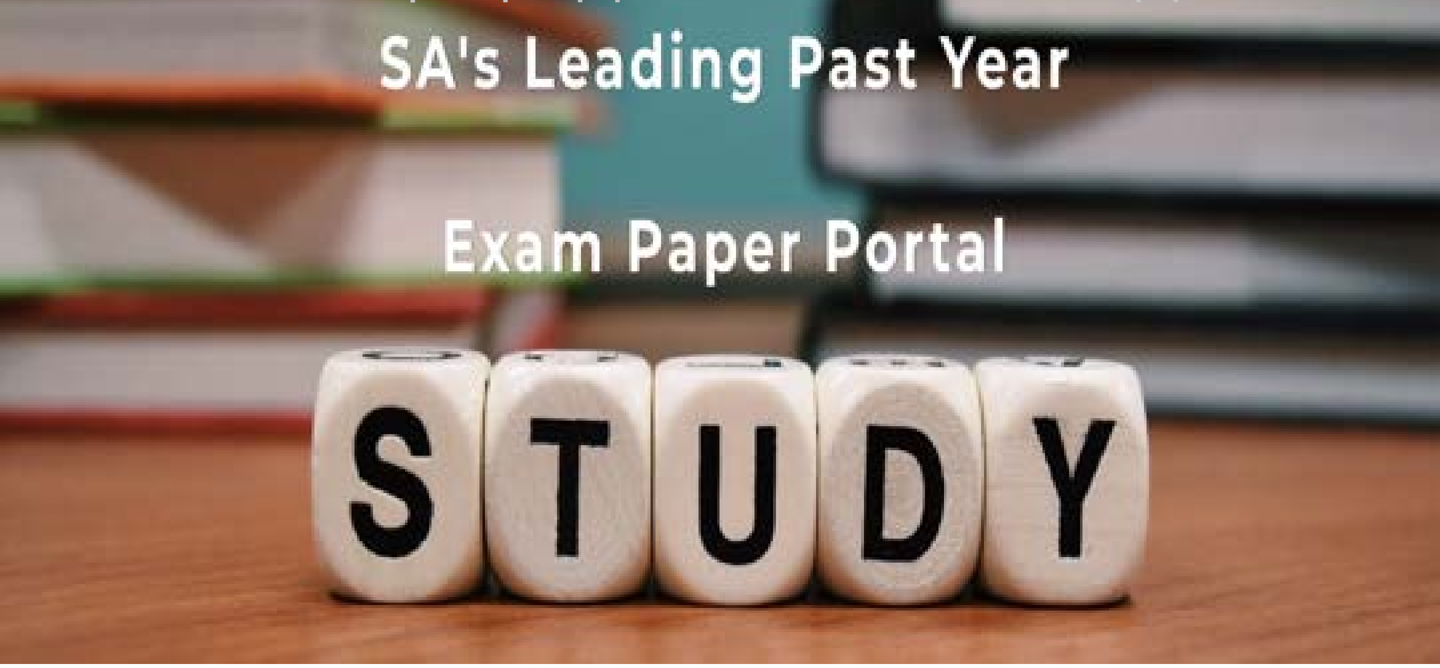
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FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2024

MARKING GUIDELINES

MARKS: **150**

TIME: **3 hours**

These marking guidelines consist of 17 pages.

NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	**A mark for a correct statement (A statement mark is independent of a reason.)**
R	**A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)**
S/R	**Award a mark if the statement AND reason are both correct.**

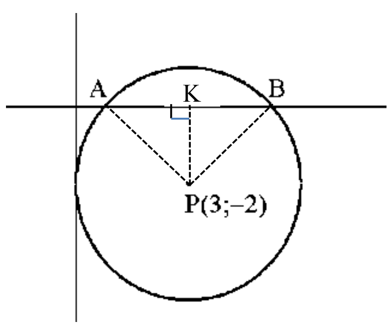
QUESTION 1

1.1	$m_{EF}=\frac{-8-0}{1-(-5)}$ $=-\frac{4}{3}$ Answer only: Full marks	A✓ substitution CA✓ answer (2)
1.2	$\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $=\left(\frac{-5+1}{2};\frac{0+(-8)}{2}\right)$ $=(-2;-4)$ Answer only: Full marks	A ✓ x-coordinate A ✓ y-coordinate (2)
1.3	$m_{GH}\times\left(-\frac{4}{3}\right)=-1$ $m_{GH}=\frac{3}{4}$ Substitute $(-2;-4)$ and $m_{GH}=\frac{3}{4}$ into $y=mx+c$: $-4=\frac{3}{4}(-2)+c$ $c=-\frac{5}{2}$ $y=\frac{3}{4}x-\frac{5}{2}$ If $m=-\frac{4}{3}$ is used: maximum 1 mark	CA✓ value of m_{GH} CA✓ substitution of point and gradient CA✓ answer (3)
1.4	$m_{DG}=m_{EF}=-\frac{4}{3}$ $m_{DG}=\tan\beta=-\frac{4}{3}$ $\beta=126,87°$	CA✓ value of m_{DG} CA✓ $\tan\beta=-\frac{4}{3}$ CA✓ answer (3)

1.5	$O\hat{J}K = 126,87° - 90°$ [exterior ∠ of ΔOJK] $= 36,87°$ If answer is a negative angle: 0/2	CA✓ method CA✓ answer (2)
1.6	$DE = \sqrt{(x-(-5))^2 + (7-0)^2} = 5\sqrt{2}$ $(x-(-5))^2 + (7-0)^2 = 50$ $x^2 + 10x + 24 = 0$ $(x+6)(x+4) = 0$ $x = -6$ or $x = -4$ $x = -4$ only	A✓ substitution in distance formula and equating to $5\sqrt{2}$ CA✓ squaring both sides CA✓ standard form CA✓ both x-values CA✓ selecting the x-value > – 5 (5)
	OR Equation of DG: $y = -\frac{4}{3}x + \frac{5}{3}$ Substitute $y = 7$: $7 = -\frac{4}{3}x + \frac{5}{3}$ $\frac{4}{3}x = -7 + \frac{5}{3}$ $\frac{4}{3}x = \frac{-16}{3}$ $\therefore x = \frac{-16}{3} \times \frac{3}{4}$ $= -4$	**OR** CA✓ equation of DG CA✓ substitute $y = 7$ CA✓ simplification CA✓✓ answer $x = -4$ (5)

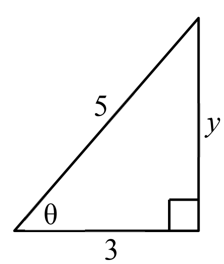
1.7	Equation of DG: $y=-\frac{4}{3}x+\frac{5}{3}$	CA✓ equation of DG
	For x-coordinate of K: $0=-\frac{4}{3}x+\frac{5}{3}$	CA ✓ substitution of $y=0$
	$x=\frac{5}{4}=1,25$	CA✓ value of x-coordinate of K
	Area of ΔDEK $=\frac{1}{2}\times\text{base}\times\text{height}$	
	$=\frac{1}{2}\times\left(5+\frac{5}{4}\right)\times 7$	CA✓ substitution to calculate area of ΔDEK
	$=\frac{175}{8}$	
	Area of ΔOJK $=\frac{1}{2}\times\frac{5}{4}\times\frac{5}{3}$	CA✓ substitution to calculate area of ΔOJK
	$=\frac{25}{24}$	
	Area of DEOJ $=\frac{175}{8}-\frac{25}{24}=\frac{125}{6}=20,83\text{ units}^2$	CA✓ area of DEOJ (6)
	OR	**OR**
	Equation of DG: $y=-\frac{4}{3}x+\frac{5}{3}$	CA✓ equation of DG
	For x-coordinate of K: $0=-\frac{4}{3}x+\frac{5}{3}$	CA ✓ substitution of $y=0$
	$x=\frac{5}{4}=1,25$	CA✓ value of x-coordinate of K
	$DK=\sqrt{\left[1,25-(-4)\right]^2+(0-7)^2}=\sqrt{\frac{1225}{16}}=8,75$	
	$EK=1,25-(-5)=6,25$	
	Area of ΔDEK $=\frac{1}{2}\times DK\times EK\times\sin J\hat{K}O$	
	$=\frac{1}{2}\times 8,75\times 6,25\times\sin 53,13°$	CA✓ substitution to calculate area of ΔDEK
	$=21,87$	
	Area of ΔOJK $=\frac{1}{2}\times\frac{5}{4}\times\frac{5}{3}$	CA✓ substitution to calculate area of ΔOJK
	$=\frac{25}{24}$	
	Area of DEOJ $=21,87-\frac{25}{24}=20,83\text{ units}^2$	CA✓ area of DEOJ (6)
		[23]

QUESTION 2

2.1.1	radius = 3 units	A✓ answer (1)
2.1.2	$(x-3)^2+(y+2)^2=3^2$	CA✓ $(x-3)^2+(y+2)^2=3^2$ (1)
2.2	For x-intercepts, let $y=0$: $(x-3)^2+(0+2)^2=3^2$ $(x-3)^2=5$ $x-3=+\sqrt{5}$ or $x-3=-\sqrt{5}$ $x=3+\sqrt{5}=5,24$ or $x=3-\sqrt{5}=0,76$ $AB=3+\sqrt{5}-(3-\sqrt{5})$ $=2\sqrt{5}=4,47$ units	CA✓ substitute $y=0$ CA✓ values of x CA✓ subtraction CA✓ answer (4)
	OR	**OR**
	For x-intercepts, let $y=0$: $(x-3)^2+(0+2)^2=3^2$ $(x-3)^2=5$ $x^2-6x+9=5$ $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}$ $x=3+\sqrt{5}=5,24$ or $x=3-\sqrt{5}=0,76$ $AB=3+\sqrt{5}-(3-\sqrt{5})$ $=2\sqrt{5}=4,47$ units	CA✓ substitute $y=0$ CA✓ values of x CA✓ subtraction CA✓ answer **(4)**
	OR	**OR**
	 $AP^2=AK^2+PK^2$ [Pythagoras] $3^2=AK^2+2^2$ $AK=\sqrt{5}$ $AK=BK$ [line from centre ⊥ to chord] $\therefore AB=2\sqrt{5}$	CA✓ applying Theorem of Pythagoras CA✓ substitution CA✓ length of AK CA✓ answer (4)

2.3	$x^2+2x+y^2-8y-8=0$ $x^2+2x+1+y^2-8y+16=8+1+16$ $(x+1)^2+(y-4)^2=5^2$ radius = 5 units centre: $(-1\ ;\ 4)$	A✓ completing the square A✓ $(x+1)^2+(y-4)^2=5^2$ CA✓ radius CA✓ coordinates of centre (4)
2.4	Sum of radii $=3+5=8$ Distance between centres $=\sqrt{(-1-3)^2+(4-(-2))^2}$ $=\sqrt{52}=7,21$ units $\therefore$ Distance between centres $<$ Sum of radii $\therefore$ The circles will intersect	CA✓ sum of radii CA✓substitution CA✓ distance between centres CA✓ distance between centres < sum of radii CA✓ conclusion (5)
2.5	$y=3$ $y=-3$	A✓ answer A✓answer (2)
		[17]

QUESTION 3

3.1.1	$\cos\theta=\frac{3}{5}$ $y^2=5^2-3^2$ [Pythagoras] $y=4$ $\tan\theta=\frac{4}{3}$ 	A✓ $\cos\theta=\frac{3}{5}$ A✓ $y=4$ CA✓ answer (3)
3.1.2	$\sin 2\theta=2\sin\theta\cos\theta$ $=2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$ $=\frac{24}{25}$	A✓ expansion CA✓ substitution CA✓ answer (3)

3.1.3	$\text{RO} = \sqrt{k^2 + 6^2}$ [Pythagoras] $\sin(90° + \theta) = \cos\theta$ $\frac{6}{\sqrt{k^2 + 6^2}} = \frac{3}{5}$ $\sqrt{k^2 + 6^2} = 10$ $k^2 + 6^2 = 100$ $k^2 = 64$ $k = -8$	A✓ $\text{RO} = \sqrt{k^2 + 6^2}$ A✓ $\sin(90° + \theta) = \cos\theta$ CA✓ substitution CA✓ simplification CA✓answer (5)
	OR	**OR**
	$\text{RO} = \sqrt{k^2 + 6^2}$ [Pythagoras] $\cos(90° + \theta) = -\sin\theta$ $\frac{k}{\sqrt{k^2 + 6^2}} = -\frac{4}{5}$ $-4\sqrt{k^2 + 6^2} = 5k$ $16(36 + k^2) = 25k^2$ $576 + 16k^2 = 25k^2$ $9k^2 = 576$ $k^2 = 64$ $k = -8$	A✓ $\text{RO} = \sqrt{k^2 + 6^2}$ A✓ $\cos(90° + \theta) = -\sin\theta$ CA✓ substitution CA✓ simplification CA✓answer (5)
	OR	**OR**
	$\text{RO} = \sqrt{k^2 + 6^2}$ [Pythagoras] $\text{RP}^2 = \text{OP}^2 + \text{OR}^2$ [Pythagoras in ΔPOR] $(k-3)^2 + (6-4)^2 = 5^2 + (k^2 + 36)$ $k^2 - 6k + 9 + 4 = 25 + k^2 + 36$ $-6k = 48$ $k = -8$	A✓ $\text{RO} = \sqrt{k^2 + 6^2}$ A✓ Pythagoras in ΔPOR CA✓ substitution CA✓ simplification CA✓answer (5)
	OR	**OR**
	$m_{OP} = \frac{4}{3}$ $m_{OR} = \frac{6-0}{k-0}$ $m_{OP} \times m_{OR} = -1$ [OR $\perp$ OP] $\therefore \frac{4}{3} \times -\frac{6}{k} = -1$ $k = -8$	A✓ m of OP A✓ m of OR A✓ condition for gradients of $\perp$ lines CA✓ substitution CA✓answer (5)

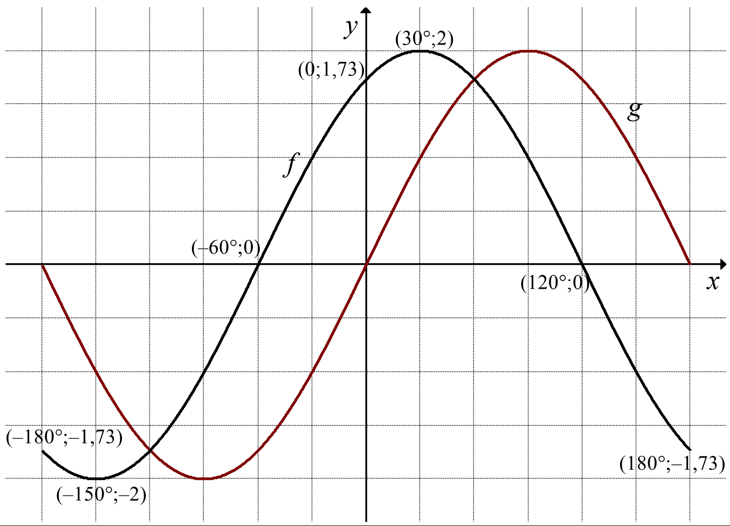
3.2	$\cos(385°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos(25°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\cos[90°-(55°+\beta)]$ $=\cos(25°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\cos(35°-\beta)$ $=\sin(25°+\beta+35°-\beta)$ $=\sin 60°$ $=\frac{\sqrt{3}}{2}$	A✓ $\cos(25°+\beta)$ A✓ $\cos(35°-\beta)$ A✓ applying compound angle identity A✓ answer (4)
	OR	**OR**
	$\cos(385°+\beta).\sin(35°-\beta)+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos(25°+\beta).\cos[90°-(35°-\beta)]+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos(25°+\beta).\cos(55°+\beta)+\sin(25°+\beta).\sin(55°+\beta)$ $=\cos[25°+\beta-(55°+\beta)]$ $=\cos(-30°)$ $=\frac{\sqrt{3}}{2}$	A✓ $\cos(25°+\beta)$ A✓ $\sin(55°+\beta)$ A✓ applying compound angle identity A✓ answer (4)
3.3.1	$\sin 3\theta$ $=\sin(2\theta+\theta)$ $=\sin 2\theta\cos\theta+\cos 2\theta\sin\theta$ $=2\sin\theta\cos^2\theta+(2\cos^2\theta-1)\sin\theta$ $=2\sin\theta\cos^2\theta+2\sin\theta\cos^2\theta-\sin\theta$ $=4\sin\theta\cos^2\theta-\sin\theta$	A✓ replace 3θ by $(2\theta+\theta)$ A✓ compound angle expansion A✓sine double angle expansion A✓ cosine double angle expansion A✓ simplification (5)
	OR	**OR**
	$\sin 3\theta$ $=\sin(2\theta+\theta)$ $=\sin 2\theta\cos\theta+\cos 2\theta\sin\theta$ $=2\sin\theta\cos^2\theta+(1-2\sin^2\theta)\sin\theta$ $=2\sin\theta\cos^2\theta+\sin\theta-2\sin^3\theta$ $=2\sin\theta\cos^2\theta+\sin\theta-2\sin\theta(1-\cos^2\theta)$ $=2\sin\theta\cos^2\theta+\sin\theta-2\sin\theta+2\sin\theta\cos^2\theta$ $=4\sin\theta\cos^2\theta-\sin\theta$	A✓ replace 3θ by $(2\theta+\theta)$ A✓ compound angle expansion A✓sine double angle expansion A✓ cosine double angle expansion A✓ simplification (5)

	OR $\sin 3\theta$ $= \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos^2\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$ $= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$ $= 3\sin\theta\cos^2\theta - \sin\theta(1-\cos^2\theta)$ $= 3\sin\theta\cos^2\theta - \sin\theta + \sin\theta\cos^2\theta$ $= 4\sin\theta\cos^2\theta - \sin\theta$	**OR** A✓ replace 3θ by $(2\theta+\theta)$ A✓ compound angle expansion A✓sine double angle expansion A✓ cosine double angle expansion A✓ simplification (5)
3.3.2	$\frac{\sin 3\theta + \sin\theta}{2 + 2\cos 2\theta}$ $= \frac{4\sin\theta\cos^2\theta - \sin\theta + \sin\theta}{2+2\cos 2\theta}$ $= \frac{4\sin\theta\cos^2\theta}{2+2(2\cos^2\theta - 1)}$ $= \frac{4\sin\theta\cos^2\theta}{2+4\cos^2\theta - 2}$ $= \frac{4\sin\theta\cos^2\theta}{4\cos^2\theta}$ $= \sin\theta$	A✓ numerator simplified A✓ cos double angle expansion A✓ denominator simplified (3)
3.3.3	$2 + 2\cos 2\theta = 0$ $\cos 2\theta = -1$ $\therefore 2\theta = 180° + k.360°, \quad k \in Z$ $\theta = 90° + k.180°, \quad k \in Z$ Penalty of 1 if $k \in Z$ is omitted	A✓ equating denominator to 0 A✓ $\cos 2\theta = -1$ CA✓ $2\theta = 180° + k.360°$ CA✓ $\theta = 90° + k.180°, \quad k \in Z$ (4)
3.4	Minimum value of $\cos 3x = -1$ $\therefore$ Minimum value of $\cos 3x - 5 = -6$ Answer only: Full marks	A✓ Minimum value of $\cos 3x = -1$ A✓ answer (2)
		[29]

QUESTION 4

4.1	$\hat{RAB} = \hat{RBA} = \beta$ [$\Delta RAT \equiv \Delta RBT$] $\therefore \hat{ARB} = 180° - (\hat{RAB} + \hat{RBA})$ [sum of $\angle$s of ΔRAB] $= 180° - 2\beta$	A✓ answer (1)
4.2	$\hat{RBT} = \alpha$ [alt. $\angle$s; ∥ lines] $\frac{h}{BR} = \sin\alpha$ $BR = \frac{h}{\sin\alpha}$ $\frac{AB}{\sin \hat{ARB}} = \frac{BR}{\sin \hat{RAB}}$ $AB = \frac{BR.\sin \hat{ARB}}{\sin \hat{RAB}}$ $= \frac{h\sin(180° - 2\beta)}{\sin\alpha \sin\beta}$ $= \frac{h\sin 2\beta}{\sin\alpha\sin\beta}$ $= \frac{h(2\sin\beta\cos\beta)}{\sin\alpha\sin\beta}$ $= \frac{2h\cos\beta}{\sin\alpha}$	A✓ $\frac{h}{BR} = \sin\alpha$ A✓ BR subject of formula A✓ applying sine rule A✓ AB subject of formula A✓ $\sin 2\beta$ A✓ $2\sin\beta\cos\beta$ (6)
	OR $\hat{RBT} = \alpha$ [alt. $\angle$s; ∥ lines] $\frac{h}{BR} = \sin\alpha$ $BR = \frac{h}{\sin\alpha}$ $AB^2 = BR^2 + AR^2 - 2BR.AR.\cos(180° - 2\beta)$ $= 2BR^2 - 2BR^2.\cos(180° - 2\beta)$ $= 2BR^2 + 2BR^2.\cos 2\beta$ $= 2BR^2(1 + \cos 2\beta)$ $= 2BR^2(1 + 2\cos^2\beta - 1)$ $= 4BR^2\cos^2\beta$ $\therefore AB = 2BR\cos\beta$ $= \frac{2h\cos\beta}{\sin\alpha}$	**OR** A✓ $\frac{h}{BR} = \sin\alpha$ A✓ BR subject of formula A✓ applying cosine rule A✓ $-\cos(180° - 2\beta) = \cos 2\beta$ A✓ $\cos 2\beta = 2\cos^2\beta - 1$ A✓ square root on LHS and RHS (6)
4.3	$5{,}4 = \frac{2h\cos 65°}{\sin 51°}$ $h = \frac{5{,}4 \times \sin 51°}{2\cos 65°}$ $h = 5$ units Accept 4,96 units as answer	A✓ substitution A✓ h subject of formula CA✓ answer (3)
		[10]

QUESTION 5

5.1	$a = 2$	A✓ answer (1)
5.2		A✓ shape A✓ turning points A✓ x-intercepts A✓ y-intercept (4)
5.3.1	$y \in [-2\ ;\ 2]$ OR $-2 \le y \le 2$ Penalty of 1 mark if one or both end points are excluded	A✓ A✓ answer (2)
5.3.2	$\text{period} = \frac{360°}{3}$ $= 120°$ Answer only: Full marks	A✓ $\frac{360°}{3}$ A✓ answer (2)
5.4	$2\sin x = 2\cos(x - 30°)$ $\sin x = \cos(x - 30°)$ $\sin x = \sin[90° - (x - 30°)]$ $\sin x = \sin(-x + 120°)$ $x = -x + 120° + k.360°$ or $x = 180° - (-x + 120°) + k.360°$, $k \in Z$ $2x = 120° + k.360°$ $x = 300° + x + k.360°$ $x = 60° + k.180°$ no solution In the interval $x \in [-180°\ ;\ 180°]$: $x = 60°$ or $x = -120°$	A✓ equating A✓ co-function A✓ both solutions CA✓ $x = 60°$ CA✓ $x = -120°$ (5)

	OR $2\sin x = 2\cos(x-30°)$ $\sin x = \cos(x-30°)$ $\cos(90°-x) = \cos(x-30°)$ $90°-x = x-30°+k.360°$ or $90°-x = 360°-(x-30°)+k.360°$, $k \in Z$ $2x = 120°+k.360°$ $x = 300°+x+k.360°$ $x = 60°+k.180°$ no solution In the interval $x \in [-180° ; 180°]$: $x = 60°$ or $x = -120°$	**OR** A✓ equating A✓ co-function A✓ both solutions CA✓ $x = 60°$ CA✓ $x = -120°$ (5)
	OR $2\sin x = 2\cos(x-30°)$ $\sin x = \cos(x-30°)$ $\sin x = \cos x\cos 30° + \sin x \sin 30°$ $\sin x = \cos x.\frac{\sqrt{3}}{2} + \sin x.\frac{1}{2}$ $\frac{1}{2}\sin x = \cos x.\frac{\sqrt{3}}{2}$ $\sin x = \sqrt{3}\cos x$ $\tan x = \sqrt{3}$ $x = 60°+k.180°$ In the interval $x \in [-180° ; 180°]$: $x = 60°$ or $x = -120°$	**OR** A✓ equating A✓ compound angle expansion A✓ $\tan x = \sqrt{3}$ CA✓ $x = 60°$ CA✓ $x = -120°$ (5)
5.5.1	$x = -90°$ or $x = 90°$	A✓A✓ (2)
5.5.2	$x \in (-120° ; 60°)$ OR $-120° < x < 60°$ Penalty of 1 mark if one or both end points are included	CA✓CA✓ (2)
		[18]

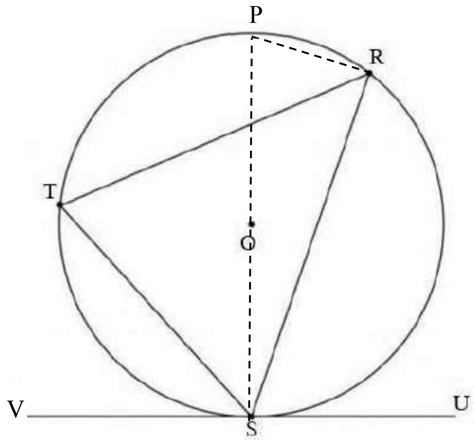
QUESTION 6

6.1.1	$P\hat{R}S = 90°$ [tangent ⊥ diameter]	S✓R✓ (2)
6.1.2 (a)	$P\hat{S}R = \hat{R}_1$ [tan-chord-theorem] $= 48°$ $\therefore \hat{P} = 180° - (P\hat{S}R + P\hat{R}S)$ [sum of ∠s of ΔPSR] $= 180° - (48° + 90°)$ $= 42°$ **OR** $\hat{T}_1 = 90°$ [∠ in semi-circle] $\hat{P} = \hat{T}_1 - \hat{R}_1$ [exterior ∠ of ΔPTR] $= 90° - 48°$ $= 42°$	S/R✓ A✓ $P\hat{S}R = 48°$ CA✓ answer (3) **OR** S/R✓ S✓ A✓ answer (3)
6.1.2 (b)	$\hat{R}_2 = 90° - 48° = 42°$ $\hat{V}_1 = \hat{R}_2$ [∠s in same segment] $= 42°$	A✓ $\hat{R}_2 = 42°$ R✓ CA✓ answer (3)
6.1.3	$P\hat{Q}S = 90° + \hat{S}_1$ [exterior ∠ of ΔQSR] $\hat{T}_1 = 90°$ [∠ in semi-circle] $\therefore V\hat{T}S = 90° + \hat{T}_2$ But: $\hat{T}_2 = \hat{S}_1$ [∠s in same segment] $\therefore P\hat{Q}S = V\hat{T}S$ **OR** $\hat{V}_1 = \hat{P}$ [both $= 42°$] PTVQ is a cyclic quadrilateral [converse: ext. ∠ = opp int ∠] $\therefore P\hat{Q}S = V\hat{T}S$ [ext. ∠ = opp int ∠] **OR** $P\hat{Q}S = 180° - (\hat{P} + \hat{S}_2)$ [sum of ∠s of ΔPQS] $V\hat{T}S = 180° - (\hat{V}_1 + \hat{S}_2)$ [sum of ∠s of ΔVTS] $\therefore P\hat{Q}S = V\hat{T}S$ [$\hat{V}_1 = \hat{P}$; proved above]	S/R✓ S✓ $\hat{T}_1 = 90°$ S✓ $V\hat{T}S = 90° + \hat{T}_2$ S/R✓ (4) **OR** S✓ S✓R✓ R✓ (4) **OR** S✓ R✓ S/R✓ R✓ (4)

6.2	$\hat{O}_1 = 2 \times \hat{A}$ $= 2 \times 66° = 132°$ $\hat{C}_1 = \hat{E}$ $= 42°$ $\hat{B}_2 = \hat{O}_1 - \hat{C}_1$ $= 132° - 42° = 90°$ $\therefore AB = BC$	[$\angle$ at centre = 2× $\angle$ at circumf.] [ext. $\angle$ of cyclic quad.] [ext. $\angle$ of ΔOBC] [line from centre $\perp$ to chord]	S/R✓ A✓ answer S/R✓ ✓A answer ✓A $132° - 42° = 90°$ R✓ (6)
			[18]

QUESTION 7

7.1

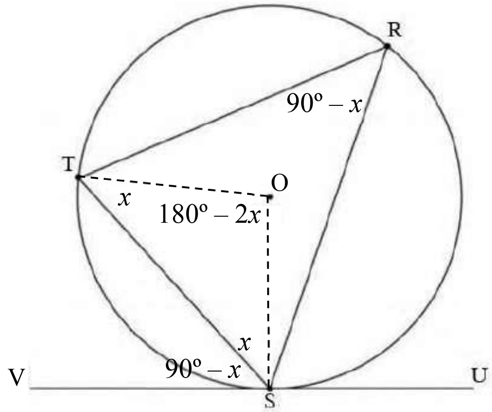


NOTE:
If there is no construction: 0/5 marks

Draw diameter SP
Join P to R — ✓ construction

$\hat{PSV} = 90°$ [tangent ⊥ diameter] — S/R✓
$\hat{VST} = 90° - \hat{TSP}$
$\hat{SRP} = 90°$ [∠ in semi-circle] — S/R✓
$\hat{TRS} = 90° - \hat{PRT}$
But: $\hat{TSP} = \hat{PRT}$ [∠s in same segment] — S✓R✓
$\therefore \hat{VST} = \hat{TRS}$ OR $\hat{VST} = \hat{R}$ — (5)

OR — **OR**



Draw radii OT and OS. — ✓ construction
Let $\hat{OST} = x$.
$\hat{OTS} = x$ [∠s opposite equal radii] — S/R✓
$\hat{TOS} = 180° - 2x$ [sum of ∠s of a Δ]
$\hat{TRS} = 90° - x$ [∠ at centre = 2× ∠ at circumf.] — S✓R✓
$\hat{OSV} = 90°$ [tangent ⊥ radius] — S/R✓
$\hat{VST} = 90° - x$
$\hat{VST} = \hat{TRS}$ OR $\hat{VST} = \hat{R}$ [both $= 90° - x$] — (5)

7.2.1	$\hat{F}_2 = \hat{G}_4 = x$ [tan-chord-theorem] $\hat{P} = \hat{G}_4 = x$ [tan-chord-theorem] $\hat{G}_2 = \hat{P} = x$ [alt. $\angle$s; GE ∥ HP]	S✓R✓ S✓R✓ S✓R✓ (6)
7.2.2	In ΔHMG and ΔEFG : 1. $\hat{H}_2 = \hat{E}$ [$\angle$s in the same segment] 2. $\hat{G}_3 = \hat{G}_1$ [$\angle$s subtended by = chords] 3. $\hat{M}_3 = E\hat{F}G$ [sum of $\angle$s of a Δ] $\therefore \Delta HMG \mid\mid\mid \Delta EFG$ [$\angle\angle\angle$]	S✓R✓ S✓ R✓ R✓ (5)
7.2.3	$\frac{HM}{EF} = \frac{HG}{EG}$ [∣∣∣ Δs] But: EF = PH [given] $\therefore \frac{HM}{PH} = \frac{HG}{EG}$ And PH.HG = EG.HM	S✓R✓ S✓ $\frac{HM}{PH} = \frac{HG}{EG}$ (3)
		[19]

QUESTION 8

8.1	$AB^2 = AC^2 - BC^2$ [Pythagoras] $= 6{,}5^2 - 6^2$ $\therefore AB = 2{,}5$ units	S/R✓ using Theorem of Pythagoras A✓ (2)
8.2	In ΔCBA and ΔCEB: 1. $\hat{C} = \hat{C}$ [common] 2. $A\hat{B}C = C\hat{E}B$ [both $= 90°$; given] 3. $\hat{A} = C\hat{B}E$ [sum of $\angle$s of a Δ] $\therefore \Delta CBA \;\|\|\|\; \Delta CEB$ [$\angle\angle\angle$] $\therefore \frac{CB}{CA} = \frac{CE}{CB}$ [$\|\|\|$ Δs] $\therefore CB^2 = CA.CE$ and $CB = \sqrt{CA.CE}$ **OR** $\Delta CBA \;\|\|\|\; \Delta CEB$ [perpendicular from right $\angle$ vertex to hypotenuse] $\therefore \frac{CB}{CA} = \frac{CE}{CB}$ [$\|\|\|$ Δs] $\therefore CB^2 = CA.CE$ and $CB = \sqrt{CA.CE}$	S✓ identifying triangles S ✓ S/R ✓ R ✓ S✓ S✓ $CB^2 = CA.CE$ (6) **OR** S✓ $\Delta CBA \;\|\|\|\; \Delta CEB$ R ✓✓✓ S✓ S✓ $CB^2 = CA.CE$ (6)
8.3	$6 = \sqrt{6{,}5.CE}$ $36 = 6{,}5.CE$ $CE = 5{,}5$ units Penalty of 1 mark for incorrect rounding off (5,54 units)	A✓ substitution A✓ answer (2)
8.4	AE = 6,5 – 5,5 = 1 unit Accept 0,96 units	CA✓ answer (1)
8.5	$\frac{BD}{BC} = \frac{AE}{AC}$ [prop. theorem; AB ‖ ED] or [line ‖ to side of Δ] $= \frac{1}{6{,}5}$ Also: $\frac{BD}{BC} = \frac{EF}{EC}$ [prop. theorem; EB ‖ FD] or [line ‖ to side of Δ] $\therefore \frac{1}{6{,}5} = \frac{EF}{5{,}5}$ Penalise only once if parallel lines are left out in reason $\therefore EF = \frac{1 \times 5{,}5}{6{,}5}$ $\therefore EF = \frac{11}{13}$ units = 0,85 units Accept 0,82 units	S✓R✓ S/R✓ CA✓ substitution CA✓ answer (5)
		[16]

TOTAL: 150