education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

OMNIA LABOR VINCIT

| NATIONAL |
| SENIOR CERTIFICATE |

| GRADE 12 |

| MATHEMATICS P2 (FINAL) |
| JUNE 2024 |

MARKS: 150

TIME: 3 HOURS

This question paper consists of 14 pages including 1 information sheet
and an answer book is provided.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Write neatly and legibly.

QUESTION 1

Nine learners wrote a test out of 30 their marks are as follows

7	9	9	13	17	21	24	26	27

1.1   Write down the five number summary of the data.   (4)

1.2   Draw a box and whisker diagram to represent the data.   (3)

1.3   Comment on the skewness of the data.   (1)

1.4   If a learner's mark lies below one standard deviation from the mean, the learner must attend an intervention class. How many learners will not attend the intervention class.   (2)

1.5   The teacher omitted a question with a total of two marks. Determine the actual

     1.5.1   Standard deviation   (1)

     1.5.2   Mean   (1)

     1.5.3   Upper quartile   (1)

                                                                [13]

QUESTION 2

The ogive below shows the money collected by parents during a fundraising event at a school.
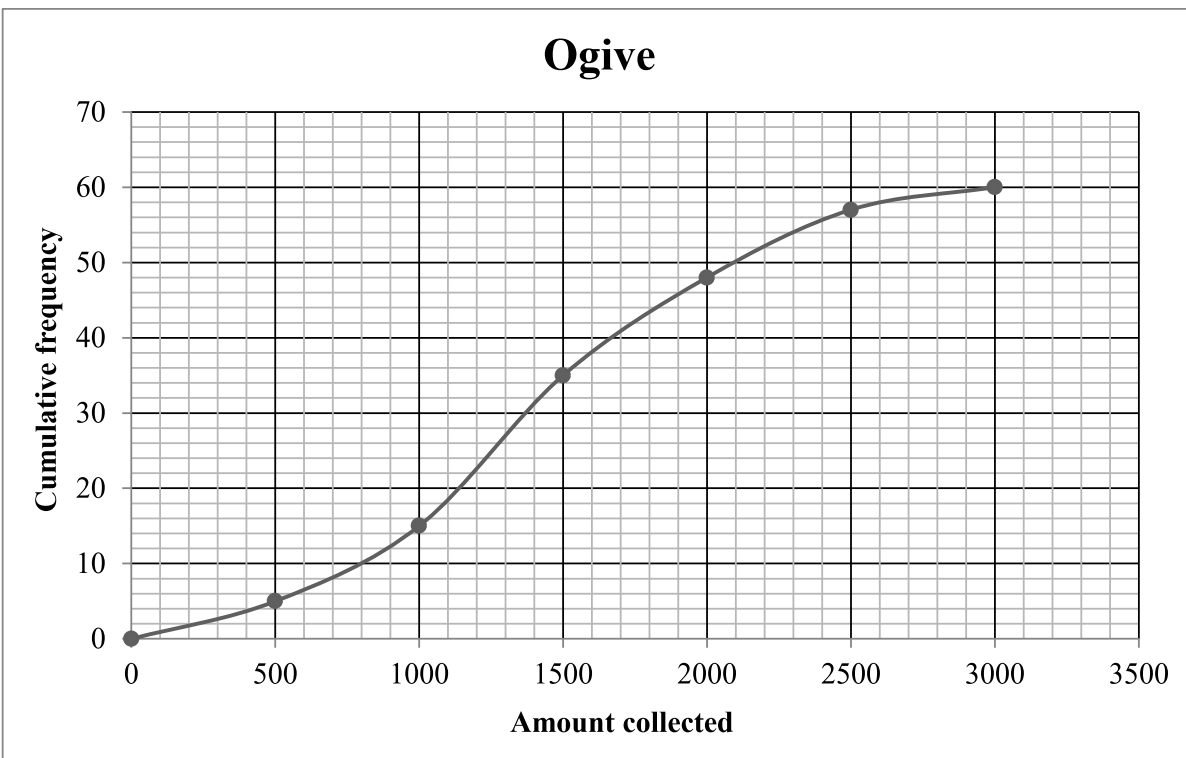
2.1   How many parents were present.   (1)
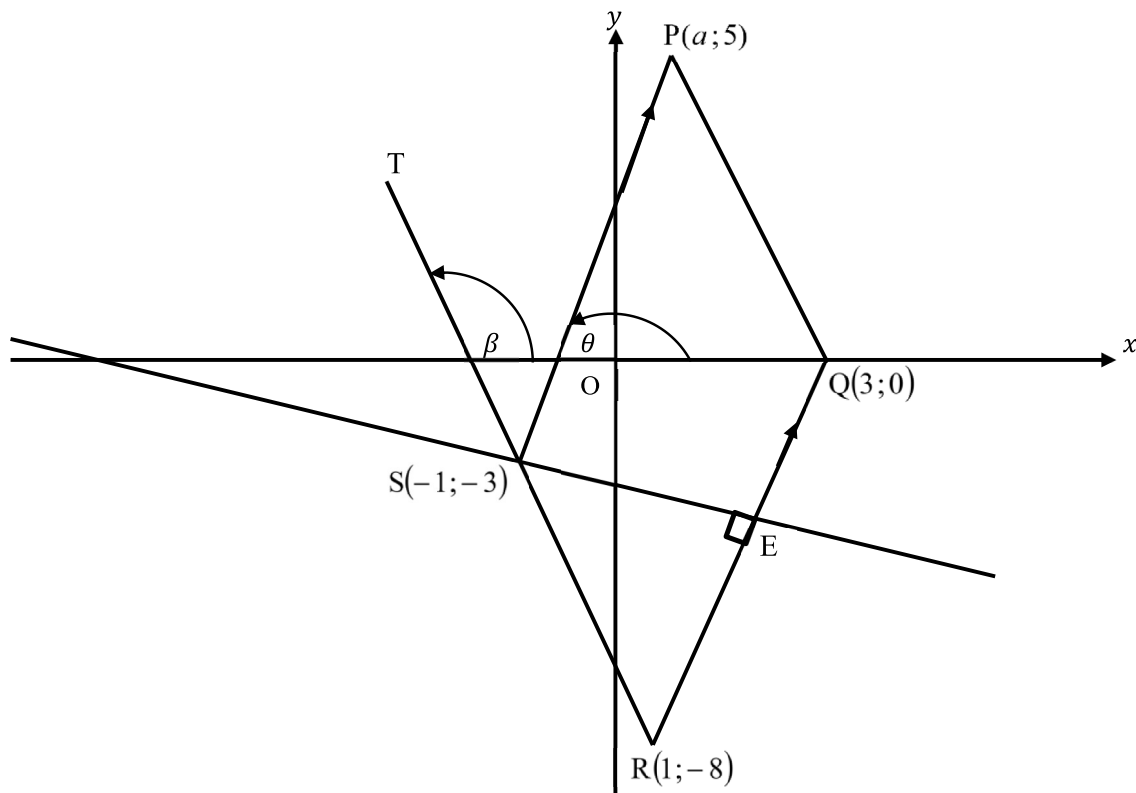
2.2   Write down the modal class.   (1)

2.3   Estimate the number of parents that can contribute R1 900.   (2)

2.4   Estimate the semi-interquartile range.   (3)

[7]

QUESTION 3

In the diagram below, PQRS is parallelogram with SP ∥ QR and vertices P$(a;5)$, Q$(3;0)$, R$(1;-8)$ and S$(-1;-3)$. SE ⊥ QR.

3.1   Determine the length of QR.   (2)

3.2   Determine the gradient of QR.   (2)

3.3   Determine the equation of SE   (3)

3.4   Determine the equation of a circle passing through SER   (3)

3.5   Calculate the value of a.   (3)

3.6   Calculate the value of:

       3.6.1 θ (2)

       3.6.2 $S\hat{R}Q$ (5)

3.7   Determine the area of PSRQ   (3)
                                                                                                                            [23]

QUESTION 4

A circle with equation $x^2 + 2x + y^2 + 2y = 4$ has centre C. FAE is a tangent to the circle at $A(-3;1)$ with $E\hat{A}B = 33°$
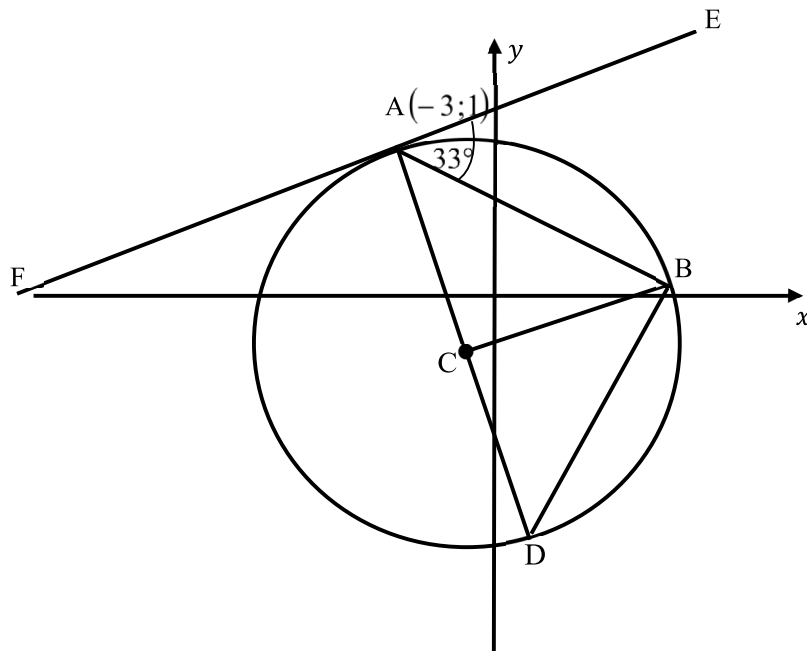
4.1   Determine the coordinates of C.   (2)

4.2   Determine the equation of the tangent to the circle at A.   (5)

4.3   Determine the value of k if $y = x + k$ will be a secant to the circle.   (3)

4.4   Calculate the size of $A\hat{C}B$.   (4)

4.5   Another circle with equation $(x+3)^2 + (y+2)^2 = 4$ is drawn, will the two circles   (4)
        intersect

                                                                                        [18]

QUESTION 5

5.1   Given $\cos 11° = k$. Determine the following in terms of k, **without using a calculator**.

     5.1.1 $\sin 11°$ (2)

     5.1.2 $\sin 22°$ (2)

     5.1.3 $\cos 19°$ (3)

     5.1.4 $\cos 5{,}5°$ (3)

5.2   Simplify the following expression without a calculator:

$$\frac{\sin 25° \cos(x - 180°) \sin 2x}{\sin^2(90 + x)\cos 65°}$$ (5)

5.3   Given: $\dfrac{\sin x - \cos x}{\sin x + \cos x} - \dfrac{\sin x + \cos x}{\cos x - \sin x} = 2\tan 2x$

     5.3.1   Prove the identity.   (5)

     5.3.2   For which values of x will $\dfrac{\sin x - \cos x}{\sin x + \cos x} - \dfrac{\sin x + \cos x}{\cos x - \sin x}$ be undefined?   (4)

[24]

QUESTION 6

Consider the functions: $f(x) = \cos 2x$ and $g(x) = \sin(x - 45°)$, where $x \in [-90°; 180°]$

6.1   On the same set of axis sketch the graphs of f and g (5)

6.2   Determine the general solution of $f(x) = g(x)$.   (4)

6.3   Determine the $x - \text{value}(x)$ in the interval $x \in [0°; 180°]$ for which:

     6.3.1 $f(x) > g(x)$ (2)

     6.3.2 $x \, f(x). g(x) > 0$ (2)
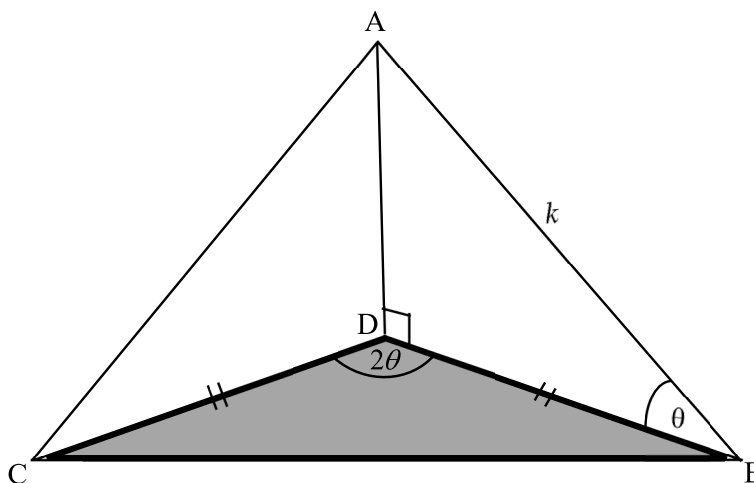
                                                               [13]

QUESTION 7

AD represents a vertical tower, with A at the top and D at the foot of the tower. D is a point on the tower below A. On the same horizontal level as C is point B, AB = k metres. $\hat{CDB} = 2\theta$ and $\hat{ABD} = \theta$.
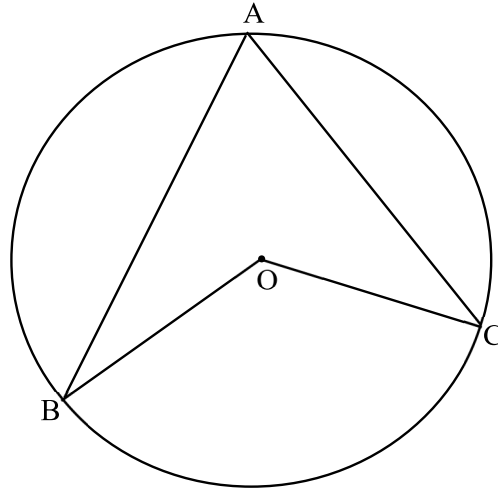

7.1   Show that $CB = k.\sin 2\theta$

(7)

7.2   If $k = 10$ units and $\theta = 30°$ calculate the area of $\triangle CBD$

(4)

[11]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

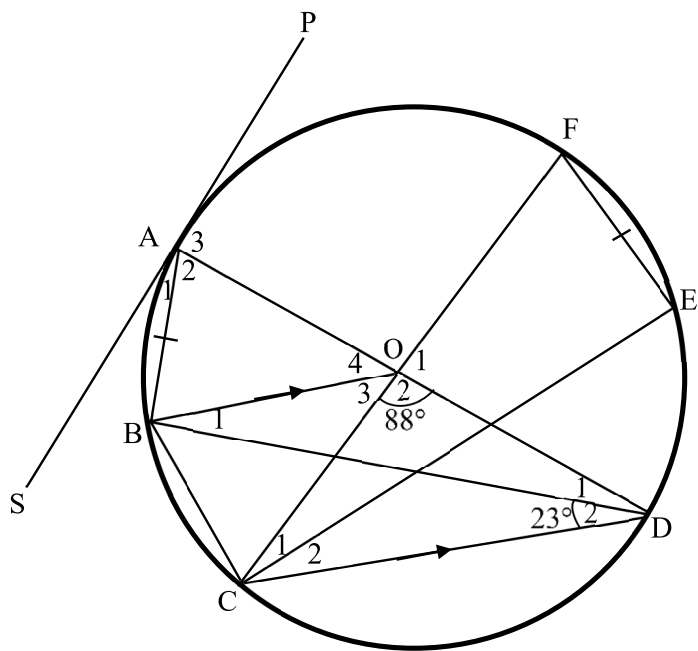8.1   In the diagram below O is the centre of the circle

        Prove that $B\hat{O}C = 2B\hat{A}C$ (5)

8.2   In the diagram below O is a centre of the circle with FE = AB and BO ∥ CD. $\hat{O}_2 = 88°$ and $\hat{D}_2 = 23°$. Tangent PAS touches the circle at A.

Determine with reasons the size of the following angles:

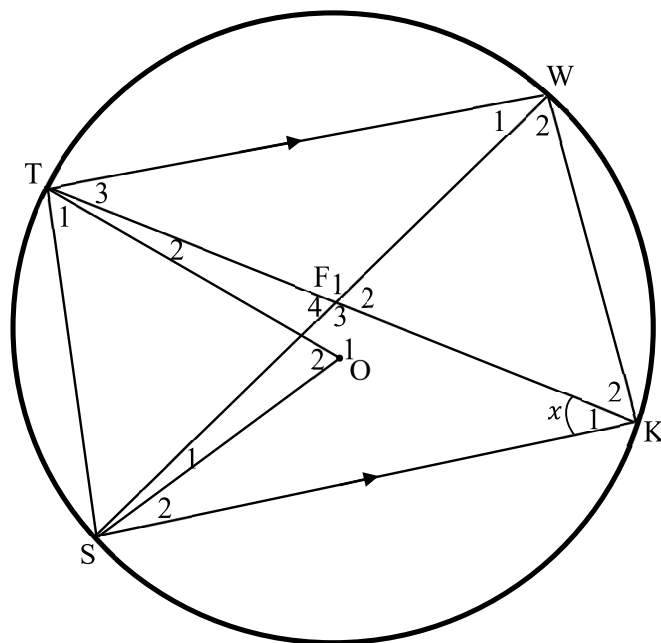8.2.1 $\hat{O}_3$ (2)

8.2.2 $\hat{D}_1$ (3)

8.2.3 $\hat{A}_1$ (2)

8.2.4 $\hat{C}_1$ (2)

8.2.5 $\hat{C}_2$ (2)

[16]

QUESTION 9

In the diagram below TWSK lies on the the circumference of the circle, such that TW || SK with O the centre on the circle. $K_1 = x$

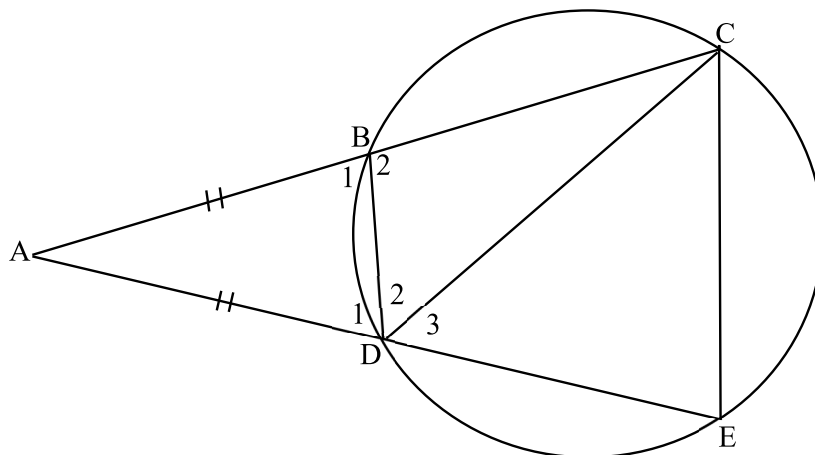9.1   Determine with reasons, THREE other angles equal to x.   (4)

9.2   Prove that SOFT is a cyclic quadrilateral   (4)

9.3   Prove that ST $= 2$SO.$\sin x$ (4)

[12]

QUESTION 10

In the diagram below BCED is a cyclic quadrilateral with AB =AD , AB = 2 cm, BC = 3 cm and BD = 4 cm.

10.1   Prove that BD || CE   (3)

10.2   Calculate the length of DE   (2)

10.3   Prove that ΔADB |||ΔAEC   (3)

10.4   Calculate the length of EC   (2)

10.5   Calculate the value of $\dfrac{\text{Area of }\Delta ECD}{\text{Area of}\Delta ABD}$ (3)

[13]

TOTAL: 150

FORMULA SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad\qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n - 1)d \qquad\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i} \qquad\qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$In \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \qquad\qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= 1 - 2\sin^2\alpha$$

$$= 2\cos^2\alpha - 1$$

$$\bar{x} = \frac{\sum fx}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_1 - x)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$