FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

COMMON TEST

JUNE 2024

MARKING GUIDELINES

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 150

These marking guidelines consist of 13 pages.

SA EXAM
PAPERS

QUESTION 1

1.1.1	$x = 0$ or $-\dfrac{7}{2}$ or 8	✓ A answer ✓ A answer ✓A answer (3)
1.1.2	$(x+1)(x+12) = 0$ $x = -1$ or $x = -12$	✓A factors ✓CA answer ✓CA answer (3)
1.1.3	$5x^2 - 7x + 8 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{7 \pm \sqrt{(-7)^2 - 4(5)(8)}}{2(5)}$ $x = \dfrac{7 \pm \sqrt{-111}}{10}$ No real values of x Accept: No solution	✓A substituting in formula ✓CA answer (2)
1.1.4	$\sqrt{x-2} + 2 = x$ $\left(\sqrt{x-2}\right)^2 = (x-2)^2$ $x - 2 = x^2 - 4x + 4$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ Answers only: 2 marks $x = 2$ or $x = 3$	✓A isolating $\sqrt{x-2}$ ✓CA squaring both sides ✓CA standard form ✓CA answer ✓CA answer (5)
1.1.5	$x^2 - x - 20 < 0$ $(x+4)(x-5) < 0$ (graph of parabola with roots at -4 and 5) **OR** (number line with open circles at -4 and 5) $-4 < x < 5$ OR $x \in (-4 ; 5)$ Penalty of 1 mark if one or both end points are included	✓A standard form ✓CA critical values CA✓ CA✓ answer (4)
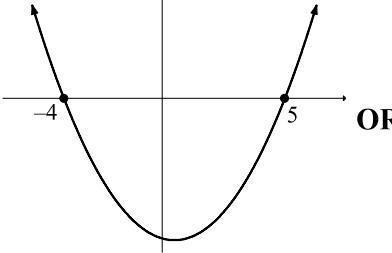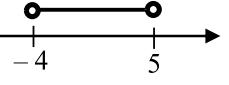
1.1.6	$2^{1-2x} + 7.2^{-x} - 4 = 0$ $2.2^{-2x} + 7.2^{-x} - 4 = 0$ $(2.2^{-x} - 1)(2^{-x} + 4) = 0$ $2^{-x} = \dfrac{1}{2}$ or $2^{-x} = -4$ $2^{-x} = 2^{-1}$ no solution $x = 1$ **OR** $2^{1-2x} + 7.2^{-x} - 4 = 0$ $2.2^{-2x} + 7.2^{-x} - 4 = 0$ Let $2^{-x} = k$ $(2k - 1)(k + 4) = 0$ $k = \dfrac{1}{2}$ or $k = -4$ $2^{-x} = 2^{-1}$ no solution $x = 1$	✓A splitting exponents ✓A factor ✓A factor ✓A answer ✓A no solution (5) **OR** ✓A splitting exponents ✓A factor ✓A factor ✓A answer ✓A no solution (5)
1.2	$22 - 3m \geq 0$ $22 \geq 3m$ $\dfrac{22}{3} \geq m$ $m = 7$	✓A $22 - 3m \geq 0$ ✓CA $\dfrac{22}{3} \geq m$ ✓CA answer (3)
1.3	$\dfrac{\sqrt{9.9^{2023}}}{\sqrt{9^{2023}} - \sqrt{9^2.9^{2023}}}$ $= \dfrac{3\sqrt{9^{2023}}}{\sqrt{9^{2023}}(1 - 9)}$ $= -\dfrac{3}{8}$	✓A simplifying numerator ✓A simplifying denominator ✓CA answer (3)
1.4	$y = 2x - 3$ $4x^2 + (2x - 3)^2 - 2x(2x - 3) - 7 = 0$ $4x^2 + 4x^2 - 12x + 9 - 4x^2 + 6x - 7 = 0$ $4x^2 - 6x + 2 = 0$ $2x^2 - 3x + 1 = 0$ $(2x - 1)(x - 1) = 0$ $x = \dfrac{1}{2}$ or $x = 1$ $y = 2\left(\dfrac{1}{2}\right) - 3$ or $y = 2(1) - 3$ $y = -2$ $y = -1$	✓A making y the subject of the formula ✓CA substitution ✓CA standard form ✓CA factors ✓CA x-values ✓CA y-values
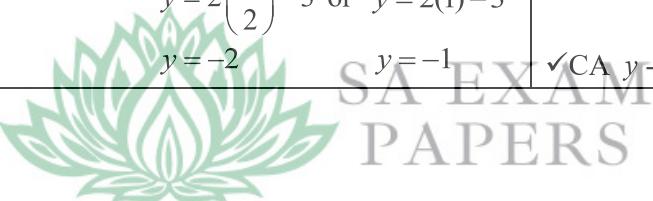
	OR $x = \dfrac{y+3}{2}$ $4\left(\dfrac{y+3}{2}\right)^2 + y^2 - 2y\left(\dfrac{y+3}{2}\right) - 7 = 0$ $y^2 + 3y + 2 = 0$ $(y+1)(y+2) = 0$ $y = -2 \quad \text{or} \quad y = -1$ $x = 1 \quad \text{or} \quad x = \dfrac{1}{2}$	✓A making x the subject of the formula ✓CA substitution ✓CA standard form ✓CA factors ✓CA y-values ✓CA x-values (6)
		[34]

QUESTION 2

2.1	$-5 \; ; \; 12 \; ; \; 27 \; ; \; 40 \; ; \; 51. \; . \; . \; .$ $\quad 17 \; ; \; 15 \; ; \; 13 \; ; \; 11 \; ; \; . \; . \; .$ $\quad\quad -2 \; ; -2 \; ; -2 \; ; \; . \; . \; .$ The next two terms are 40 and 51	$\checkmark$A $\checkmark$A answer (2)
2.2	$-5 \; ; \; 12 \; ; \; 27 \; ; \; 40 \; ; \; 51. \; . \; . \; .$ $\quad 17 \; ; \; 15 \; ; \; 13 \; ; \; 11 \; ; \; . \; . \; .$ $\quad\quad -2 \; ; -2 \; ; -2 \; ; \; . \; . \; .$ $\quad\quad 2a = -2$ $\quad\quad\; a = -1$ $\quad 17 = 3a + b$ $\quad 17 = 3(-1) + b$ $\quad\quad b = 20$ $\quad -5 = -1 + 20 + c$ $\quad\quad c = -24$ $\quad T_n = -n^2 + 20n - 24$	$\checkmark$ A $\; a = -1$ $\checkmark$CA value of b $\checkmark$CA value of c $\checkmark$CA answer (4)
2.3	$\quad\quad\quad 51 = -n^2 + 20n - 24$ $\quad\; n^2 - 20n + 75 = 0$ $\left(n - 5\right)\left(n - 15\right) = 0$ $\quad\quad n = 5 \text{ or } n = 15$ $\quad\quad \therefore T_5 \text{ and } T_{15}$ If stopping at values of n, still award the last mark	$\checkmark$CA equating T_n to 51 $\checkmark$CA standard form $\checkmark$CA answers (3)
2.4	$\displaystyle\sum_{n=3}^{10} T_n - \sum_{n=11}^{17} T_n$ Using symmetry: $\; T_3 = T_{17} \; ; \; T_4 = T_{16} \; \; T_5 = T_{15} \; ;$ etc. $(T_3 - T_{17}) + (T_4 - T_{16}) + (T_5 - T_{15}) + (T_6 - T_{14}) +$ $(T_7 - T_{13}) + (T_8 - T_{12}) + (T_9 - T_{11}) + T_{10}$ $\; = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 76$ $\; = 76$ **OR** Listing all the terms from $\displaystyle\sum_{n=3}^{10} T_n$. Listing all the terms from $\displaystyle\sum_{n=11}^{17} T_n$ $\displaystyle\sum_{n=3}^{10} T_n - \sum_{n=11}^{17} T_n = 76$	$\checkmark$CA$\checkmark$CA $\;=$ terms, using symmetry $\checkmark$CA answer (3) **OR** $\checkmark$CA Listing all the terms from $\displaystyle\sum_{n=3}^{10} T_n$ $\checkmark$CA Listing all the terms from $\displaystyle\sum_{n=11}^{17} T_n$ $\checkmark$CA answer (3)
		[12]

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QUESTION 3

3.1.1	$-\dfrac{7}{2}$; -3 ; $-\dfrac{5}{2}$;	
	$a = -\dfrac{7}{2}$	
	$d = \dfrac{1}{2}$	✓A value of d
	$T_n = a + (n-1)d$	
	$T_n = -\dfrac{7}{2} + (n-1)\dfrac{1}{2}$	
	$T_n = -\dfrac{7}{2} + \dfrac{1}{2}n - \dfrac{1}{2}$	
	$T_n = \dfrac{1}{2}n - 4$	✓CA answer
		(2)

3.1.2	$S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$	
	$675 = \dfrac{n}{2}\left[2\left(\dfrac{-7}{2}\right) + (n-1)\dfrac{1}{2}\right]$	✓CA substitute into formula
	$1350 = n\left(-7 + (n-1)\dfrac{1}{2}\right)$	
	$2700 = -14n + n^2 - n$	
	$0 = n^2 - 15n - 2700$	
	$(n-60)(n+45) = 0$	✓CA factors
	$n = 60$ or $n = -45$	✓CA values of n
	$\therefore n = 60$ only	✓CA answer
		(4)

3.1.3	$T_n = \left(\dfrac{1}{2}n - 4\right)^2$	✓CA squaring T_n
	$T_n = \dfrac{1}{4}n^2 - 4n + 16$	
	$n = -\dfrac{b}{2a}$	
	$n = -\dfrac{-4}{2\left(\dfrac{1}{4}\right)}$	✓CA substituting in $n = -\dfrac{b}{2a}$
	$n = 8$	✓CA answer (3)
	The 8ᵗʰ term is the smallest	**OR**
	OR	
	Smallest value of $\left(\dfrac{1}{2}n - 4\right)^2 = 0$	✓CA squaring T_n
		✓CA equating T_n to 0
	$\dfrac{1}{2}n - 4 = 0$	
	$\boxed{\text{If stopping at values of } n, \text{ still award the last mark}}$	
	$n = 8$	
	The 8ᵗʰ term is the smallest	✓CA answer (3)

3.2.1	$r = 2(x+1)$ $-1 < r < 1$ $-1 < 2(x+1) < 1$ $-\dfrac{1}{2} < x+1 < \dfrac{1}{2}$ $-\dfrac{3}{2} < x < -\dfrac{1}{2}$	✓A $r = 2(x+1)$ ✓A $-1 < r < 1$ ✓CA answer (3)
3.2.2	$a = \dfrac{-3}{4} + 1$ $a = \dfrac{1}{4}$	 ✓A answer (1)
3.2.3	$r = \dfrac{1}{2}$ $a = \dfrac{1}{4}$ $T_n = ar^{n-1}$ $T_n = \dfrac{1}{4}\left(\dfrac{1}{2}\right)^{n-1}$ $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{4}\left(\dfrac{1}{2}\right)^{n-1}$ **Answer only:** **Full marks**	✓CA value of r. ✓CA expression of T_n ✓CA answer (3)
3.2.4	$S_\infty = \dfrac{a}{1-r}$ $= \dfrac{\dfrac{1}{4}}{1-\dfrac{1}{2}}$ $= \dfrac{1}{2}$	 ✓CA substitute in S_∞ formula ✓CA answer (2)
		[18]

QUESTION 4

4.1	$f(x) = \dfrac{6}{x-4} + 3$	✓A $\dfrac{6}{x-4}$ ✓A $+3$ (2)
4.2	For x-intercept: $\quad 0 = \dfrac{6}{x-4} + 3$ $\qquad\qquad -3 = \dfrac{6}{x-4}$ $\qquad\qquad -3x + 12 = 6$ $\qquad\qquad -3x = -6$ $\qquad\qquad x = 2$ For y-intercept: $\quad y = \dfrac{6}{0-4} + 3$ $\qquad\qquad\qquad = \dfrac{3}{2}$	✓ CA equating to zero ✓ CA x-intercept ✓ CA y-intercept (3)
4.3		✓A shape ✓A asymptotes ✓CA intercepts (3)
4.4	$y = -x + c$ $3 = -4 + c$ $c = 7$ $y = -x + 7$ Answer only: Full marks	✓A substituting $m = -1$ ✓A substituting $(4\,;\,3)$ ✓CA answer (3)
4.5	$(5;10)$	✓CA ✓CA answer (2)
		[13]
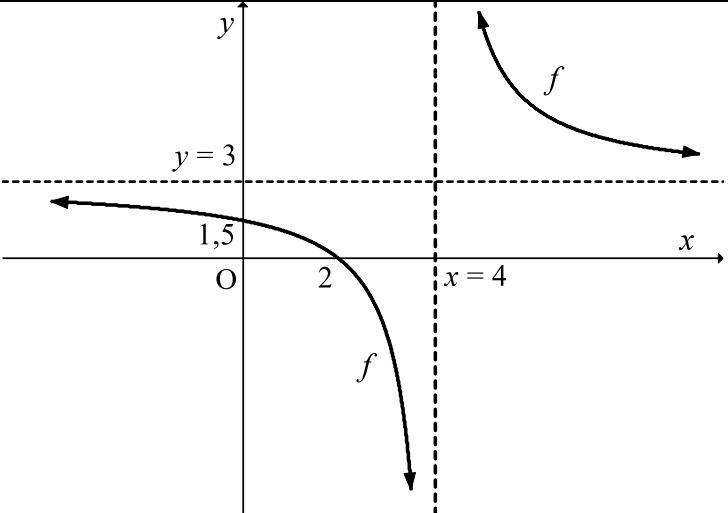
QUESTION 5

DO NOT MARK QUESTIONS 5.5, 5.6 AND 5.7.

5.1	B$(1 ; -9)$	A$\checkmark$ x-coordinate A$\checkmark$ y-coordinate (2)
5.2	$x < 1$	A$\checkmark$ answer (1)
5.3	$(x-1)^2 - 9 = 0$ $(x-1)^2 = 9$ $(x-1) = \pm 3$ $x = 1 \pm 3$ $x = 4$ or $x = -2$ $(4 ; 0)$ or $(-2 ; 0)$ **OR** $(x-1)^2 - 9 = 0$ $x^2 - 2x + 1 - 9 = 0$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4$ or $x = -2$ $(4 ; 0)$ or $(-2 ; 0)$	$\checkmark$A equating to zero $\checkmark$A taking square root on both sides $\checkmark$CA answers (3) **OR** $\checkmark$A equating to zero $\checkmark$A factors $\checkmark$CA answer (3)
5.4	$y = -a^x$ $-9 = -a^1$ $a = 9$	$\checkmark$A substituting $(1 ; -9)$ (1)
5.5	$g : y = -9^x$ $g^{-1} : x = -9^y$ $-x = 9^y$ $\therefore y = \log_9(-x)$	$\checkmark$A swapping x and y $\checkmark$A answer
5.6		$\checkmark$CA shape $\checkmark$CA x-intercept
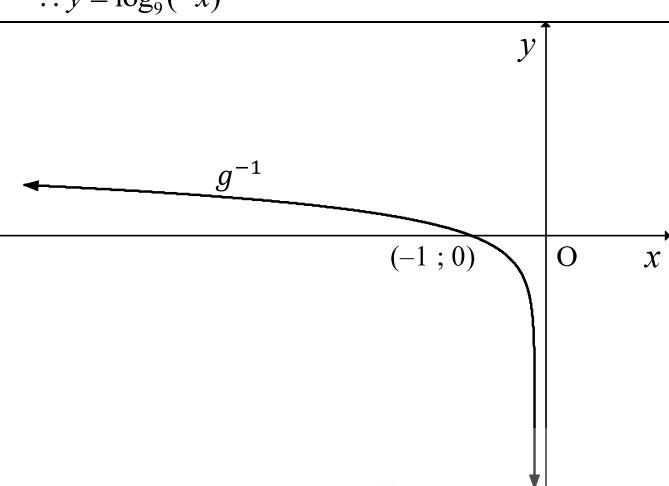
5.7	$y = \log_9(-x)$ $2 = \log_9(-x)$ $-x = 9^2$ $x = -81$ $\therefore x < -81$	✓CA $2 = \log_9(-x)$ ✓CA value of x ✓CA answer
		[7]

QUESTION 6

6.1	$y = a(x+6)(x-2)$ Substitute $(-4\,;\,6)$: $6 = a(-4+6)(-4-2)$ $6 = -12a$ $a = -\dfrac{1}{2}$ $\therefore y = -\dfrac{1}{2}(x+6)(x-2)$ $y = -\dfrac{1}{2}x^2 - 2x + 6$ $\therefore b = -2$ and $c = 6$	A✓ $y = a(x+6)(x-2)$ A✓ substitute $(-4\,;\,6)$ A✓ $6 = -12a$ A✓ substitute back $a = -\dfrac{1}{2}$ (4)
6.2	$x = -\dfrac{b}{2a}$ **OR** $= -\dfrac{(-2)}{2\left(-\dfrac{1}{2}\right)}$ $x = \dfrac{-6+2}{2}$ $= -2$ $= -2$ Maximum value $= h(-2) = -\dfrac{1}{2}(-2)^2 - 2(-2) + 6 = 8$	A ✓ substitution A✓ x-value of TP CA✓ answer (3)
6.3	$y = -\dfrac{1}{2}x + c$ Substitute $(-4\,;\,6)$: $6 = -\dfrac{1}{2}(-4) + c$ $c = 4$ $\therefore y = -\dfrac{1}{2}x + 4$ **OR** $s(x) = -\dfrac{1}{2}x + 4$	✓A substitution ✓CA answer (2)
6.4	$k < 2$	✓✓CA CA answer (2)
6.5	Translated downwards by 8 units	✓✓CA CA answer (2)
		[13]

QUESTION 7

Penalise once only for incorrect notation in Question 7.1.1

7.1.1	$f(x) = 2x^2 + 4$	
	$f(x+h) = 2(x+h)^2 + 4 = 2x^2 + 4xh + 2h^2 + 4$	✓A value of $f(x+h)$
	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	
	$f'(x) = \lim\limits_{h \to 0} \dfrac{2x^2 + 4xh + 2h^2 + 4 - 2x^2 - 4}{h}$	✓CA substitution into formula
	$f'(x) = \lim\limits_{h \to 0} \dfrac{4xh + 2h^2}{h}$	✓CA simplifying
	$f'(x) = \lim\limits_{h \to 0} \dfrac{h(4x + 2h)}{h}$	✓CA factors
	$f'(x) = \lim\limits_{h \to 0}(4x + 2h)$	
	$f'(x) = 4x$	✓CA answer (5)
7.1.2	$f'(x) = m_{\text{tangent}}$	
	$\therefore 4x = -12$	✓CA $4x = -12$
	$x = -3$	
	$y = 2(-3)^2 + 4 = 22$	
	The tangent is at $(-3 ; 22)$	✓CA coordinates of contact point
	$y = -12x + c$	
	$22 = -12(-3) + c$	✓CA substitution of point and gradient
	$c = -14$	
	$y = -12x - 14$	✓CA answer (4)
7.2.1	$f(x) = \dfrac{2x^2 - 5x - 12}{x - 4}$	
	$= \dfrac{(2x+3)(x-4)}{x-4}$	✓A factors
	$= 2x + 3$	✓CA answer
	$f'(x) = 2$	✓CA answer (3)
7.2.2	$D_x \left[x^{\frac{2}{5}} + x^2 - 9x \right]$	✓A $x^{\frac{2}{5}}$
	$= \dfrac{2}{5} x^{\frac{-3}{5}} + 2x - 9$	✓CA $\dfrac{2}{5} x^{\frac{-3}{5}}$ ✓A $+2x$ ✓A -9 (4)
7.2.3	$y = \dfrac{x}{6} - \dfrac{6}{x}$	
	$= \dfrac{x}{6} - 6x^{-1}$	✓A $-6x^{-1}$
	$\dfrac{dy}{dx} = \dfrac{1}{6} + 6x^{-2}$	✓A $\dfrac{1}{6}$ ✓CA $+6x^{-2}$ (3)

**[19]**

QUESTION 8

8.1	For x-intercepts: $-x^3 + 10x^2 - 17x - 28 = 0$ $\therefore x^3 - 10x^2 + 17x + 28 = 0$ $(x+1)(x^2 - 11x + 28) = 0$ $(x+1)(x-4)(x-7) = 0$ $\therefore x = -1 \;\; \text{or} \;\; x = 4 \;\; \text{or} \;\; x = 7$ $A(-1\,;\,0); \;\; B(4\,;\,0); \;\; C(7\,;\,0)$ Answer only: 3 marks	✓A $(x+1)$ ✓CA trinomial ✓CA factors ✓CA answer (4)
8.2	For the turning points: $f^{/}(x) = -3x^2 + 20x - 17 = 0$ $3x^2 - 20x + 17 = 0$ $(3x - 17)(x - 1) = 0$ $x = \dfrac{17}{3} \;\; \text{or} \;\; x = 1$ $y = \dfrac{400}{27} \;\; \text{or} \;\; y = -36$ $D(1\,;\,-36); \;\; E\left(\dfrac{17}{3}\,;\,\dfrac{400}{27}\right)$	✓A $f^{/}(x) = -3x^2 + 20x - 17$ ✓CA $f^{/}(x) = 0$ ✓CA coordinates of D ✓CA coordinates of E (4)
8.3.1	x-coordinate of point of inflection $= \dfrac{1 + \dfrac{17}{3}}{2}$ **OR** At point of inflection: $f^{//}(x) = -6x + 20 = 0$ $\therefore 6x = 20$ $x = \dfrac{10}{3}$ $= \dfrac{1 + \dfrac{17}{3}}{2}$ $= \dfrac{10}{3}$ Therefore: The graph is concave down for $x > \dfrac{10}{3}$	✓CA method to calculate x-value of point of inflection ✓CA x-value ✓CA answer (3)
8.3.2	$x < \dfrac{10}{3}$	✓✓ CA CA answer (2)
		[13]

QUESTION 9

	✓A shape
	✓A turning point at $x = -2$
	✓A turning point at $x = 4$
	✓A one x-intercept
	✓A y-intercept above x-axis
	(5)
	[5]

QUESTION 10

10.1	Total area $= 2\left(\dfrac{1}{2}\pi r^2\right) + (x \times 2r)$ $400 = \pi r^2 + 2xr$ $2xr = 400 - \pi r^2$ $x = \dfrac{400 - \pi r^2}{2r}$	✓A formula for area ✓A equating to 400 (2)
10.2	Length $= 2(\pi r) + 2x$ $L(r) = 2(\pi r) + 2\left(\dfrac{400 - \pi r^2}{2r}\right)$ $= 2\pi r + \dfrac{400 - \pi r^2}{r}$ $= \dfrac{2\pi r^2 + 400 - \pi r^2}{r}$ $= \dfrac{400}{r} + \pi r$	✓A formula for perimeter ✓A substitution ✓A simplification (3)

10.3	$L(r) = 400r^{-1} + \pi r$	✓A derivative
	$\dfrac{dL}{dr} = -400r^{-2} + \pi$	
	For a minimum: $\dfrac{dL}{dr} = -400r^{-2} + \pi = 0$	✓CA equating to zero
	$\dfrac{1}{r^2} = \dfrac{\pi}{400}$	
	$r^2 = \dfrac{400}{\pi}$	✓CA r^2 subject of formula
	$r = \sqrt{\dfrac{400}{\pi}}$	
	$= 11,28\,m$	✓CA answer
		(4)
		[9]

TOTAL: 143