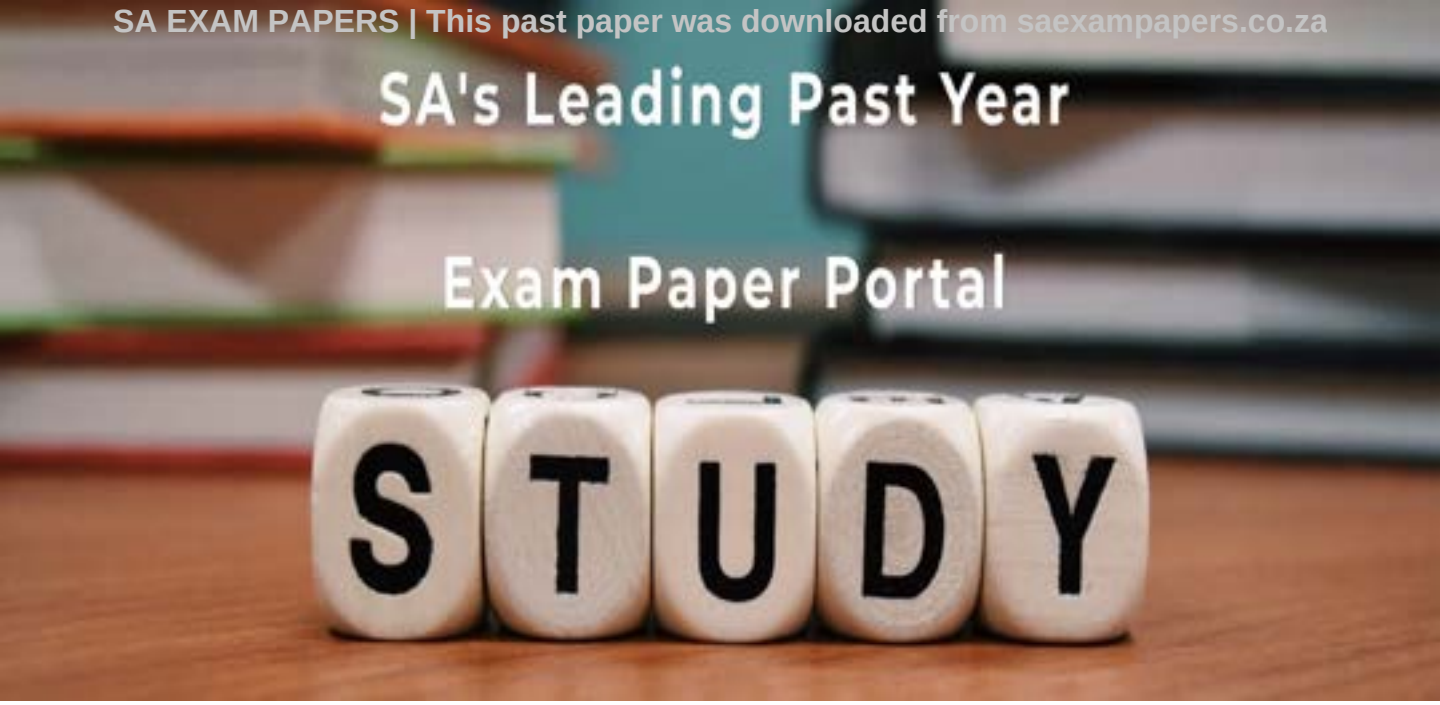


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL

SENIOR CERTIFICATE

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE 2*

JUNE 2024

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

This marking guidelines consist of 19 pages/*Hierdie nasienriglyne bestaan uit 19 bladsye*.

QUESTION/*VRAAG* 1

1.1

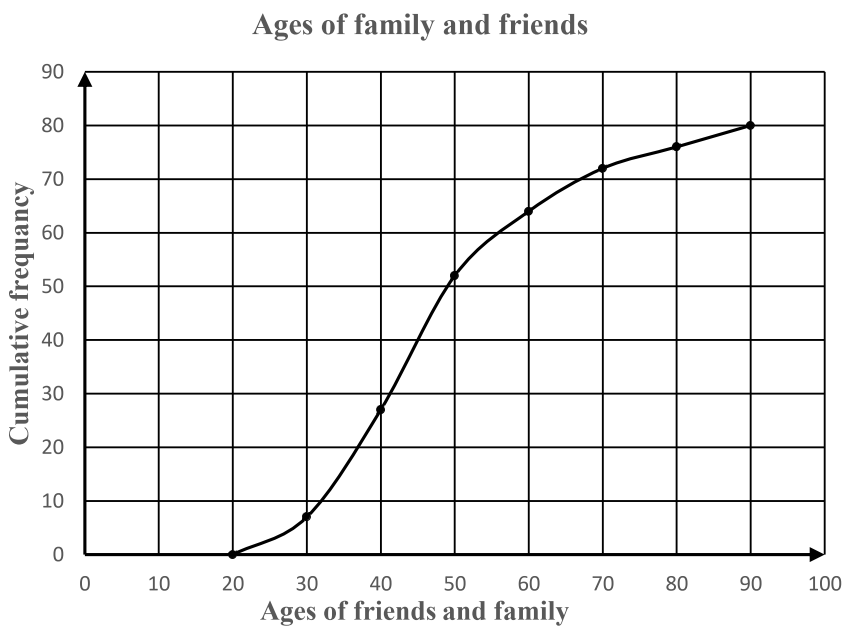
AGE OF FRIENDS AND FAMILY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$	**20**	27
$40 < x \leq 50$	25	**52**
$50 < x \leq 60$	**12**	64
$60 < x \leq 70$	**8**	72
$70 < x \leq 80$	4	**76**
$80 < x \leq 90$	**4**	80

✓ 20 & 52
✓ 12 & 8
✓ 76 & 4 (3)

1.2 80 ✓ answer (1)

1.3 $40 < x \leq 50$ ✓ answer (1)

1.4



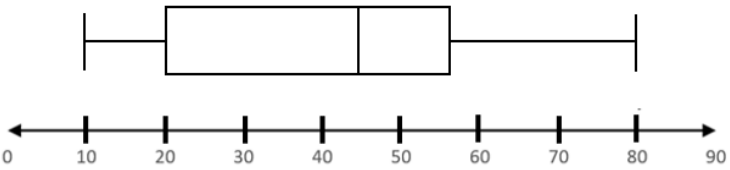
✓ grounding
✓ end point
✓ 2 points (3)

1.5 $\frac{21}{80} \times 100 = 26,25\%$

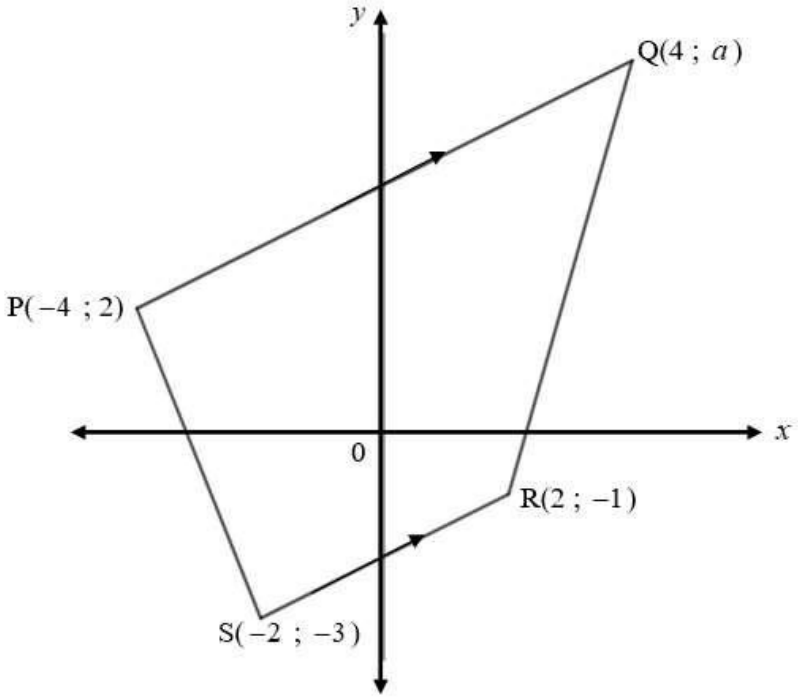
✓ accept $20-22$
✓ $\times 100$
✓ accept $25\ -27,5$ (3)

[11]

QUESTION/*VRAAG* 2

2.1		✓✓plotting values	(2)
2.2	Skew to the left OR negatively skew	✓ answer	(1)
2.3	$2^{nd} = 20$ $4^{th} = 38$ $8^{th} = 60$ $7^{th} = 56$ $6^{th}: \frac{10+20+20+38+45+x+56+60+80}{9} = 42$ $\frac{329+x}{9} = 42$ $329 + x = 378$ $x = 49$	✓ 20 ✓ 32 ✓ 60 ✓ 56 ✓ $\frac{329+x}{9} = 42$ ✓ answer	(6)
			[9]

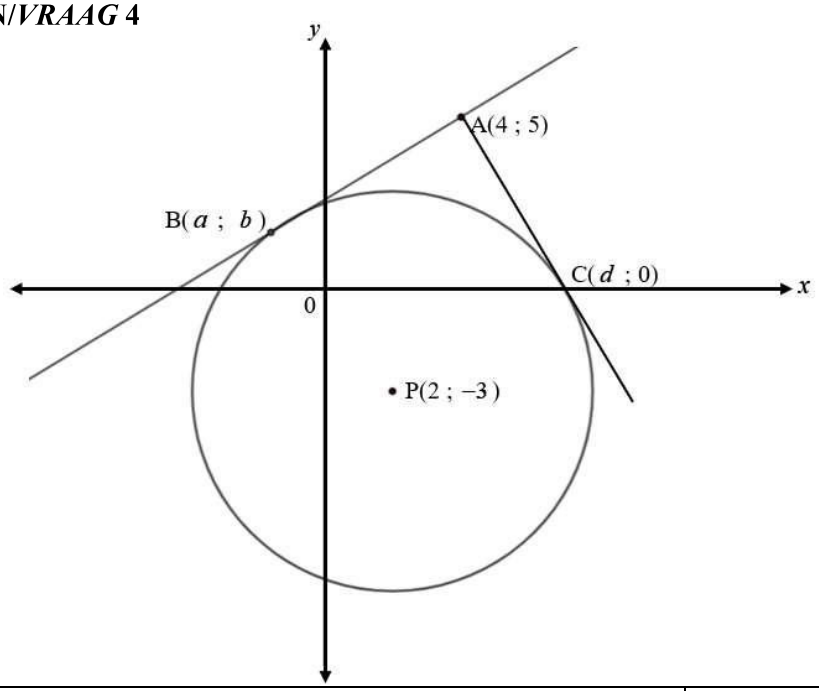
QUESTION/*VRAAG* 3



3.1	$m(PQ)=m(SR)$ $\frac{2-a}{-4-4}=\frac{-3+1}{-2-2}$ $\frac{2-a}{-8}=\frac{1}{2}$ $4-2a=-8$ $-2a=-12$ $a=6$	✓ $m(PQ)$ ✓ $m(SR)$ ✓ equating ✓ answer	(4)
3.2	$m(PR)=\frac{2+1}{-4-2}=-\frac{1}{2}$ $y=mx+c$ $2=\left(-\frac{1}{2}\right)(-4)+c$ $c=0$ $y=-\frac{1}{2}x$ OR	✓ $m(PR)$ ✓subst gradient and point ✓ answer	(3)

	$m(PR)=\frac{2+1}{-4-2}=-\frac{1}{2}$ $y-y_1=m(x-x_1)$ $y-2=\left(-\frac{1}{2}\right)(x+4)$ $y-2=-\frac{1}{2}x-2$ $y=-\frac{1}{2}x$	✓ $m(PR)$ ✓subst gradient and point ✓ answer	(3)
3.3	$m(PR)=-\frac{1}{2}$ $\quad m(QR)=\frac{6+1}{4-2}=\frac{7}{2}$ $\tan\alpha=-\frac{1}{2}$ $\quad \tan\beta=\frac{7}{2}$ $\alpha=180°-26,57°$ $\quad \beta=74,05°$ $\alpha=153,43°$ $\therefore \text{P}\hat{\text{R}}\text{Q}=79,38°$	✓ tan ratio α ✓ $\alpha=153,43°$ ✓ $m(QR)$ ✓ $\beta=74,05°$ ✓ $\text{P}\hat{\text{R}}\text{Q}$	(5)
3.4	$y=-\frac{1}{2}x$ $(-1\ ;\ t):\ t=-\frac{1}{2}(-1)$ $t=\frac{1}{2}$ OR $m(PA)=m(PR)$ $\frac{2-t}{-4+1}=-\frac{1}{2}$ $\frac{2-t}{-3}=-\frac{1}{2}$ $4-2t=3$ $-2t=-1$ $t=\frac{1}{2}$	✓ equation ✓answer ✓ equating ✓ answer	(2) (2)
			[14]

QUESTION/*VRAAG* 4



4.1	P(2 ; −3) Radius = $\sqrt{34}$	✓ centre ✓ radius	(2)
4.2	$m(PB)=\frac{b+3}{a-2}$	✓ subst in formula ✓ answer	(2)
4.3	$m(AB)=-\frac{a-2}{b+3}$ OR $m(AB)=\frac{b-5}{a-4}$	✓ answer	(1) (1)
4.4	Eq AB: $5y=3x+13$ through B(a ; b) $5b=3a+13$ $b=\frac{3}{5}a+\frac{13}{5}$ …..1 Eq PB: $y=mx+c$ $m=-\frac{5}{3}$ and P(2 ; −3) $-3=-\frac{5}{3}(2)+c$ $c=\frac{1}{3}$ $y=-\frac{5}{3}x+\frac{1}{3}$ through B(a ; b) $b=-\frac{5}{3}a+\frac{1}{3}$ …..2	✓ subt B(a ; b) in line equation ✓ $m(\text{PB})=-\frac{5}{3}$ ✓subst gradient and P(2 ; −3) ✓ equation PB	

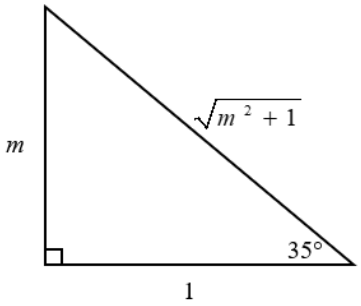
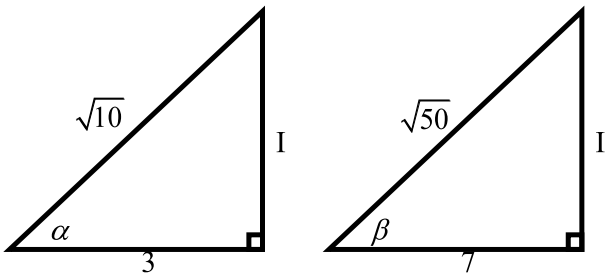
	Subs 1 in 2: $\frac{3}{5}a+\frac{13}{5}=-\frac{5}{3}a+\frac{1}{3}$ $9a+39=-25a+5$ $34a=-34$ $a=-1$ $b=2$ OR Eq AB: $5y=3x+13$ through B(a ; b) $5b=3a+13$ $b=\frac{3}{5}a+\frac{13}{5}$ …..1 $m(\text{PB})\times m(\text{BA})=-1$ $\frac{b+3}{a-2}\times\frac{b-5}{a-4}=-1$ $(b+3)(b-5)=-1(a-2)(a-4)$ …..2 Subs 1 in 2: $\left(\frac{3}{5}a+\frac{13}{5}+3\right)\left(\frac{3}{5}a+\frac{13}{5}-5\right)=-1(a-2)(a-4)$ $\left(\frac{3}{5}a+\frac{28}{5}\right)\left(\frac{3}{5}a+\frac{12}{5}\right)=-a^2+6a-8$ $\frac{9}{25}a^2+\frac{48}{25}a-\frac{336}{25}=-a^2+6a-8$ $34a^2-102a-136=0$ $a^2-3a-4=0$ $(a-4)(a+1)=0$ $a=4$ or/*of* $a=-1$ NA $b=2$	✓ equate equations ✓ simplification ✓ subt B(a ; b) in line equation ✓ $m(\text{PB})\times m(\text{BA})=-1$ ✓ subst gradients in formula ✓ subst b value in eq ✓ simplification ✓ simplification	(6) (6)

	OR Subst B(a ; b) in $5y=3x+13$: $5b=3a+13$ .....1 $y=\frac{3}{5}x+\frac{13}{5}$ $\therefore \frac{3}{5}=-\frac{a-2}{b+3}$ $3(b+3)=-5(a-2)$ $3b+9=-5a+10$ $3b=-5a+1$ $b=-\frac{5}{3}a+\frac{1}{3}$ .....2 Subst 2 in 1: $5\left(-\frac{5}{3}a+\frac{1}{3}\right)=3a+13$ $-\frac{25}{3}a+\frac{5}{3}=3a+13$ $-25a+5=9a+39$ $-34a=34$ $a=-1$ $b=2$	✓ subt B(a ; b) in line equation ✓ equate gradients ✓ simplification ✓ equation 2 ✓ subst ✓ simplification	(6)
4.5	$AB=\sqrt{(-1-4)^2+(2-5)^2}$ $AB=\sqrt{34}$	✓ subst in dist formula ✓ answer	(2)
4.6	$(x-2)^2+(y+3)^2=34$ through (d ; 0) $(d-2)^2+(0+3)^2=34$ $(d-2)^2+9=34$ $(d-2)^2=25$ $d-2=\pm 5$ $d=7$ or $d=-3$ NA	✓ Subst in formula ✓ simpification ✓ factors ✓ answers	(4)

	OR $(x-2)^2+(y+3)^2=34$ through $(d\ ;0)$ $(d-2)^2+(0+3)^2=34$ $d^2-4d+4+9=34$ $d^2-4d-21=0$ $(d-7)(d+3)=0$ $d=7$ or $d=-3$ NA OR $m(PC)\times m(AC)=-1$ $\frac{-3-0}{2-d}\times\frac{5-0}{4-d}=-1$ $\frac{-3}{2-d}\times\frac{5}{4-d}=-1$ $\frac{-15}{8-6d+d^2}=-1$ $-15=-8+6d-d^2$ $d^2-6d-7=0$ $(d-7)(d+1)=0$ $d=7$ or $d=-1$ NA	✓ subst in formula ✓ standard form ✓ factors ✓ select answer ✓ subst in formula ✓ standard form ✓ factors ✓ select answer	(4) (4)
4.7	$m(PB)=\frac{2+3}{-1-2}=-\frac{5}{3}$ $m(PC)=\frac{-3-0}{2-7}=\frac{3}{5}$ $\therefore m(PB)\times m(PC)=-\frac{5}{3}\times\frac{3}{5}=-1$ $\therefore \hat{BPC}=90°$	✓ $m(PB)$ ✓ $m(PC)$ ✓ subst in formula	(3)

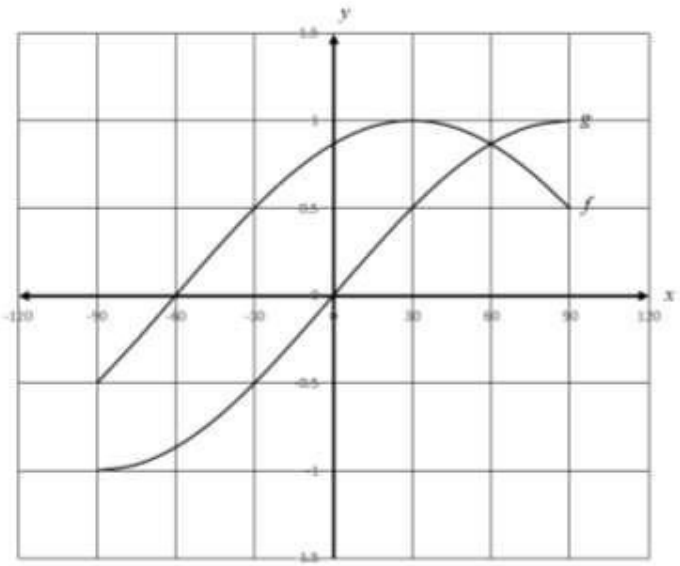
4.8	y-intercept of AB: Let $x = 0$ in $5y = 3x + 13$ $5y = 13$ $y = \frac{13}{5}$ $D\left(0\ ;\ \frac{13}{5}\right)$ Equation of CP: $y = mx + c$ $-3 = \frac{3}{5}(2) + c$ $c = -\frac{21}{5}$ $y = \frac{3}{5}x - \frac{21}{5}$ $E\left(0\ ;\ -\frac{21}{5}\right)$ $DE = \frac{34}{5} = 6,8$	✓ y - value ✓ subst m and point ✓ equation of CP ✓ y – value of E ✓ answer	(5)
			[25]

QUESTION/*VRAAG* 5

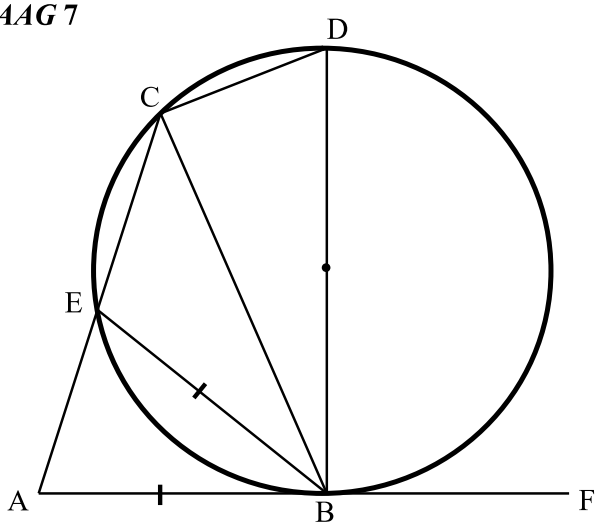
5.1.1	$\sin 215°$ $= -\sin 35°$ $= -\frac{m}{\sqrt{m^2+1}}$  	✓ $-\sin 35°$ ✓ $\sqrt{m^2+1}$ ✓ answer	(3)
5.1.2	$\sin 70° = \sin 2\times 35°$ $= 2\sin 35°\cos 35°$ $= 2\left(\frac{m}{\sqrt{m^2+1}}\right)\left(\frac{1}{\sqrt{m^2+1}}\right)$ $= \frac{2m}{m^2+1}$	✓double ∠ ✓ subst ✓ answer	(3)
5.2	 $50\times\sin(2\alpha+\beta)$ $= 50[\sin 2\alpha\cos\beta + \cos 2\alpha\sin\beta]$ $= 50\left[2\sin\alpha\cos\alpha\cos\beta + (2\cos^2\alpha - 1)\sin\beta\right]$ $= 50\left[2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right)\left(\frac{7}{\sqrt{50}}\right) + \left(2\left(\frac{3}{\sqrt{10}}\right)^2 - 1\right)\left(\frac{1}{\sqrt{50}}\right)\right]$ $= 50\left[\frac{42}{10\sqrt{50}} + \frac{8}{10}\left(\frac{1}{\sqrt{50}}\right)\right]$ $= 50\left[\frac{42}{10\sqrt{50}} + \frac{8}{10\sqrt{50}}\right]$ $= 50\left[\frac{50}{10\sqrt{50}}\right]$ $= 25\sqrt{2}$	✓ $\sqrt{10}$ & $\sqrt{50}$ ✓compound ∠ ✓ double ∠'s ✓ subst ✓ simplification ✓ answer	(6)

5.3	$\dfrac{\cos(-x)\tan(180°-x)\cos(90°-x)}{\sin(540°+x)\sin(180-x)}$ $=\dfrac{(\cos x)(-\tan x)(\sin x)}{(-\sin x)(\sin x)}$ $=\dfrac{-\tan x}{-\tan x}$ $=1$	✓ $\cos x$ ✓ $-\tan x$ ✓ $\sin x$ ✓ $-\sin x$ ✓ $\sin x$ ✓ answer	(6)
5.4.1	$\dfrac{\sin x}{1-\sin x}+\dfrac{\sin x}{1+\sin x}=\dfrac{2\tan x}{\cos x}$ LHS $=\dfrac{\sin x}{1-\sin x}+\dfrac{\sin x}{1+\sin x}$ $=\dfrac{\sin x(1+\sin x)+\sin x(1-\sin x)}{(1-\sin x)(1+\sin x)}$ $=\dfrac{\sin x+\sin^2 x+\sin x-\sin^2 x}{1-\sin^2 x}$ $=\dfrac{2\sin x}{\cos^2 x}$ $=\dfrac{2\sin x}{\cos x\cos x}$ $=\dfrac{2\tan x}{\cos x}=$ RHS	✓ LCM ✓ simplify numerator ✓ $1-\sin^2 x$ ✓ $\cos^2 x$	(4)
5.4.2	$x=90°$ and $x=270°$	✓ $90°$ ✓ $270°$	(2)
			[24]

QUESTION/*VRAAG* 6

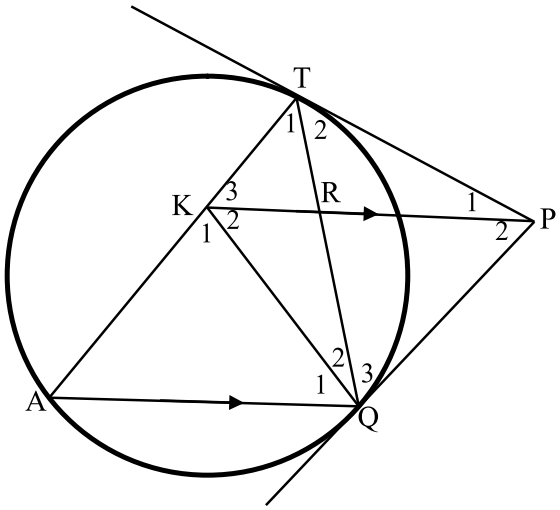
6.1	$\cos(x-30°)=\sin x$ $\cos(x-30°)=\cos(90°-x)$ $x-30°=90°-x+k.360°$ or $x-30°=360°-(90°-x)+k.360°$ $2x=120°+k.360°$ $\quad x-30°=360°-90°+x+k.360°$ $x=60°+k.180°, k\in Z$ $\quad$ NA $x=60°$	✓ co-function ✓ both equations ✓ $x=60°+k.180°, k\in Z$ ✓ $x=60°$	(4)
6.2.1		✓✓ f ✓✓ g	(4)
6.2.2	$y\in[-1\ ;\ 1]$	✓ critical values ✓ notation	(2)
6.2.3	$x\in(30°\ ;\ 90°]$	✓ critical values ✓ notation	(2)
6.2.4	$x\in[-90°\ ;\ -60°],\ x=0°$	✓✓ $x\in[-90°\ ;\ -60°]$ ✓ $x=0°$	(3)
6.2.5	$x\in(60°\ ;\ 90°]$	✓ critical values ✓ notation	(2)
			[17]

QUESTION/*VRAAG* 7



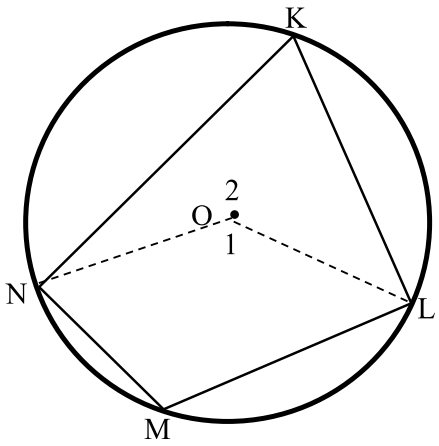
7.1.1	$\hat{A} = A\hat{E}B$ ∠'s opp equal sides $2\hat{A} = 180° - 2x$ ∠'s of Δ $\hat{A} = 90° - x$	✓ S ✓ R	(2)
7.1.2	$A\hat{C}B = 2x$ tan- chord theorem	✓ S ✓ R	(2)
7.2	In ΔECB: $\frac{BC}{\sin(90°+x)} = \frac{h}{\sin 2x}$ $BC = \frac{h\cos x}{2\sin x\cos x}$ $BC = \frac{h}{2\sin x}$	✓ subst in sine rule ✓ co-function ✓ double ∠	(3)
7.3	In ΔBCD: $\hat{C} = 90°$ $\frac{BD}{\sin 90°} = \frac{\frac{h}{2\sin x}}{\sin(90°-x)}$ $BD = \frac{h}{2\sin x\cos x}$ $BD = \frac{h}{\sin 2x}$ OR $\sin(90°-x) = \frac{\frac{h}{2\sin x}}{BD}$ $BD = \frac{h}{2\sin x\cos x} = \frac{h}{\sin 2x}$	✓ $\hat{C} = 90°$ ✓ subst in sine rule ✓ co-function ✓ $\hat{D} = 90° - x$ ✓ subst in sine rule ✓ co-function	(3) (3)
			[10]

QUESTION/*VRAAG* 8

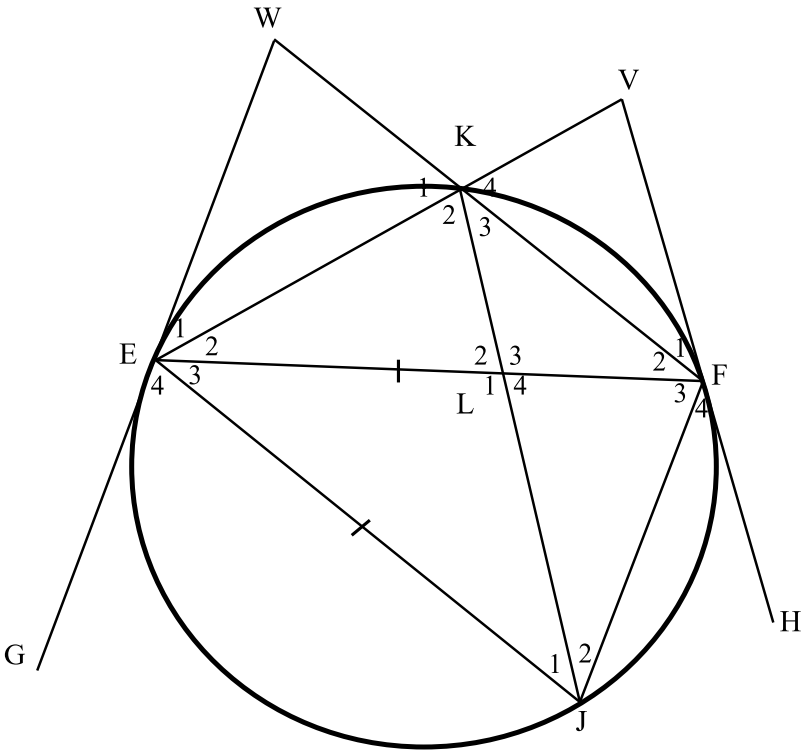


8.1	$\hat{T}_2 = \hat{A} = 50°$ tan-chord theorem	✓ S ✓ R	(2)
8.2	$\hat{A} = \hat{K}_3 = 50°$ corresp $\angle$'*s* , AQ ⫴ KP	✓ S ✓ R	(2)
8.3	$\hat{A} = \hat{Q}_3 = 50°$ tan-chord theorem OR $\hat{T}_2 = \hat{Q}_3 = 50°$ $\angle$'*s* opp equal tangents	✓ S ✓ R ✓ S ✓ R	(2) (2)
8.4	Line subtend equal $\angle$'*s* OR converse $\angle$'*s* in the same segment	✓ R	(1)
8.5	$\hat{T}_2 = \hat{K}_2 = 50°$ $\angle$'*s* in same segment $\hat{K}_2 = \hat{Q}_1 = 50°$ alt $\angle$'*s* , AQ ⫴ KP	✓ S ✓ R ✓ R	 (3)
			[10]

QUESTION/*VRAAG* 9

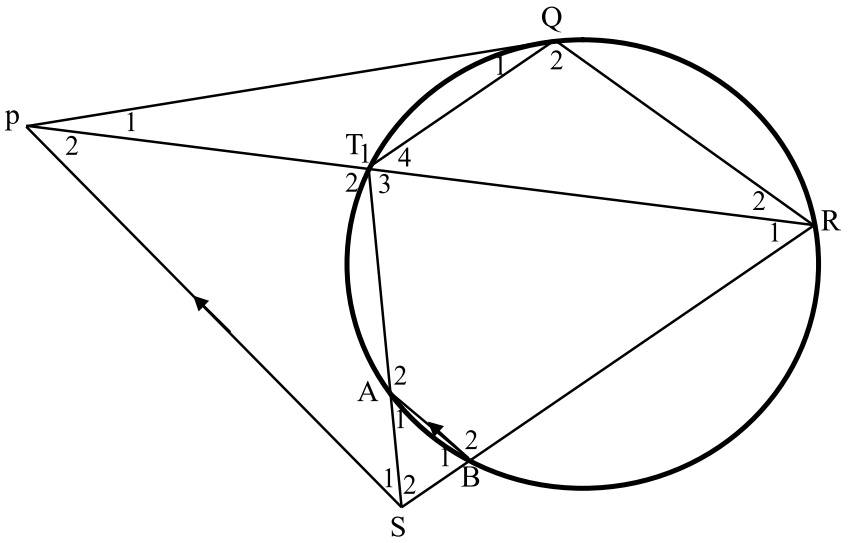


9.1	Construction: Draw radii ON and OL $\hat{O}_1 = 2 \times \hat{K}$ midpt $\angle = 2 \times$ circumf $\angle$ $\hat{O}_2 = 2 \times \hat{M}$ midpt $\angle = 2 \times$ circumf $\angle$ $\hat{O}_1 + \hat{O}_2 = 360°$ revolution $2\hat{K} + 2\hat{M} = 360°$ $\hat{K} + \hat{M} = 180°$	✓ Construction ✓ S ✓ R ✓ S/R ✓ S/R	(5)



9.2.1	$\hat{J} = \hat{F}_3$ $\angle's$ opp equal sides $\hat{F}_3 = \hat{K}_2$ $\angle's$ in the same segment $\therefore \hat{J} = \hat{K}_2$	✓ S ✓ R ✓ S ✓ R	(4)
9.2.2	$\hat{K}_1 = \hat{J}$ ext $\angle$ of cyclic quad $\hat{J} = \hat{K}_2$ proven $\hat{K}_1 = \hat{K}_2$	✓ S ✓ R	(2)
9.2.3	$\hat{F}_1 + \hat{F}_2 = \hat{J}$ tan-chord theorem $\hat{J} = \hat{K}_2$ proven $\hat{F}_1 + \hat{F}_2 = \hat{K}_2$ $\therefore$ KLFV is a cyclic quad converse ext $\angle$ of cyclic quad	✓ S ✓ R ✓ ✓ R	(3)
			[14]

QUESTION/*VRAAG* 10



10.1	$\hat{S}_1 = \hat{A}_1 = x$ alt $\angle$'s , PS $\parallel$ AB $\hat{A}_1 = \hat{R}_1 = x$ ext $\angle$ of cyclic quad	✓ S ✓ R ✓ S ✓ R	(4)
10.2	In ΔPQT and ΔPRQ : (i) $\hat{P}_1 = \hat{P}_1$ common (ii) $\hat{Q}_1 = \hat{R}_2$ tan-chord theorem $\therefore \Delta PQT \,\|\|\| \,\Delta PRQ$ 3 $\angle$'s OR In ΔPQT and ΔPRQ : (i) $\hat{P}_1 = \hat{P}_1$ common (ii) $\hat{Q}_1 = \hat{R}_2$ tan-chord theorem (iii) $\hat{T}_1 = P\hat{Q}R$ 3rd $\angle$ $\therefore \Delta PQT \,\|\|\| \,\Delta PRQ$ 3 $\angle$'s	✓ S ✓ S/R ✓ R ✓ S ✓ S/R ✓ S	(3) (3)
10.3	$\frac{PQ}{PR} = \frac{PT}{PQ}$ $\|\|\| \,\Delta$'s $\frac{PQ}{9} = \frac{4}{PQ}$ $PQ^2 = 36$ $\therefore PQ = 6$	✓ R ✓ subst ✓ answer	(3)

10.4	In ΔPTS and ΔPSR : (i) $\hat{P}_2 = \hat{P}_2$ common (ii) $\hat{S}_1 = \hat{R}_1 = x$ proven $\therefore \Delta PTS \text{ III } \Delta PSR$ 3 $\angle$'s OR In ΔPTS and ΔPSR : (i) $\hat{P}_2 = \hat{P}_2$ common (ii) $\hat{S}_1 = \hat{R}_1 = x$ proven (iii) $\hat{P}_2 = \hat{S}$ 3rd $\angle$ $\therefore \Delta PTS \text{ III } \Delta PSR$ 3 $\angle$'s	✓ S ✓ S ✓ R ✓ S ✓ S/R ✓ S	(3)
10.5	$\frac{PT}{PS} = \frac{PS}{PR}$ III Δ's $\therefore PS^2 = PT.PR$ $PQ^2 = PT.PR$ from 10.3 $PS^2 = PQ^2$ $\therefore PS = PQ$	✓ S/R ✓ S ✓ S	(3)
			[16]

TOTAL/*TOTAAL* : 150