education

Department of Education
FREE STATE PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 1/WISKUNDE VI

JUNE/JUNIE 2024

MARKING GUIDELINES/NASIENRIGLYNE

This marking guideline consists of 14 pages

QUESTION 1

1.1.1	$(x+2)(x-5)=0$ $x=-2$ or $x=5$	✓ $x=-2$ ✓ $x=5$ (2)
1.1.2	$x(2x+3)=3$ $2x^2+3x-3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-3)}}{2(2)}$ $x=\frac{-3\pm\sqrt{33}}{4}$ $x=-2{,}19$ or $x=0{,}69$	✓ standard form ✓ substitution ✓ $x=\frac{-3\pm\sqrt{33}}{4}$ ✓ $x=-2{,}19$ ✓ $x=0{,}69$ (5)
1.1.3	$(x-1)(2-x)\geq 0$ $(x-1)(x-2)\leq 0$ CV: 1 and 2 $1\leq x\leq 2$	✓ method ✓ ✓ $1\leq x\leq 2$ (3)
1.1.4	$x+3\sqrt{x-1}=1$ $(3\sqrt{x-1})^2=(1-x)^2$ $9x-9=1-2x+x^2$ $x^2-11x+10=0$ $(x-10)(x-1)=0$ $x=10$ or $x=1$ $\therefore x=1$	✓ squaring both sides ✓ standard form ✓ factors ✓ both answers ✓ $x=1$ (5)
1.2.1	$\left(\frac{1}{81}\right)^{-x}=9^{y+3}$ $(9^{-2})^{-x}=9^{y+3}$ $9^{2x}=9^{y+3}$ $2x=y+3$ $y=2x-3$	✓ same base ✓ equating the exponents (2)
1.2.2	$y^2+x^2-3x=-1$ $(2x-3)^2+x^2-3x=-1$ $4x^2-12x+9+x^2-3x+1=0$ $5x^2-15x+10=0$ $x^2-3x+2=0$ $(x-2)(x-1)=0$ $x=2$ or $x=1$ $y=2(2)-3=1$ $y=2(1)-3=-2$	✓substitution ✓ standard form ✓ factors ✓ x-values ✓y-values (5)

1.3	$x^2+px^2+2px+p=1$ $x^2+px^2+2px+p-1=0$ $x^2(1+p)+2px+(p-1)=0$ $x^2(1+p)+2px+(p-1)=0$ $b^2-4ac=(2p)^2-4(1+p)(p-1)$ $=4p^2-4p^2+4$ $=4$ $4>0$ Always two distinct roots except when $p=-1$	✓ substitution ✓ $=4>0$ ✓ $p=-1$ Answer only (1 mark) (3)
		[25]

QUESTION 2

2.1.1	$(m+1);(m^2+m);(3m^2-m-4)$ differences : $(m^2-1);(2m^2-2m-4)$ $m^2-1=2m^2-2m-4$ $m^2-2m-3=0$ $(m+1)(m-3)=0$ $m=-1$ or $m=3$	✓ equating ✓ factors ✓ answers (3)
2.1.2	$(m+1);(m^2+m);(3m^2-m-4)$ $3+1;(3)^2+3;3(3)^2-3-4$ 4 ; 12 ; 20	✓ *substitution* ✓ answer (2)
2.2.1	-35	✓ answer (1)
2.2.2	$1;-5;-13;-23.....$ $2a=-2$ $a=-1$ $3a+b=-6$ $b=-3$ $a+b+c=1$ $(-1)+(-3)+c=1$ $c=5$ $T_n=-n^2-3n+5$	✓ a ✓ b ✓ c ✓ n[th] term (4)
2.2.3	$T_n=-n^2-3n+5$	

	$-n^2-3n+5=-299$ $n^2+3n-304=0$ $(n-16)(n+19)=0$ $n=16$ or $n=-19$ $n=16$	✓ equating ✓ factors ✓ $n=16$ (3)
2.2.4	The maximum value of T_n is 1.	✓✓ answer (2)
		[15]

QUESTION 3

3.1.1	___; 6 ; 12 ; 24 ; 48 ; .... $r=2$	✓ $r=2$ (1)
3.1.2	$T_6=96$	✓ (1)
3.1.3	$T_n=ar^{n-1}$ $T_n=3(2)^{n-1}$	✓ a ✓ $T_n=3(2)^{n-1}$ (2)
3.2	$a+42\ ;\ ar+32\ ;\ ar^2+2$ $ar^2+2=ar+32$ $ar^2-ar=32-2$ $ar(r-1)=30..........(i)$ $ar+32=a+42$ $ar-a=42-32$ $a(r-1)=10.........(ii)$ $ar(r-1)=30.......(i)$ $a(r-1)=10.......(ii)$ $r=3$ $a(r-1)=10$ $a(3-1)=10$ $2a=10$ $a=5$ $5+42\ ;\ 5(3)+32\ ;\ 5(3)^2+2$ $5\ ;\ 15\ ;\ 45$	✓ method ✓ value of a ✓ value of r ✓ first 3 terms (4)

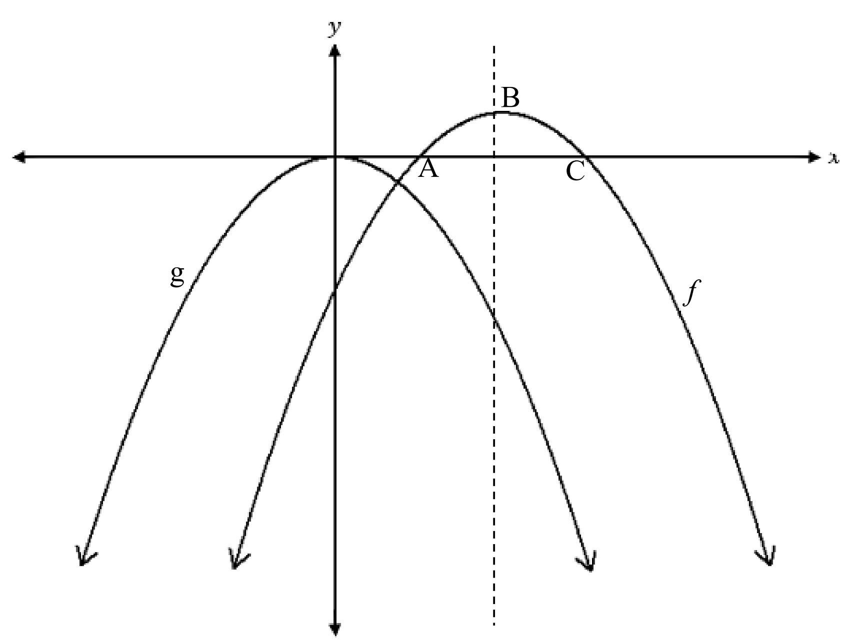
3.3.1	The first term of an arithmetic sequence is 51 and the eighth term is 100. $a = 51$ $a + 7d = 100$ $51 + 7d = 100$ $7d = 49$ $d = 7$	✓ substitution ✓ answer (2)
3.3.2	$T_n = a + (n-1)d$ $T_{20} = 51 + (20-1)(7)$ $T_{20} = 184$	✓ substitution ✓ answer (2)
3.4	7 ; 14 ; 21.......994 $T_n = a + (n-1)d$ $994 = 7 + (n-1)(7)$ $994 = 7n$ $n = 142$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{142}{2}[2(7) + (142-1)(7)]$ $S_n = 121121$	✓ substitution ✓ $n = 109$ ✓ substitution ✓ Sum (4)
		[14]

QUESTION 4

4.1.1	$81x^2 + 27x^3 + 9x^4 + \ \ldots$ $r = \frac{27x^3}{81x^2} = \frac{x}{3}$ For a series to converge $-1 < r < 1$ $-1 < \frac{x}{3} < 1$ $-3 < x < 3$	✓ r ✓ *substitution* ✓ answer (3)
4.1.2		

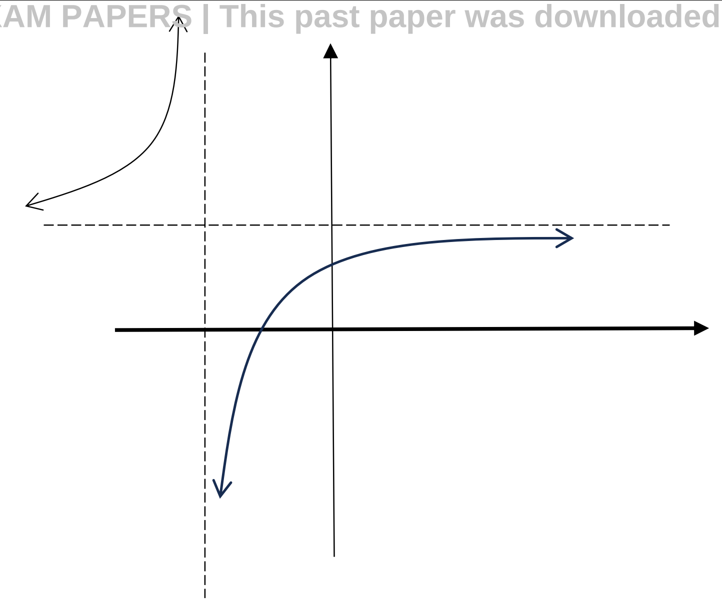
	$S_\infty = \dfrac{a}{1-r}$ $S_\infty = \dfrac{\frac{81}{4}}{1-\frac{1}{6}}$ $S_\infty = \dfrac{243}{10}$	✓ r ✓ substitution ✓ answer (3)
4.2	$80 = \sum_{n=1}^{20}(25+np)$ $(25+p)+(25+2p)+(25+3p)+..........+(25+20p)$ $S_n = \dfrac{n}{2}[a+l]$ $S_n = \dfrac{142}{2}[25+p+25+20p]$ $80 = \dfrac{142}{2}[25+p+25+20p]$ $50+21p=8$ $p=-2$ **Or** $S_n = \dfrac{n}{2}[2a+(n-1)d]$ $80 = \dfrac{20}{2}[2(25+p)+(20-1)p]$ $8=[50+2p+19p]$ $8-50=21p$ $-42=21p$ $p=-2$	✓ series ✓ substitution ✓ equating ✓ answer (4) ✓ series ✓ substitution ✓ equating ✓ answer (4)
		[16]

QUESTION 5

5.1	$B(2;1)$	✓ $B(2;1)$ (1)
5.2	$x=2$	✓ substitution (1)
5.3	Range $y\in(-\infty;2]$ / $y\le 2$	✓ ✓ answer (2)
5.4	$y=-x^2+4x-3$ $x^2-4x+3=0$ $(x-1)(x-3)=0$ $x=1$ or $x=3$ $A(3;0)$ and $B(1;0)$	✓ factors ✓ $A(3;0)$ ✓ $B(1;0)$ (3)
5.5	$x\le 1$ or $x\ge 3$	✓ ✓ answer (2)
5.6	average gradient $=\dfrac{1-0}{2-3}=-1$ $=-1$	✓ substitution ✓ answer (2)
5.7	$g(x)=-x^2$ 	✓ $g(x)=-x^2$ ✓ shape ✓ intercepts (3)
		[14]

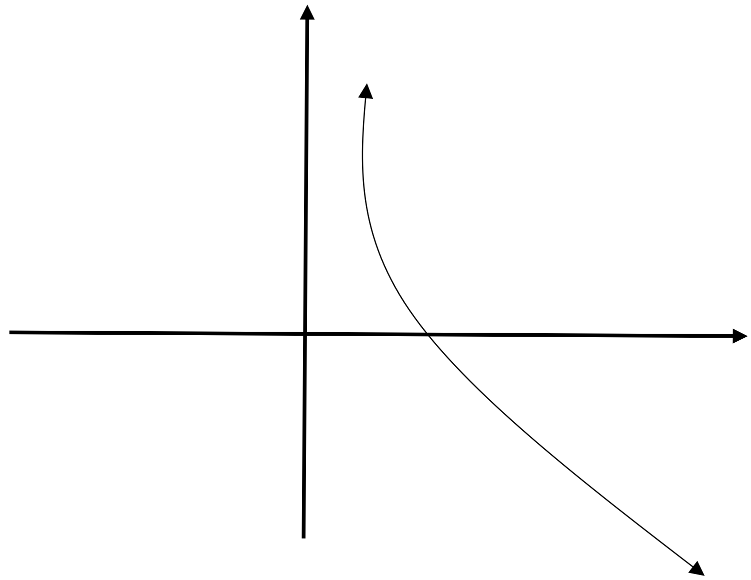
QUESTION 6

6.1.1	$h(x) = -x + c$ $y = -x + c$ $2 = -(3) + c$ $c = 5$ $h(x) = -x + 5$	✓ subst. ✓ answer (2)
6.1.2	$y = -x + 5$ $4 = -x + 5$ $x = 1$ $A(1;4)$	✓ x-value ✓ coordinates (2)
6.1.3	$f(x) = \frac{a}{x+p} + 4$ $y = \frac{a}{x-1} + 4$ $-2 = \frac{a}{0-1} + 4$ $a = 6$ $f(x) = \frac{6}{x-1} + 4$	✓ subst. ✓ a value ✓ equation (3)
6.1.4	$f(x) = \frac{6}{(x+1)-1} + 4$ $f(x) = \frac{6}{x} + 4$ $y = 4$ $x = 0$	✓✓ asymptotes (2)
6.1.5	$D'\left(-\frac{9}{4};\frac{5}{8}\right)$	✓✓ (2)

6.2		✓ asymptotes ✓ intercepts ✓ shape (3)
		[14]

QUESTION 7

7.1	$f(x)=k^x,\ \left(2;\frac{1}{9}\right)$ $y=k^x$ $\frac{1}{9}=k^2$ $k=\pm\frac{1}{3}$ $k=\frac{1}{3}$ $f(x)=\left(\frac{1}{3}\right)^x$	✓ substitution ✓ answer (2)
7.2	$y\in(0;\infty)\,/\,y>0$	✓ answer (1)
7.3	By reflecting graph across the line $y = x$.	✓ answer (1)
7.4	$y=\left(\frac{1}{3}\right)^x$ $x=\left(\frac{1}{3}\right)^y$ $\qquad y=\log_{\frac{1}{3}} x$	✓ swap x and y ✓ answer (2)

<table>
<tr><td>7.5</td><td></td><td>✓ shape
✓asymptote
✓ x-intercept

(3)</td></tr>
<tr><td>7.6</td><td>$\left[f(x)\right]^2-\left[f(-x)\right]^2=f(2x)-f(-2x)$

LHS $=\left[\left(\frac{1}{3}\right)^x\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]^2$
$=3^{-2x}-3^{2x}$

RHS $=\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$
$=3^{-2x}-3^{2x}$

$\therefore LHS=RHS$</td><td>✓ substitution LHS
✓ substitution RHS

✓ simplification

(3)</td></tr>
<tr><td></td><td></td><td>**[12**</td></tr>
</table>

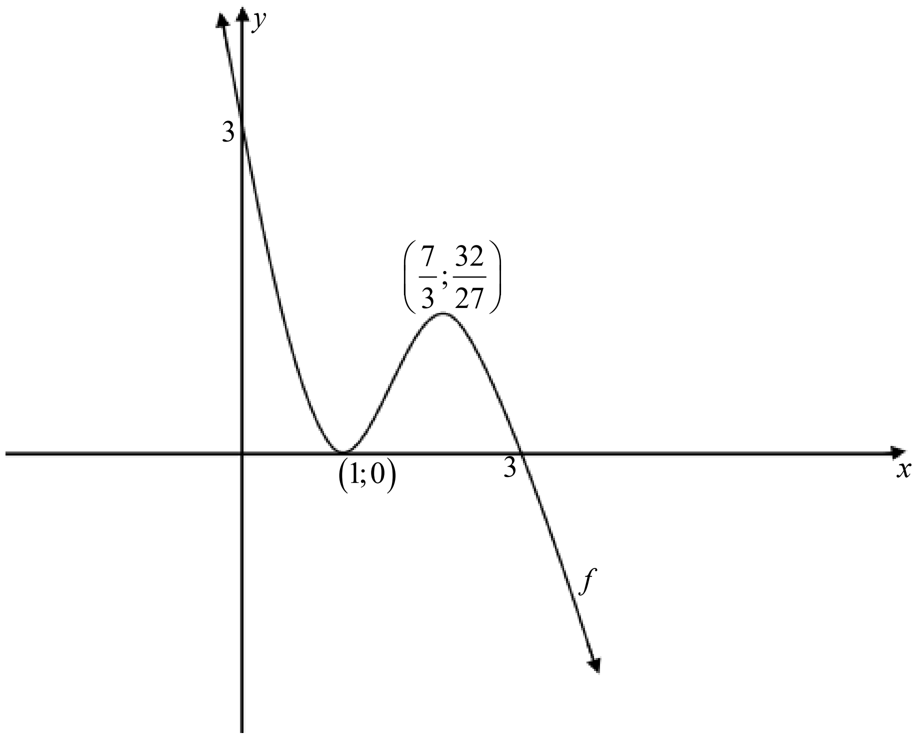
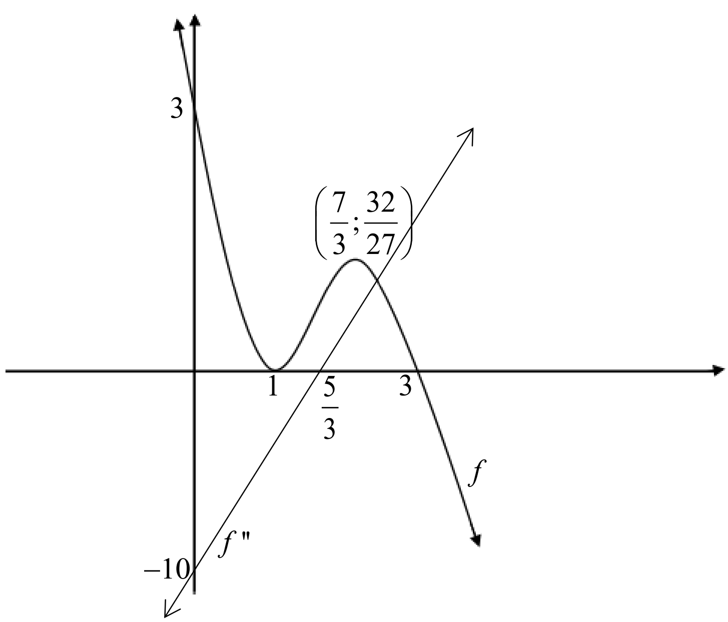
QUESTION 8

8.1	$f(x)=x^2-3$ $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\frac{(x+h)^2-3-(x^2-3)}{h}$ $f'(x)=\lim\limits_{h\to 0}\frac{x^2+2xh+h^2-3-x^2+3}{h}$ $f'(x)=\lim\limits_{h\to 0}\frac{2xh+h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\frac{h(2x+h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(2x+h)$ $f'(x)=2x$	✓ substitution ✓ simplification ✓ common factor ✓ $=\lim\limits_{h\to 0}(-2x-h)$ ✓ answer (5)
8.2	$y=\frac{9x^4-6}{3x}$ $y=\frac{9x^4}{3x}-\frac{6}{3x}$ $y=3x^2-2x^{-1}$ $\frac{dy}{dx}=6x+2x^{-2}$	✓ $y=3x^2-2x^{-1}$ ✓ $6x$ ✓ $2x^{-2}$ (3)
8.3	$\frac{d}{dx}\left[\frac{\sqrt[3]{x^3}-2x\sqrt{x}}{3x}\right]$ $=\frac{d}{dx}\left[\frac{x-2x^{\frac{3}{2}}}{3x}\right]$ $=\frac{d}{dx}\left[\frac{1}{3}-\frac{2}{3}x^{\frac{1}{2}}\right]$ $=\frac{d}{dx}\left[\frac{1}{3}\right]-\frac{d}{dx}\left[\frac{2}{3}x^{\frac{1}{2}}\right]$ $=0-\frac{1}{3}x^{-\frac{1}{2}}$ $=-\frac{1}{3\sqrt{x}}$	✓ x ✓ $2x^{\frac{3}{2}}$ ✓ $0-\frac{1}{3}x^{\frac{1}{2}}$ ✓ answer (4)

8.4	$f(x)=x^3-2x+1$ $f'(x)=3x^2-2$ $3=3x^2-2$ $\frac{5}{3}=x^2$ $x=\pm\sqrt{\frac{5}{3}}$	✓ $3=3x^2-2$ ✓ $x=\pm\sqrt{\frac{5}{3}}$ (2)
		[14]

QUESTION 9

9.1.1	$f(x)=-x^3+5x^2-7x+3$ $(0;3)$	✓ $(0;3)$ (1)
9.1.2	$f(x)=-x^3+5x^2-7x+3$ $(x-1)(-x^2+4x-3)=0$ $(x-1)(x-1)(x-3)=0$ $x=1$ or $x=3$	✓ linear factor ✓ quadratic factor ✓ factors ✓ answer (4)
9.1.3	$f(x)=-x^3+5x^2-7x+3$ $f'(x)=-3x^2+10x-7$ $-3x^2+10x-7=0$ $3x^2-10x+7=0$ $(3x-7)(x-1)=0$ $x=\frac{7}{3}$ or $x=1$ $f\left(\frac{7}{3}\right)=-\left(\frac{7}{3}\right)^3+5\left(\frac{7}{3}\right)^2-7\left(\frac{7}{3}\right)+3=\frac{32}{27}$ $f(1)=-(1)^3+5(1)^2-7(1)+3=0$ $(1,0)$ and $\left(\frac{7}{3},\frac{32}{27}\right)$	✓ $f'(x)=0$ ✓ factors ✓ x-values ✓ coordinates (4)

9.1.4		✓ y-intercept ✓ x-intercepts ✓ turning points ✓ shape (4)
9.1.5	$f'(x)=-3x^2+10x-7$ $f''(x)=-6x+10$ $-6x+10=0$ $-6x=-10$ $x=\frac{5}{3}$	✓ $f''(x)=-6x+10$ ✓ $-6x+10=0$ ✓ $x=\frac{5}{3}$ (3)
9.1.6		✓ intercepts ✓ shape (2)

9.1.7	$f''(x) > 0$ $-6x+10 > 0$ $-6x > 10$ $x < \frac{5}{3}$	✓✓ answer (2)
9.1.8	$x \in \left(1;\frac{7}{3}\right) \cup (3;\infty)$	✓✓ answer (2)
9.2	$f(x) = x^3 + 3x^2 - 24x + 20$ $x^3 + 3x^2 - 24x + 20 = -8$ $x^3 + 3x^2 - 24x + 28 = 0$ $(x-2)(x-2)(x+7) = 0$ $x = 2$ or $x = -7$ $P(-7,-8)$	✓ equating ✓ factors ✓ x-values ✓ coordinates of P (4)
9.3.1	$x=1$ and $x=5$	✓✓ answer (2)
9.3.2	The graph of *f* is decreasing on the intervals $x \in (0,1)$ and $x \in (5,6)$.	✓✓ notation, end points ✓✓ notation, end points (4)
		[32]