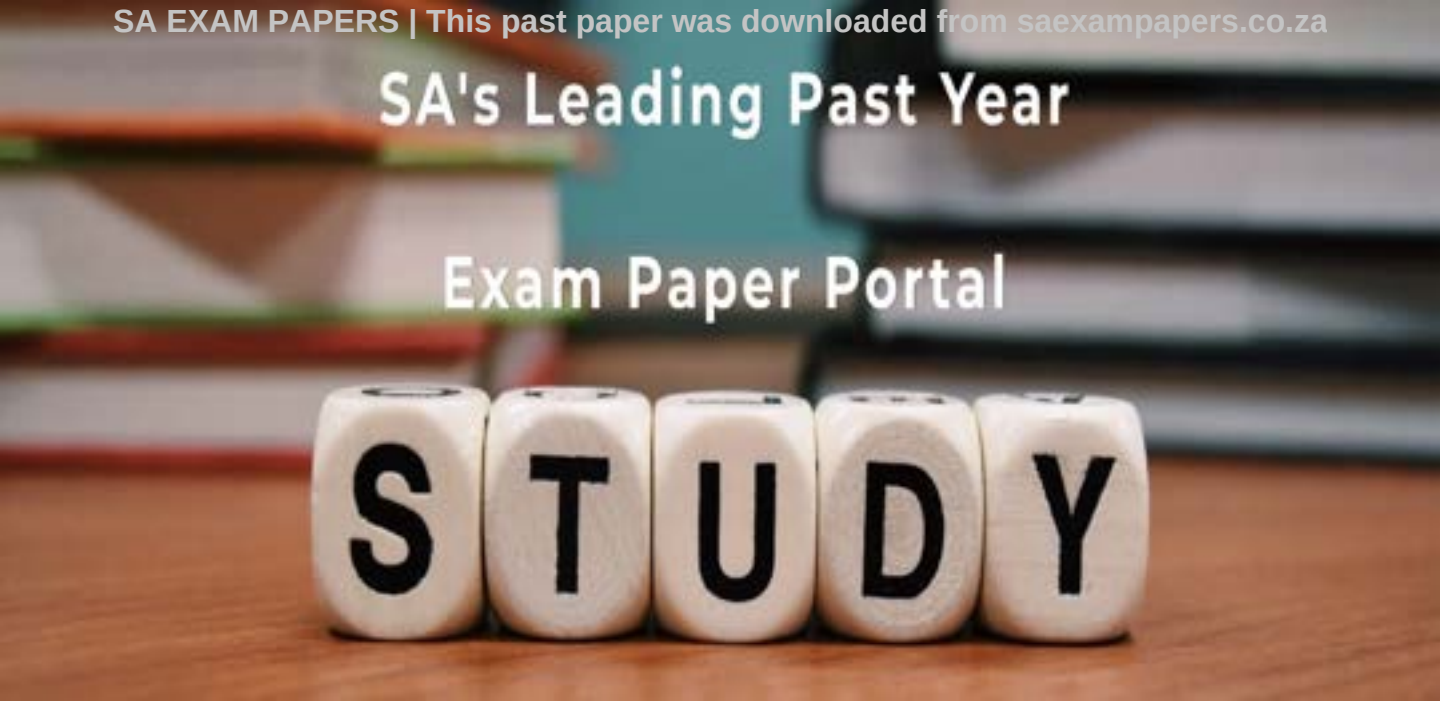


education

Department of Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS P2

JUNE EXAMINATION
2024

MARKING GUIDELINES

MARKS: 150

This Marking Guidelines consists of 19 pages

QUESTION/ VRAAG 1

1.1	$C(0;6)$	$C(0;6)$✓ (1)
1.2	$tanCX = m$ $tanC\hat{B}X = -1$ $C\hat{B}X = 135°$	Substitution ✓ $135°$ ✓ (2)
1.3	$D(0;10)$ $CD = 10 - 6$ $CD = 4$ **OR** $CD = \sqrt{(0-0)^2 + (6-10)^2}$ $CD = 4$	$CD = 10 - 6$ ✓ $CD = 4$ ✓ **OR** $CD = \sqrt{(0-0)^2 + (6-10)^2}$ ✓ $CD = 4$ ✓ (2)
1.4	E(– 4;8) $\quad$ E$\left(-\frac{4}{3};\frac{22}{3}\right)$ $M\left(\frac{-4+6}{2};\frac{8+0}{2}\right)$ $\quad$ $M\left(\frac{\frac{-4}{3}+6}{2};\frac{\frac{22}{3}+0}{2}\right)$ $M(1;4)$ $\quad$ $M\left(\frac{7}{3};\frac{11}{3}\right)$ $y - y_1 = m(x - x_1)$ $y - 4 = 2(x-1)$ $\quad$ $-\frac{11}{3} = 2\left(-\frac{7}{3}\right) + c$ $y = 2x + 2$ $\quad$ $y = 2x + 1$	Correct substitution ✓ answer ✓ Correct substitution ✓ equation✓ (3)
1.5	$tanE\hat{A}B = 2$ $E\hat{A}B = 63.43°$ $A\hat{E}B = 135° - 63.43°$ $A\hat{E}B = 71.57°$	$tanE\hat{A}B = 2$ ✓ $E\hat{A}B = 63.43°$ ✓ $A\hat{E}B = 135° - 63.43°$ ✓ $71.57°$ ✓ (4)
1.6	$2x = -10$ $x = -5$ $A(-5;0)$	$A(-5;0)$ ✓

	Using E(– 4 ; 8) midpoint EB = midpoint of AG. $1 = \frac{-5+x}{2}$ $\quad 4 = \frac{0+y}{2}$ $x = 7$ $\quad y = 8$ $G(7;\ 8)$ **OR** $2x = -10$ $x = -5$ $A(-5; 0)$ By using translation geometry : A → E $(x\,;y) \rightarrow \quad (x+1\,;y+8)$ ∴ B → G $B(6\,;0) \rightarrow \quad G(6+1\,;0+8)$ $\therefore\ G(7\,;8)$ **Using E** $\left(\frac{7}{3};\frac{11}{3}\right)$ midpoint EB = midpoint of AG. $\frac{7}{3} = \frac{-5+x}{2}$ $\quad \frac{11}{3} = \frac{0+y}{2}$ $x = \frac{19}{3}$ $\quad y = \frac{22}{3}$ $G\left(\frac{19}{3};\frac{22}{3}\right)$ **OR** $2x = -10$ $x = -5$ $A(-5; 0)$ By using translation geometry : A → E $(x\,;y) \rightarrow \quad (x+0{,}33\,;y+7{,}33)$ ∴ B → G $B(6\,;0) \rightarrow \quad G(6+0{,}33\,;0+7{,}33)$ $G\left(\frac{19}{3};\frac{22}{3}\right)$	Method ✓ Midpoint of diagonals $x-$ Co-ordinate. ✓ $y-$ Co-ordinate. ✓ **OR** $A(-5; 0)$ ✓ Method ✓ $x-$ Co-ordinate. ✓ $y-$ Co-ordinate. ✓ (4)

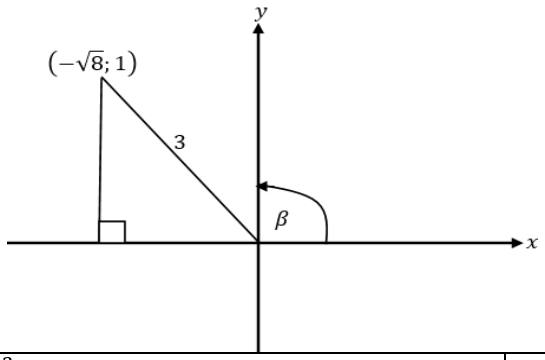
1.7	**Use G(7 ; 8)** In parm ABGE Perp h = 8 *units* Base = $x_B - x_A = 6 - (-5) = 11$ *units* Area parm ABGE = (perp h)(base) $= (8)(11) =$ 88 units^2 OR In ΔAEB Perp h = 8 *units* Base = $x_B - x_A = 6 - (-5) = 11$ *units* Area $\Delta AEB = \frac{1}{2}(b)(h)$ $= \frac{1}{2}(11)(8)$ $= 44\ units^2$ Area parm ABGE = (2)(Area ΔAEB) $= (2)44 = 88 units^2$ **Use** $G\left(\frac{19}{3};\frac{22}{3}\right)$ In parm ABGE Perp h = $\frac{22}{3}$ units Base = $x_B - x_A = 6 - (-5) = 11$ *units* Area parm ABGE = (perp h)(base) $= 11 \times \frac{22}{3} = 80,67$ units^2	Perp h = 8 ✓ Base $= 11$ *units* ✓ Substitution ✓ Area ✓ OR Perp h = $\frac{22}{3}$ ✓ Base $= 11$ *units* ✓ Substitution ✓ Area parm ABGE ✓ (4)
1.8	Midpoint of CD $(0 ; 8)$ M$(3 ; 4) \rightarrow$ C$(0; 8)$ M must be translated 3 units left and 4 units up.	midpoint ✓ ✓✓3 units left ✓✓4 units up (5)
		[24]

QUESTION/ VRAAG 2

2.1	$ML = 4 - (-5) = 9$ units. $(x-3)^2 + (y-4)^2 = 9^2$ $(x-3)^2 + (y-4)^2 = 81$	 Substitution ✓ Answer ✓ (2)
2.2	T(– 1;8) $m_r = \frac{8-4}{-1-3} = -1$ $m_{rad} \times m_{tan} = -1$ $m_{tan} = 1$ T(– 4 ; 8) $m_r = \frac{8-4}{-4-3} = -\frac{4}{7}$ $m_{rad} \times m_{tan} = -1$ $m_{tan} = \frac{7}{4}$ $y - y_1 = m(x - x_1)$ $y - (8) = (x - (-1))$ $8 = \frac{7}{4}(-4) + c$ $y = x + 7$ $y = \frac{7}{4}x + 15$ **T**$(-1; 4+\sqrt{65})$ $m = \frac{4+\sqrt{65}-4}{-1-3}$ $m = -2,02$ $m_{rad} \times m_{tan} = -1$ $m = \frac{4}{\sqrt{65}}$ $y = \frac{4}{\sqrt{65}}x + 8 + \sqrt{65}$	m_{rad} ✓ $m_{rad} \times m_{tan} = -1$ ✓ m_{tan}✓ Substitution ✓ equation✓ (5)
2.3	$(x-3)^2 + (y-4)^2 = {r_2}^2$ $(7-3)^2 + (3-4)^2 = 17$ ${r_2}^2 < {r_1}^2$ $17 < 81$ $\therefore$ *Inside the circle* **OR**	Substitution ✓ ${r_2}^2 = 17$ ✓ $17 < 81$ ✓ *Inside* ✓ (4) **OR**

	$d = \sqrt{(3-7)^2 + (4-3)^2}$ $d = \sqrt{17} = 4.12$ $d < r$ $\therefore$ *Inside the circle*	Substitution ✓ $d = \sqrt{17}$ ✓ $d < r$ ✓ *Inside* ✓ (4)
2.4	T(– 1;8) T(– 4 ; 8) $y_k = y_l = -5$ $y_k = y_l = -5$ $m_{rad} = m_{MK} = -1$ (collinear points) $-1 = \frac{4-(-5)}{3-x_k}$ $-\frac{4}{7} = \frac{4-(-5)}{3-x}$ $-3 + x = 9$ $-12 + 4x = 36$ $12 = x_k$ $12 = x_k$ **T**$(-1; 4+\sqrt{65})$ $y_k = y_l = -5$ $m_{rad} = m_{MK} = -1$ $-\frac{\sqrt{65}}{4} = \frac{9}{3-x}$ $-3\sqrt{65} + \sqrt{65}x = 36$ $7{,}47 = x_k$	$y_k = -5$ ✓ $m_{rad} = m_{MK}$ ✓✓ substitution✓ x-value✓ (5)
		[14]

QUESTION/VRAAG 3

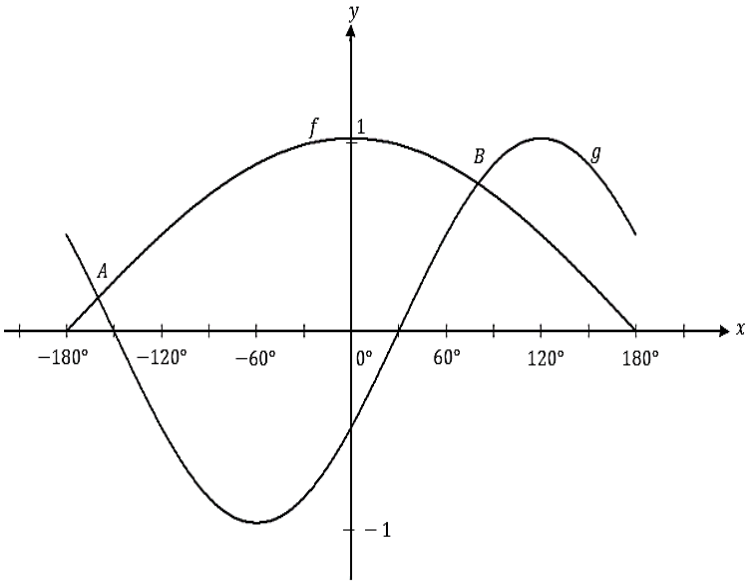


3.1.1	$r^2 = x^2 + y^2$ pyth $3^2 = x^2 + 1^2$ $x = -\sqrt{8}$ $tan\beta = -\frac{1}{\sqrt{8}}$	Method ✓ $x = -\sqrt{8}$ ✓ answer ✓ (3)
3.1.2	$cos2\beta = 1 - 2sin^2\beta$ $= 1 - 2\left(\frac{1}{3}\right)^2$ $= \frac{7}{9}$	double angle identity ✓ answer ✓ (2)
3.1.3	$\cos\left(-\beta - 450^\circ\right) = \cos(-\beta - 90°)$ $= -\cos(90° + \beta)$ $= \sin\beta = \frac{1}{3}$	$-\cos(90° + \beta)$ ✓ Answer ✓ (2)
3.2	$\frac{4\cos(-x).\cos(90^\circ + x)}{\sin(30^\circ - x).\cos x + \cos(30^\circ - x).\sin x}$ $= \frac{-4\cos x \sin x}{\frac{1}{2}\left(\cos^2 x + \sin^2 x\right)}$ $= \frac{-4\cos x \sin x}{\frac{1}{2}\left(1 - \sin^2 x + \sin^2 x\right)}$ $= -8\ sinxcosx = -4\sin 2x$	$4cosx$ ✓ $-sinx$ ✓ $\left(\cos^2 x + \sin^2 x\right)$ ✓ $\frac{1}{2}$ ✓ $-8sinx\cos x$ ✓ $-4sin2x$ ✓ (6)
3.3.1	$tan203°$	tan 23° ✓

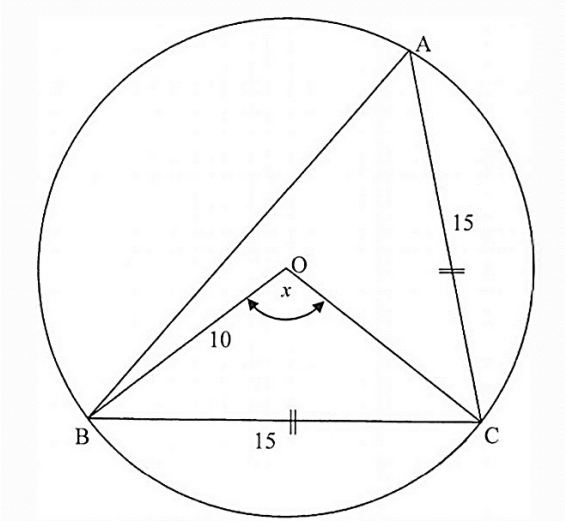
	$tan23° = \frac{\sqrt{1-a^2}}{a}$	$\sqrt{1-a^2}$ ✓ Answer ✓ (3)
3.3.2	$sin46°$ $= \sin 2(23°)$ $= 2sin23°cos23°$ $= 2a\sqrt{1-a^2}$	 double angle ✓ Expansion ✓ Answer ✓ (3)
3.4.1	$\sin(60° + 45°)$ $= sin60°.cos45° + cos60°.sin45°$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{\sqrt{3}+1}{2\sqrt{2}}$ OR $\frac{\sqrt{6}+\sqrt{2}}{4}$ OR $= \sin(180° - 75°)$ $= sin75°$ $= sin30°.cos45° + cos30°.sin45°$ $= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{\sqrt{3}+1}{2\sqrt{2}}$ OR $\frac{\sqrt{6}+\sqrt{2}}{4}$	compound angles ✓ expansion ✓ substitution ✓ answer ✓ (4) OR $sin75°$✓ expansion ✓ substitution ✓ answer ✓
3.4.2	$cos69°.cos9 + cos81°.cos21°$ $= cos69°.cos9° + sin9°.sin69°$ $= cos60°$ $= \frac{1}{2}$	co-function : $cos81°.cos21° = sin9°.sin69°$ ✓ $cos60°$✓ answer ✓ OR

	OR $sin21°.cos9° + cos21°.sin9°$ $= sin30°$ $= \frac{1}{2}$	co-function $cos69°.cos9° = sin21°.cos9°$ ✓ $sin30°$✓ answer ✓ (3)
3.5	$LHS = \frac{2sinx.cosx-cosx}{1-(1-2sin^2x)-sinx}$ $= \frac{2sinx.cosx-cosx}{2sin^2x-sinx}$ $= \frac{cosx(2sinx-1)}{sinx(2sinx-1)}$ $= \frac{cosx}{sinx}$ $= \frac{1}{\frac{sinx}{cosx}}$ $\frac{1}{tanx} = RHS$	$2sinx.cosx$ ✓ $1-2sin^2x$ ✓ factorization ✓ simplification ✓ identity for $\tan x$ ✓ (5)
3.6	$\cos 2x = \cos x + 2$ $2\cos^2 x - 1 - \cos x - 2 = 0$ $2\cos^2 x - \cos x - 3 = 0$ $(2\cos x - 3)(\cos x + 1) = 0$ $\cos x \neq \frac{3}{2}$ or $\cos x = -1$ $x = 180°$	✓ expansion ✓ standard form ✓factors ✓✓answers ✓ rejection ✓ $x = 180°$ (7)
		[38]

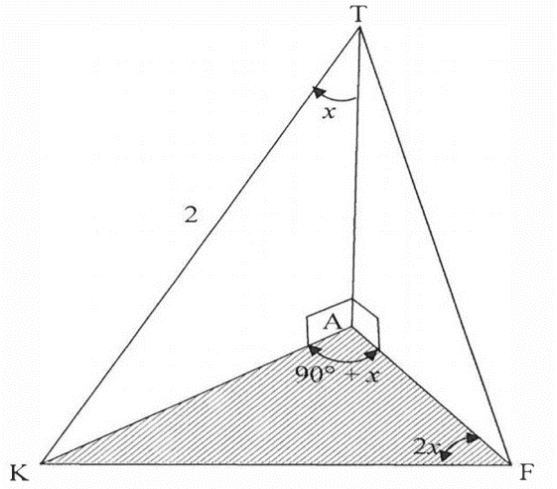
QUESTION/VRAAG 4

4		
4.1	$f(0°) - g(0°) = 1 - \left(-\frac{1}{2}\right) = \frac{3}{2}$ units	✓Answer ✓ (1)
4.2	Period of $f(4x) = \frac{360°}{4\left(\frac{1}{2}\right)}$ $= 180°$	✓substitution ✓answer (2)
4.3	Range : $y \in [-4\,;\,4]$ or $-4 \leq y \leq 4$	✓ ✓ answer (2)
4.4	$x_A = -160°$ $x_B = 80°$	$x_A = -160°$ ✓ $x_B = 80°$ ✓ (2)
4.5.1	$-160° < x < 80°$	✓ ✓ answer (2)
4.5.2	$0° < x < 30°$	✓ ✓ answer (2)
		[11]

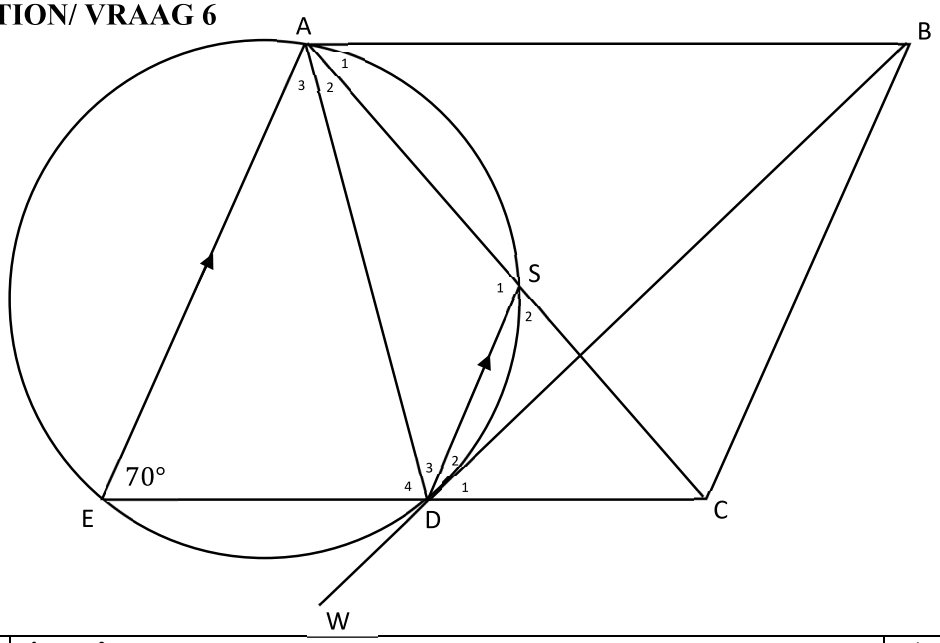
QUESTION/VRAAG 5



5.1.1	In ΔBOG : $15^2 = 10^2 + 10^2 - 2(10)(10)cosx$ $225 = 100 + 100 - 200cosx$ $\cos x = -\frac{25}{200}$ $x = 97{,}18°$	Substitution into cosine rule ✓ simplification ✓ Answer ✓ (3)
5.1.2	$\hat{A} = \frac{1}{2}(\hat{O}) = 48{,}59°$ ∠ center = 2 ∠ circumference $\hat{C} = 180° - 2(48{,}59°) = 82{,}82°$ Area $= \frac{1}{2}(15)(15)sin82{,}82°$ $= 111{,}62units^2$	$\hat{A} = 48{,}59°$ ✓ $\hat{C} = 82{,}82°$ ✓ Area rule ✓ Answer ✓ (4)

5.2		
5.2.1	$\frac{AK}{2} = sinx$ $AK = 2\sin x$	$\frac{AK}{2} = sinx$ ✓ Answer ✓ (2)
5.2.2	$\frac{KF}{\sin(90°+x)} = \frac{AK}{sin2x}$ $\frac{KF}{\cos x} = \frac{2sinx}{2sinxcosx}$ $\frac{KF}{\cos x} = \frac{1}{cosx}$ KF = 1 unit	Substitution into sine rule ✓ $cos\ x$ ✓ $2sinxcosx$ ✓ Simplification ✓ Answer ✓ (5)
		[14]

QUESTION/ VRAAG 6

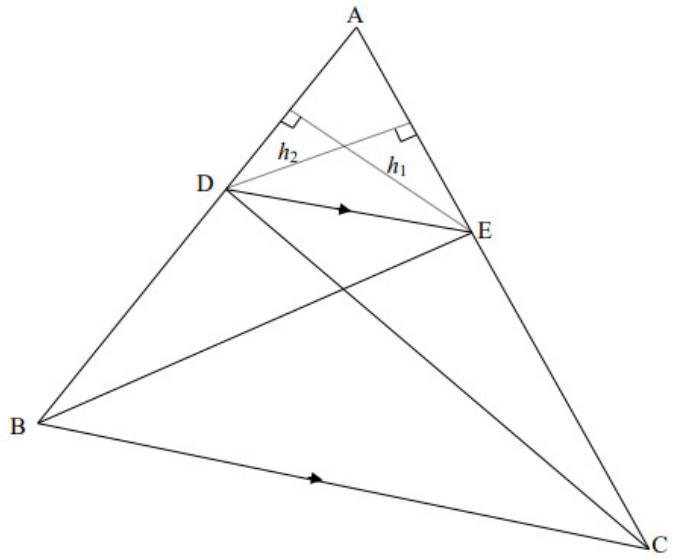


6.1.1	$\hat{S}_2 = \hat{E} = 70°$	ext ∠ of cyclic quad / buite ∠van kvh	✓ S ✓ R (2)
6.1.2	$\hat{S}_2 = \hat{A}_2 + \hat{A}_3 = 70°$ $\hat{A} + \hat{E} = 180°$ $\therefore \hat{A}_1 = 40°$	corresp ∠s; AE || DS / ooreenk. ∠e ; AE || DS co-int ∠s; AB || EC / ko-binne ∠e ; AB || EC	✓S/R ✓S/R ✓S (3)
6.1.3	$\hat{D}_3 = \hat{A}_1 = 40°$	tan chord theorem / raakl koord stelling	✓S ✓R (2)
6.1.4	AE = BC ED=DC and AE//SD ∴ AS = SC ∴ SD = ½ AE SD = 8 cm	Opp sides of parm / Teenoorst. Sye parm Given / gegee Converse midpt th. / Omgekeerde middelpt st.	✓S ✓S ✓ 8 cm (3)

6.2.1	$\hat{D}_3 = \hat{D}_1 = 40°$ $\therefore\ \hat{D}_1 = \hat{A}_1$ $\therefore\ ABCD$ is cq / kvh	Given / Gegee converse ∠s in the same seg omgekeerde ∠e in dies. Θ segm	✓S ✓R (2)
6.2.2	$\hat{B} = \hat{E} = 70°$ $A\hat{D}C + \hat{B} = 180°$ $\therefore \hat{D}_2 = 30°$ $\hat{S}_2 = \hat{A}_2 + \hat{D}_3$ $\therefore \hat{A}_2 = 30°$ $\therefore \hat{D}_2 = \hat{A}_2$ $\therefore DB$ is a tangent / DB is 'n raaklyn	opp ∠s of parm / teenoorst. ∠e van parm. opp ∠s of cyclic quad / teenoorst. ∠e van kvh ext ∠ of Δ/ buite ∠ van Δ converse tan chord theorem / omgekeerde raakl. koord stelling	✓S/R ✓S 30° ✓S / R ✓S ✓R (5) **[17]**

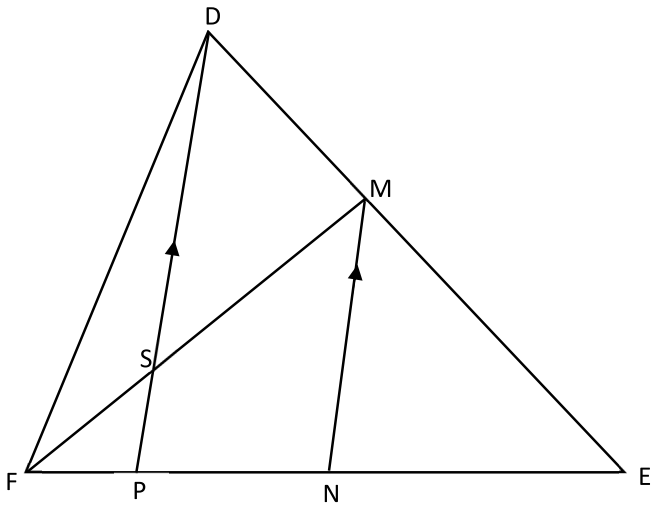
QUESTION / VRAAG 7

7.1



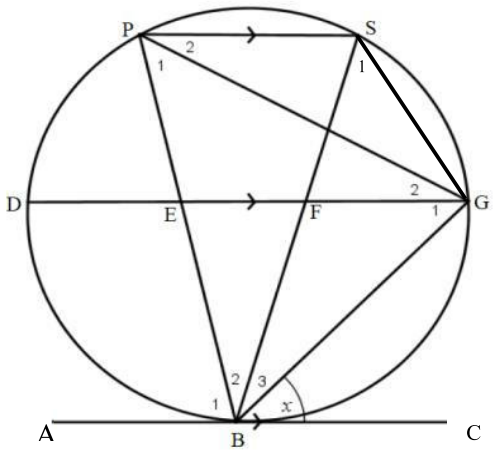
	Constr: Draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ But area ΔBDE = area ΔDEC [same base & height or DE $\parallel$ BC/ *dies basis & hoogte*; of DE $\parallel$ BC] $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ constr/*konstr* **OR** **reason: common vertex or same height** ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1}$ ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{AE}{EC}$ ✓ S / R ✓ S (5)

7.2



7.2.1	$\frac{PN}{NE} = \frac{DM}{ME}$ $\frac{PN}{NE} = \frac{2}{3}$ $\therefore PN = 35 \times \frac{2}{5}$ $PN = 14\ cm$	line || one side of Δ / lyn || een sy van Δ **or / of** prop theorem,DP//MN eweredighst. DP//MN	✓ S/ R ✓ 14 cm (2)
7.2.2	$\frac{FP}{FN} = \frac{FS}{FM}$ $FP = \frac{14}{2}$ $FP = 7$	line || one side of Δ / lyn || een sy van Δ _____	✓S / R ✓S ✓ A (3) **[10]**

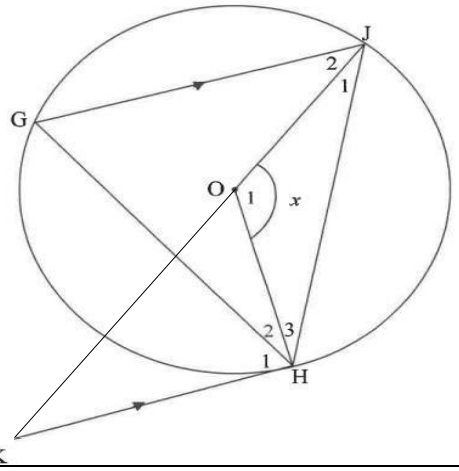
7.3



7.3.1	In ΔPGB and ΔGEB • $\hat{B}$ *is common* / gemeenskaplik - $\hat{G}_1 = x$ alt ∠s; DG || AC / verw. ∠e ; DG || AC - $\hat{P}_1 = x$ tan chord theorem / raakl. koord stelling • $\therefore \hat{P}_1 = \hat{G}_1$ • $P\hat{G}B = G\hat{E}B$ 3rd ∠s / 3de ∠e $\therefore \Delta PGB \text{ III } \Delta GEB$ $(\angle\angle\angle)$	✓R ✓R ✓R ✓R ∠∠∠ (4)
7.3.2	$\frac{PG}{GE} = \frac{GB}{EB} = \frac{PB}{GB}$ $\therefore GB^2 = PB.EB$ In ΔSGB and ΔGFB • $\hat{B}_3$ *is common* gemeenskaplik - $\hat{S}_1 = \hat{P}_1 = x$ ∠s in same segm. / ∠e in dies. segm. • $\therefore \hat{S}_1 = \hat{G}_1$ • B$\hat{F}G = S\hat{G}B$ 3rd ∠s / 3de ∠e $\therefore$ ΔSGB III ΔGFB $(\angle\angle\angle)$ $\therefore GB^2 = SB.FB$ $\therefore SB.FB = EB.PB$	$GB^2 = PB.EB$✓ S/R✓ $GB^2 = SB.FB$ ✓ (3)

7.3.3	$\frac{PG}{GE} = \frac{GB}{EB}$ $\therefore \frac{PG}{9} = \frac{9}{0,6PG}$ $0,6PG^2 = 81$ $PG = \sqrt{135}$ $PG = 11,62cm$	sub 9 ✓ sub 0,6PG ✓ 11,62cm✓ (3) **[10]**

QUESTION/ VRAAG 8



8.1.1	$2\hat{G} = \hat{O}_1$ $\hat{G} = 60^0$	∠ at centre = 2 ×∠ at circumference Midpts∠ = 2 x Omtreks∠	✓ R ✓ answer (2)
8.1.2	$\hat{H}_3 = \hat{J}_1$ $\hat{H}_3 + \hat{J}_1 + \hat{O}_1 = 180°$ $\hat{H}_3 = 30°$	∠s opp equal radii / ∠e teenoor gelyke rad ∠ sum in Δ / binne ∠e van Δ	✓S/R ✓S ✓answer (3)
8.2	$\hat{K} = 120 - 90°$ $\sin(120 - 90°) = \frac{OH}{OK}$ $OH = OJ$ $cosx = -\frac{OJ}{OK}$	ext ∠ of ΔKOH / buite ∠ van ΔKOH ΔOKH Radii / radiuses	✓S ✓trig ratio ✓S (3) **[8]**

TOTAL 150