis of symmetry. (1)

4.3.2 Determine the values of p and q. (2)

4.3.3 Calculate the coordinates of B. (2)

4.3.4 Determine the values of x for which $f(x) \times f'(x) > 0$.

(3)

[33]

QUESTION 5

5.1   November 2023 Black Friday rush dropped the price of a R7 300 stamper by 11%.

         5.1.1   Calculate 11% of R7 300.   (1)

         5.1.2   Hence or otherwise, determine the discounted price of a stamper.   (2)

5.2   Rusting of metal is an exponential process if not attended to. The car below started rusting 2 cm² area some years ago.

         Determine the number of years it has taken to rust 60 cm² area if the rate of rusting is 5% per year.   (5)

5.3   Ludwe invests R500 000 into an investment company that pays 7% per annum on simple interest. At the end of the 5th year Ludwe deposits a further R77 000 into the investment account and the interest rates changed to x% per annum, compounded monthly.

         Determine the value of x% (interest rates), if at the end of 8 years Ludwe gets a total of R880 000 from his investment account.   (6)

                                                                     [14]

QUESTION 6

6.1   Determine the derivative of $f(x) = 3ax + 5$ by using **FIRST PRINCIPLES.**   (5)

6.2   Determine:

         6.2.1 $\dfrac{dy}{dx}$ if $xy = \sqrt{x}$ (4)

         6.2.2 $D_x\left[\dfrac{3}{2x}\right]$ (2)

6.3   Determine the average gradient between points A(3 ; 5) and B (−2 ; −1).   (3)

                                                                       [14]

QUESTION 7

Consider: $f(x) = (x - 1)(x + 2)(x + 3)$

7.1	Write down the coordinates of the x-intercept of f.	(1)
7.2	Determine the y-intercept of f.	(1)
7.3	Determine the coordinates of the turning point of f.	(5)
7.4	Sketch the graph of f on the ANSWER SHEET provided. Clearly indicate your turning points and the intercepts with the axes.	(4)
7.5	Hence or otherwise, write down the values of x for which $f(x) < 0$.	(2)

[13]

SA EXAM PAPERS

QUESTION 8

The graph below shows the path of a Boeing plane that is landing after experiencing hydraulics problems.

The process of its landing is observed from a certain height, in metres, over time, in seconds.
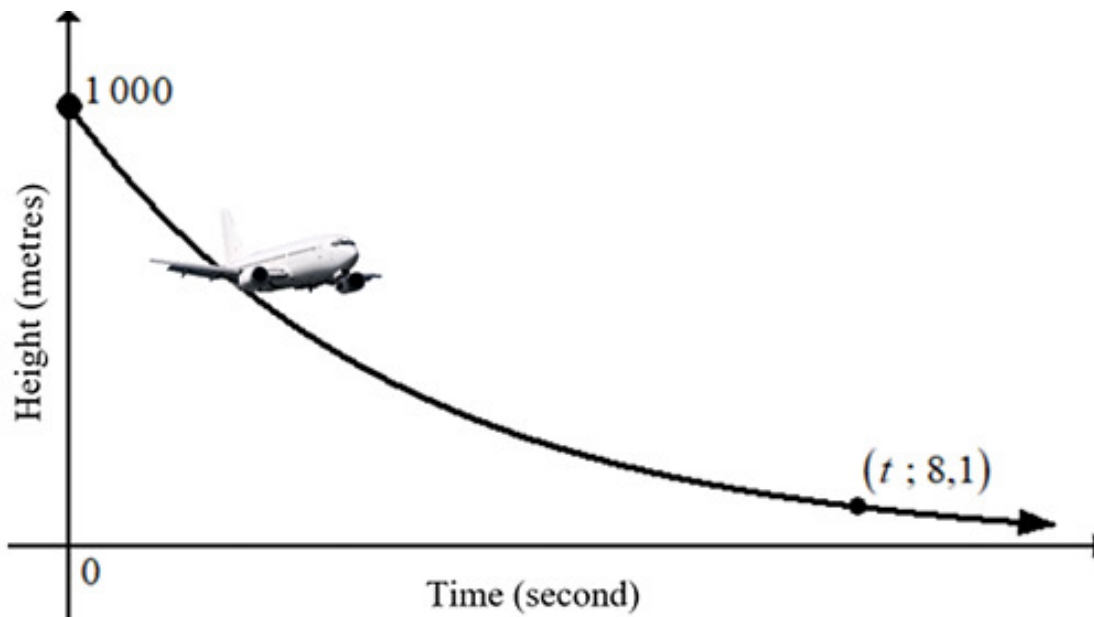
The landing path in terms of time (t) is: $h(t) = h_0 (0,09)^t$.

$h(t)$ = Final height of descend, in metres.

h_0 = Initial height of observation, in metres.

t = Time of observation, in seconds.

8.1   Write down the height of the plane, in metres, at the start of its landing observation.   (1)

8.2   Determine the height of the plane after 1 second.   (1)

8.3   Determine the time the plane took to drop to a height of 8,1 metres.   (4)

8.4   Calculate the plane's average speed of descend between 1 000 m to 8,1 m.   (2)

8.5   If the maximum landing speed of a plane is 290 km/h, indicate whether this plane's landing speed in QUESTION 8.4 was a normal speed or not. (**Show your calculations.**)   (2)
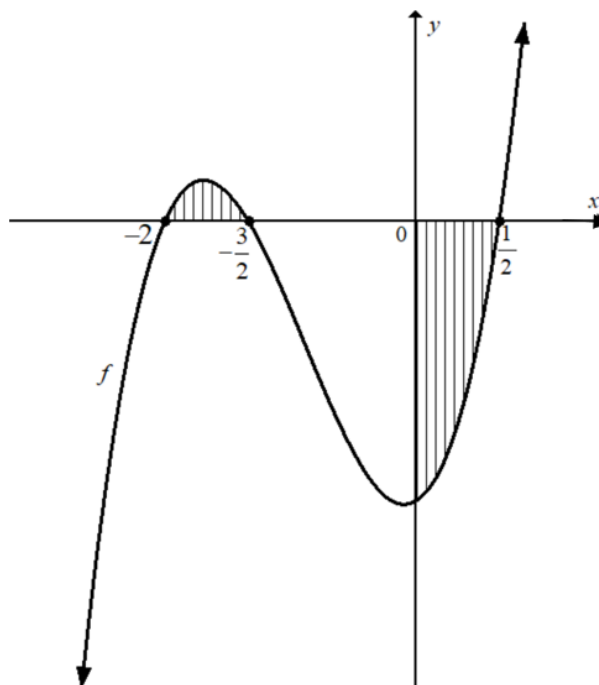**[10]**

QUESTION 9

9.1   Determine the following integrals:

   9.1.1 $\int (-3x^2 + 2x^{-1})\, dx$ (3)

   9.1.2 $\int (x-3)(x-5)\, dx$ (4)

9.2   The sketch below shows the shaded area bounded by the function g defined by:
           $g(x) = x^3 + 3x^2 + \dfrac{5}{4}x - \dfrac{3}{2}$ and the axis between the points where
           $x = -2$ and $x = -1,5$ together with $x = 0$ and $x = 0,5$.

           Determine the area of the shaded region of the graph of g bounded by the graph and
           the x-axis, between $x = -2$ and $x = -1,5$ and $x = 0$ and $x = 0,5$.   (8)
                                                                                                 [15]

                                                                         TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad\qquad x = -\frac{b}{2a} \qquad\qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad\qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad\qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \qquad\qquad \int k x^n \, dx = k.\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0 \qquad\qquad \int \frac{k}{x} \, dx = k.\ln x + C, \quad x > 0$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \quad a > 0 \qquad\qquad \int k a^{nx} \, dx = k.\frac{a^{nx}}{n \ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_2 + x_1}{2} \,;\, \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan \theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad\qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \ of \ \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad\qquad 1 + \tan^2 \theta = \sec^2 \theta \qquad\qquad 1 + \cot^2 \theta = \csc^2 \theta$$

$$\pi \, rad = 180°$$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360° \, n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate
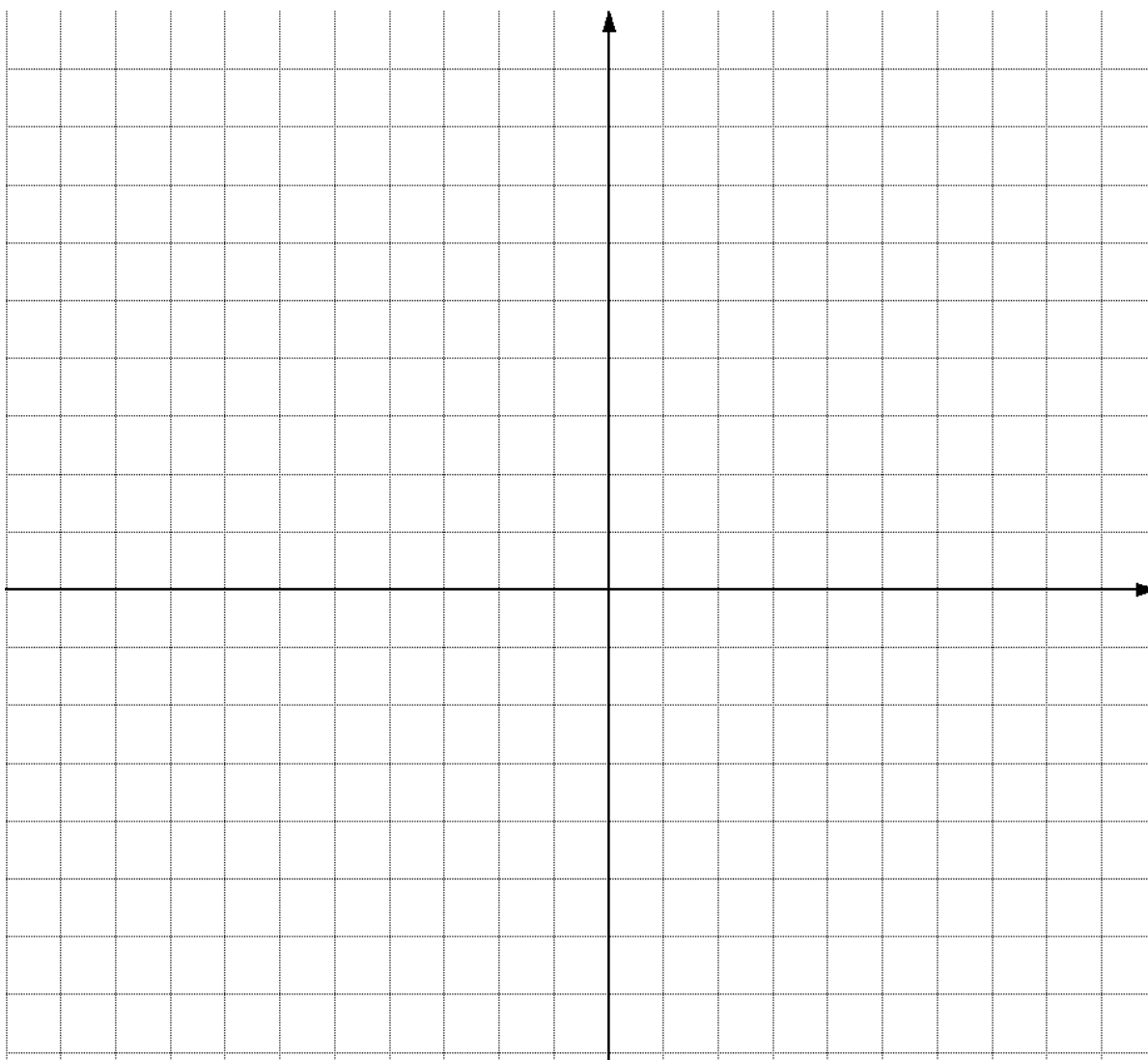
and n = number of ordinates

ANSWER SHEET

Learner Name: ………………………………………..   Class: ……………………..

School Name: …………………………………………

QUESTION 4.1.4

ANSWER SHEET

Learner Name: …………………………………………..   Class: ……………………..

School Name: …………………………………………………

QUESTION 7.4