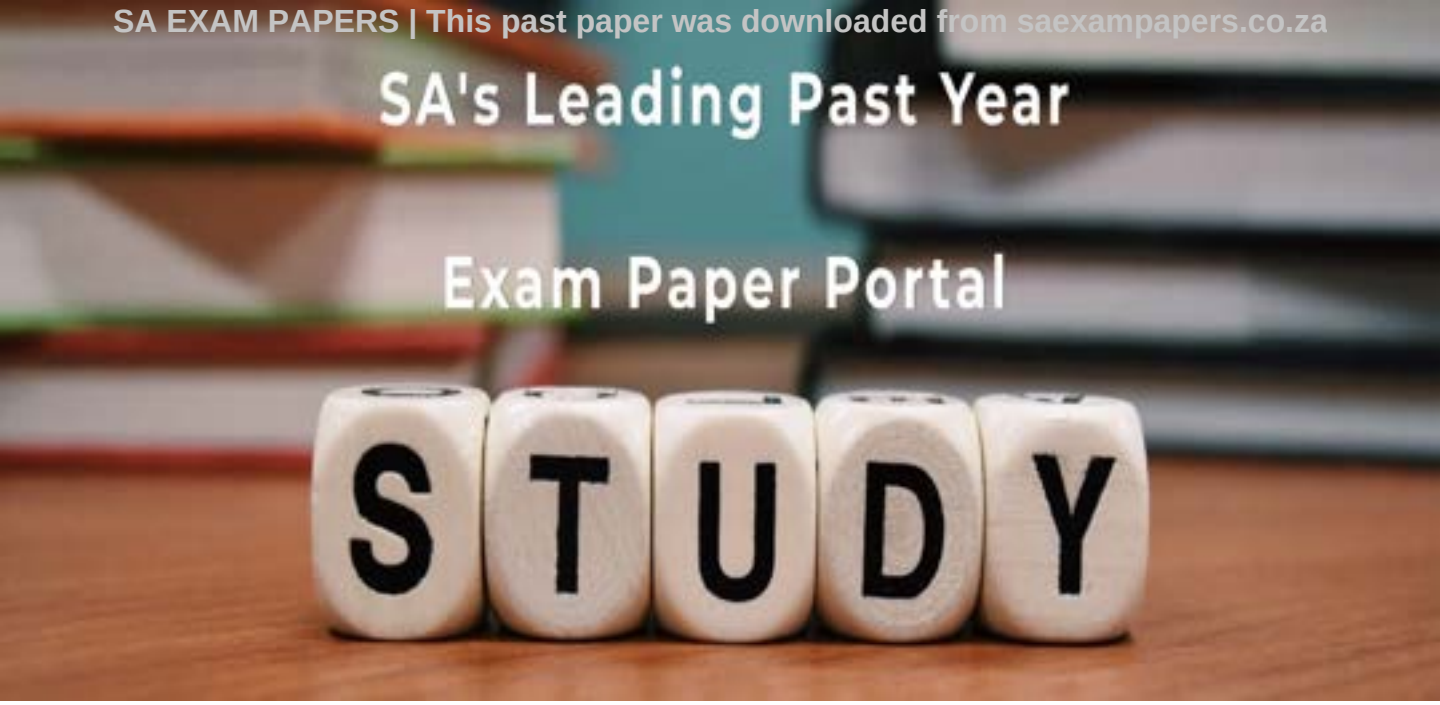


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

JUNE 2024

MARKS: 150

TIME: 3 HOURS

MEMATHP2

This question paper consists of 10 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. ANSWERS ONLY will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write legibly and present your work neatly.

QUESTION 1

In a survey, a student asked his friends and family how many marketing phone calls they receive per month. The following table represent the data:

AGE OF FRIENDS AND FAMILY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

1.1 Complete the Cumulative frequency table in the ANSWER BOOK. (3)

1.2 How many people took part in the survey? (1)

1.3 Write down the modal class. (1)

1.4 Draw the ogive in the ANSWER BOOK. (3)

1.5 Determine the percentage of marketing calls a person older that 54 will receive. (3)

[11]

QUESTION 2

The numbers {10, 20, 45, 58, 80} are the 5 number summary of a set of data with nine numbers. The second and third number of the data set are the same. The eighth number is 3 times the first quartile. The fourth number is equal to the interquartile range. The mean for the data set is 42.

2.1 Draw the box and whisker diagram in the ANSWER BOOK. (2)

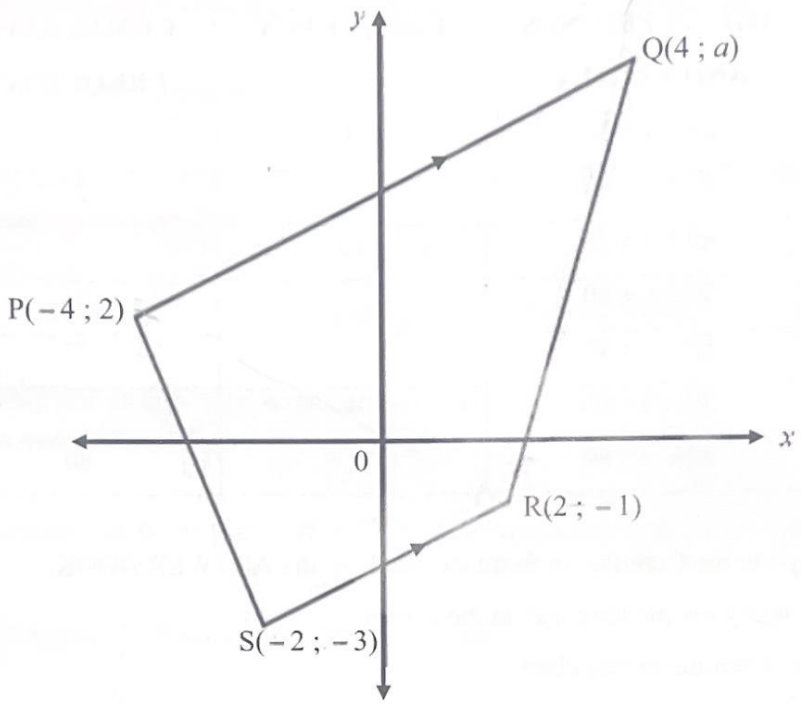
2.2 Comment on the skewness of the data. (1)

2.3 Write down a possible list of nine numbers which will result in the above box and whisker plot. (6)

[9]

QUESTION 3

In the diagram P(−4 ; 2), Q(4 ; a), R(2 ; −1) and S(−2 ; −3) are the vertices of trapezium PQRS with PQ ∥ SR.



3.1 Calculate the value of a. (4)

3.2 Find the equation of the line passing through P and R. (3)

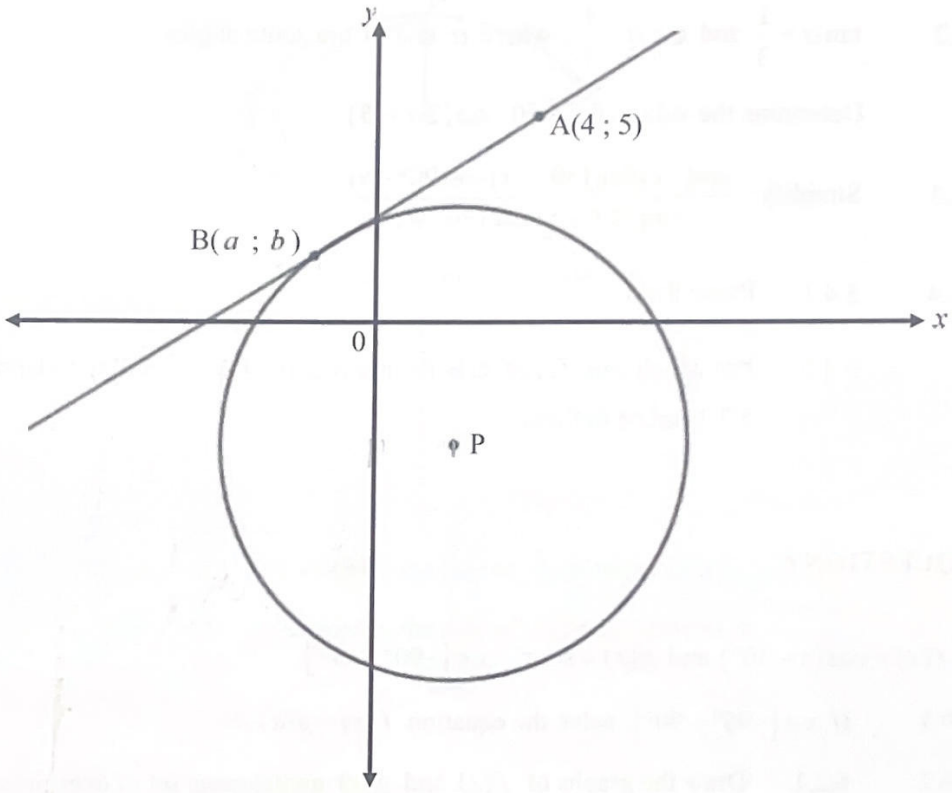
3.3 Find the size of $P\hat{R}Q$. (5)

3.4 If A(−1 ; t) and P, A and R are collinear, calculate the value of t. (2)

[14]

QUESTION 4

In the diagram the circle P with equation $(x-2)^2+(y+3)^2=34$ is drawn. The point A(4 ; 5) is a point on the tangent defined by $5y=3x+13$. B(a ; b) is the point on the circumference where the tangent touches the circle.



4.1 Determine the coordinates of P and the value of the radius of the circle. (2)

4.2 Write down the gradient of the radius PB in terms of a and b. (2)

4.3 Determine the gradient of the tangent AB in terms of a and b. (1)

4.4 Hence or otherwise, show that the values of $a=-1$ and $b=2$. (6)

4.5 Calculate the length of AB. Leave the answer in surd form. (2)

4.6 A second tangent is drawn from A to touch the circle at C(d ; 0). Determine the value of d. (4)

4.7 Prove that $B\hat{P}C=90°$. (3)

4.8 If AB intersects the $y-$axis at D and CP produced intersects the $y-$axis at E, calculate the length of DE. (5)

[25]

QUESTION 5

5.1 If $\tan 35° = m$, determine the value of the following in terms of m:

5.1.1 $\sin 215°$ (3)

5.1.2 $\sin 70°$ (3)

5.2 $\tan\alpha = \frac{1}{3}$ and $\tan\beta = \frac{1}{7}$, where α and β are acute angles.

Determine the value of: $50 \times \sin(2\alpha + \beta)$. (6)

5.3 Simplify: $\frac{\cos(-x)\tan(180° - x)\cos(90° - x)}{\sin(540° + x)\sin(180 - x)}$ (6)

5.4 5.4.1 Prove that: $\frac{\sin x}{1 - \sin x} + \frac{\sin x}{1 + \sin x} = \frac{2\tan x}{\cos x}$ (4)

5.4.2 For which value(s) of x in the interval $[0° ; 180°]$ will the identity in 5.3.1 not be defined. (2)

[24]

QUESTION 6

$f(x) = \cos(x - 30°)$ and $g(x) = \sin x$, $x \in [-90° ; 90°]$

6.1 If $x \in [-90° ; 90°]$, solve the equation $f(x) = g(x)$. (4)

6.2 6.2.1 Draw the graphs of $f(x)$ and $g(x)$ on the same set of axes provided in the ANSWER BOOK. (4)

Hence use the graphs to answer the following questions:

6.2.2 Write down the range of f. (2)

6.2.3 For which values of x will $f(x)$ decrease if x increase. (2)

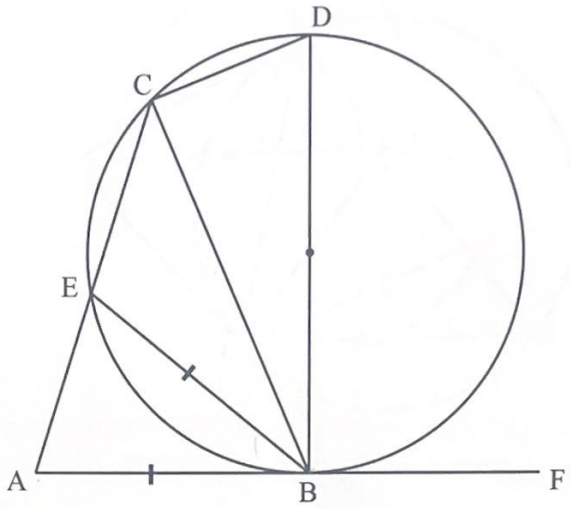
6.2.4 If $x \in [-90° ; 0°]$, for which values of x will $f(x).g(x) \geq 0$? (3)

6.2.5 Write down the value for of x where $g(x) > f(x)$. (2)

[17]

QUESTION 7

In the diagram AF is a tangent. AEC is a line intersecting the circle at E. BD is a diameter. EB, CB and CD are joined. $A\hat{B}E = 2x$ and $AB = EB = h$.



7.1 7.1.1 Write down, with reasons, the size of $\hat{A}$ in terms of x. (2)

7.1.2 Write down, with reasons, the size of $A\hat{C}B$ in terms of x. (2)

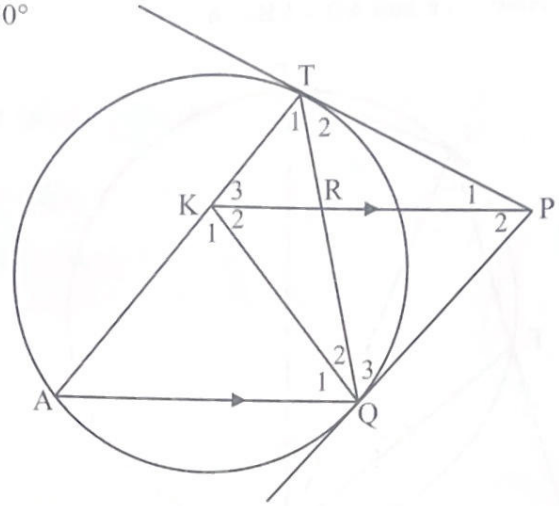
7.2 Show that $BC = \dfrac{h}{2\sin x}$. (3)

7.3 Hence show with reasons that $BD = \dfrac{h}{\sin 2x}$. (3)

[10]

QUESTION 8

In the diagram PT and PQ are tangents to the circle at T and Q. KP|| QA. Chords TQ and PK intersect at R. $\hat{T}_2 = 50°$



Find, with reasons, the sizes of:

8.1 $\hat{A}$ (2)

8.2 $\hat{K}_3$ (2)

8.3 $\hat{Q}_3$ (2)

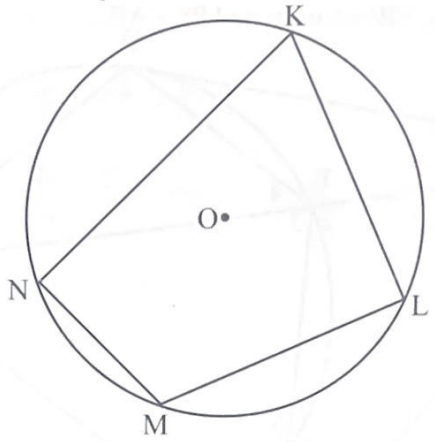
8.4 Give a reason why TKQP a cyclic quadrilateral is. (1)

8.5 $\hat{Q}_1$ (3)

[10]

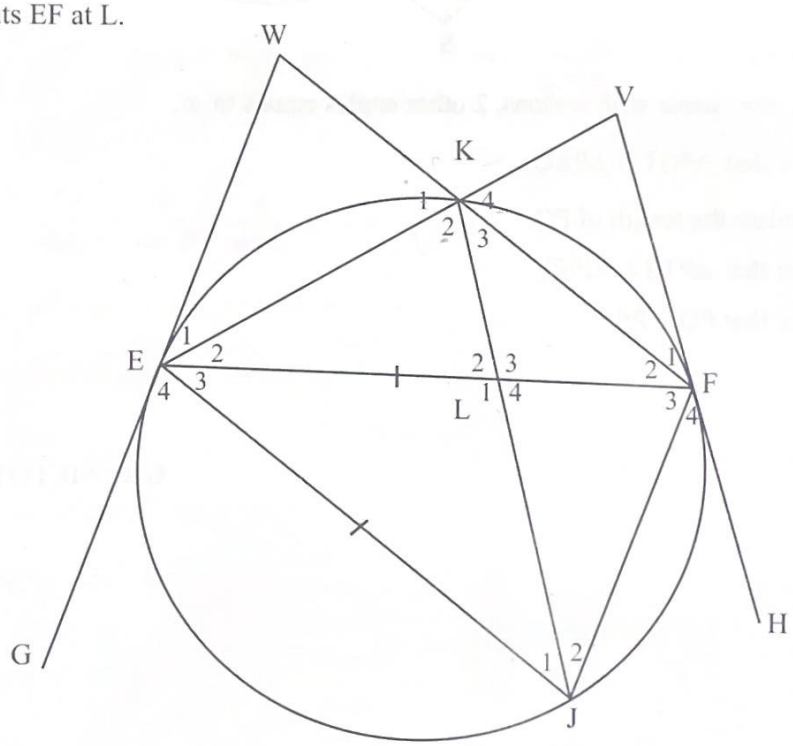
QUESTION 9

9.1 In the diagram O is the centre of the circle and KLMN is a cyclic quadrilateral. Prove the theorem stating that $\hat{K} + \hat{M} = 180°$.



(5)

9.2 In the diagram below, WEG and VFH are tangents to the circle. EJ is drawn such that EJ = EF. WF cuts the circle at K. EK produced meets VH at V. JK cuts EF at L.



Prove that:

9.2.1 $E\hat{J}F = \hat{K}_2$ (4)

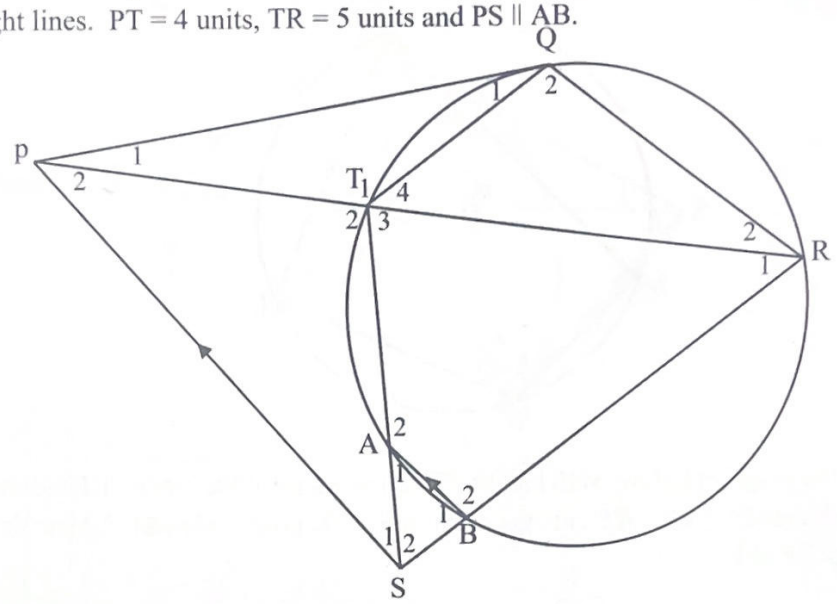
9.2.2 $\hat{K}_1 = \hat{K}_2$ (2)

9.2.3 KLFV is a cyclic quadrilateral. (3)

[14]

QUESTION 10

In the diagram PQ is a tangent to the circle at Q. T, R, A and B are points in the circumference of the circle and S is a point outside the circle. RT produced meets the tangent at P. TAS and RBS are straight lines. PT = 4 units, TR = 5 units and PS ∥ AB.



10.1 If $\hat{S}_1 = x$, name with reasons, 2 other angles equals to x. (4)

10.2 Prove that $\Delta PQT \,|||\, \Delta PRQ$. (3)

10.3 Calculate the length of PQ. (3)

10.4 Prove that $\Delta PTS \,|||\, \Delta PSR$. (3)

10.5 Prove that PQ = PS (3)

[16]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$