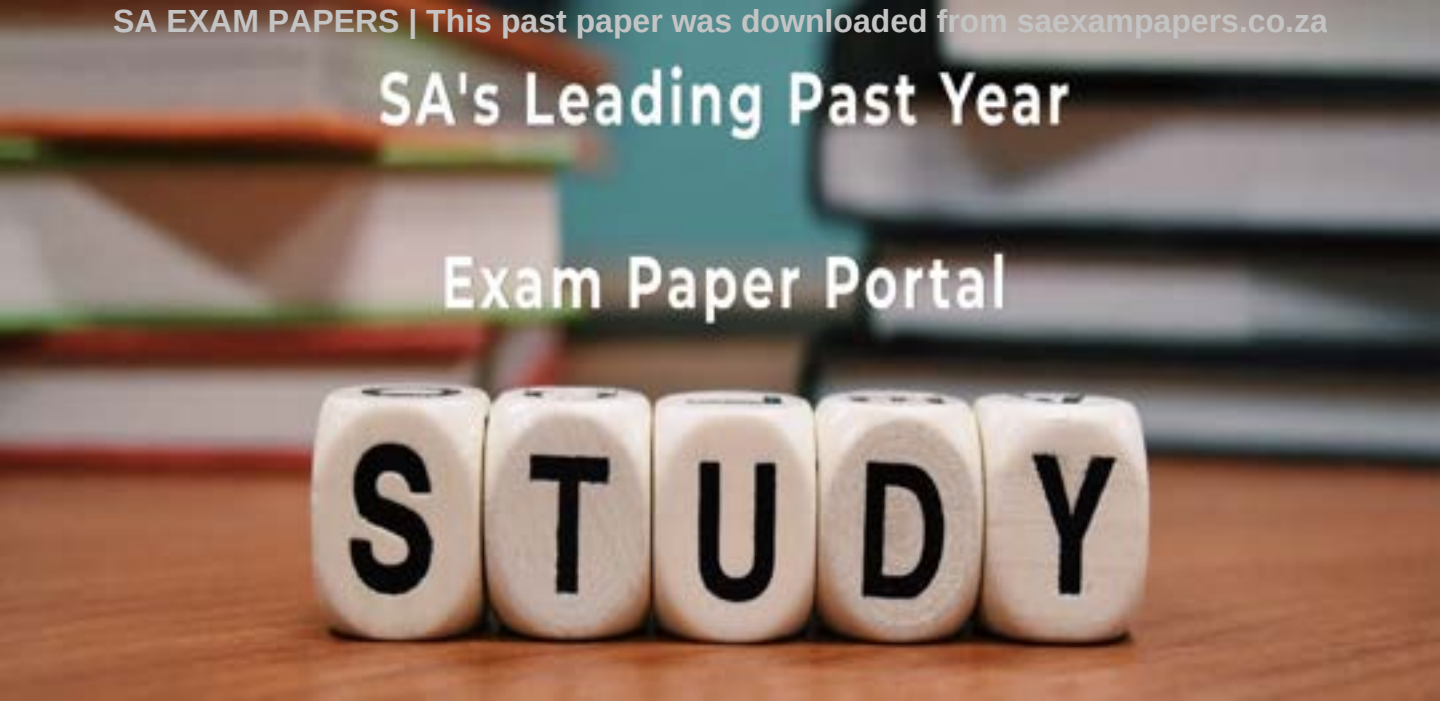


education

Department of Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS

MOCK EXAM

PAPER 1

MARKS: 150

DURATION: 3 HOURS

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have u determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwis
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included at the end of this question pape
9. Number the answers correctly according to the numbering system used question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $2x(3-x)=0$ (2)

1.1.2 $5x^2-4x=2$ (Rounede off to two decimal places) (4)

1.1.3 $\sqrt{7+3x}+2x=0$ (5)

1.1.4 $3x^2+5x\geq -2$ (4)

1.1.5 $3^{x+2}+3^{2-x}=82$ (5)

1.2 For which values of x will $\sqrt{x^2-5x+4}$ be real? (4)

1.3 Solve for x and y

$(2x-y)(x+2y-3)=0$

$x-y=1$ (5)

1.4 Show, WITHOUT using a calculator, that $\sqrt{10}\times\sqrt[3]{640}\times\sqrt[4]{810}\times\sqrt{40}=120$. (4)

[33]

QUESTION 2

2.1 The following sequence of number forms a quadratic sequence:

$-3;\ -2;\ -3;\ -6;\ldots$

2.1.1 The FIRST differences of the above sequence also form a sequence. Determine an expression for the general term of the first difference. (3)

2.1.2 Calculate the difference between 35th and 36th terms of the quadratic sequer (1)

2.1.3 Determine an expression for the nth term of the quadratic sequence. (4)

2.1.4 Show that the sequence of numbers will NEVER contain a positive term. (2)

2.2 A quadratic pattern has a constant second difference of 2 and $T_5=T_{11}=29.$

2.2.1 Does this pattern have a minimum or maximum value? Justify the answer (3)

2.2.2 Determine an expression for the nth term in the form $T_n=an^2+bn+c.$ (5)

[18]

QUESTION 3

3.1 The first four terms of an arithmetic sequence are:

65 ; 73 ; 81 ; 89 ; ...

3.1.1 Determine an expression for the nth term. (2)

3.1.2 Calculate the value of the term in the 1000^{th} position. (2)

3.1.3 Calculate the sum of the first 1000 terms. (2)

3.2 A new sequence if formed by adding together the corresponding terms of a geometr arithmetic sequence.

- The common ratio of the geometric sequence is 2.
- The common difference of the arithmetic sequence is 2.
- The first term of the new sequence is 1 and the second term is 7.

3.2.1 Calculate the third term of the new sequence. (5)

3.2.2 Determine the expression for the n^{th} term of the new sequence. (3)

[14]

QUESTION 4

Calculate the value of y if

$$\sum_{p=1}^{5}(4y+3p)+\sum_{k=4}^{7}3.(2)^{k-1}=\sum_{j=1}^{\infty}\left(\frac{1}{3}\right)^{j-1}$$

[7]

QUESTION 5

Given: $g(x) = \frac{-6}{x-3} + 1$

5.1 Determine the:

5.1.1 Equations of the asymptotes of g. (2)

5.1.2 y-intercept of g. (1)

5.1.3 x-intercept of g. (2)

5.2 For which values of x is $g(x) > 0$? (2)

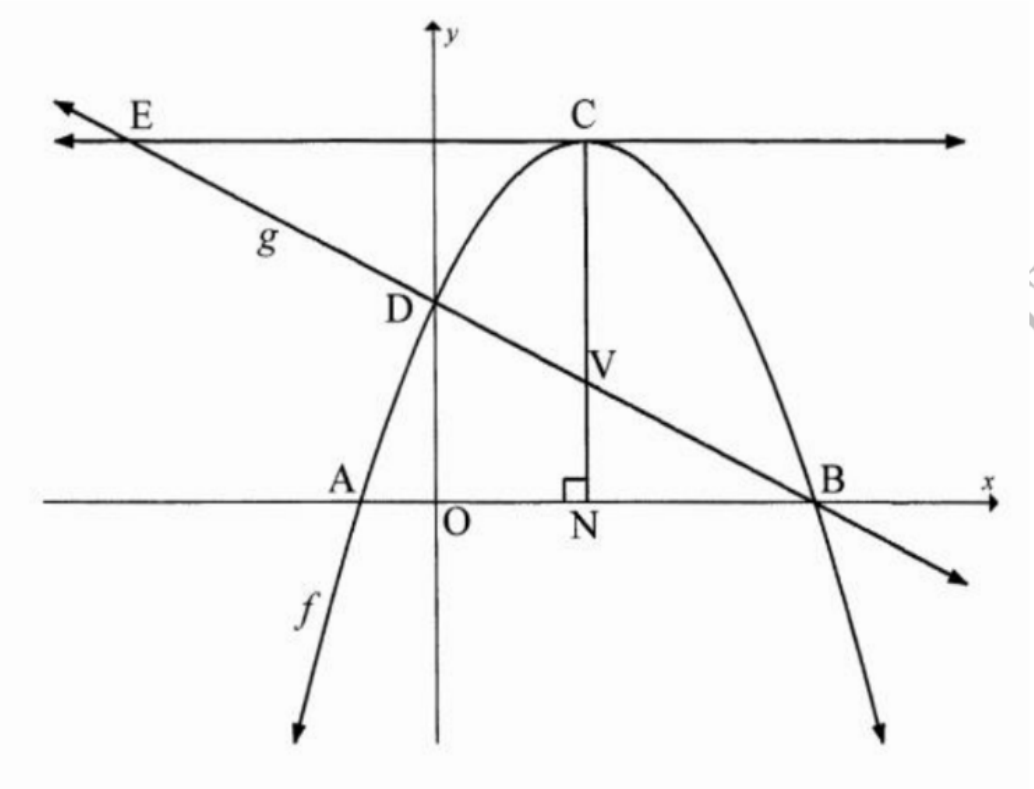
5.3 If $h(x) = x + c$ is the axis of symmetry of g, determine the value of c. (2)

5.4 Describe in words the transformation of g to $h(x) = \frac{6}{x+3} + 1$. (2)

[14]

QUESTION 6

Sketched below are the graphs of $f(x) = -x^2 + 4x + 5$ and $g(x) = mx + c$.
f and g intersect at B and D. B and D are the x– and y – intercepts of g, respectively. C is the turning point of f. V is a point on g and N is a point on the x – axis such that CVN $\perp x$ – axis. E is a point on g such that CE $\| x$ – axis. A and B are the x – intercepts of f.



6.1 Determine the co-ordinates of C, the turning point of f. (3)

6.2 Write down the range of f. (1)

6.3 Calculate the length of AB. (4)

6.4 Determine the equation of g. (2)

6.5 T is a point on f such that D and T are reflections of each other over CVN. Write dow (2)
coordinates of T.

6.6 The line EC is a tangent to f at C.

6.6.1 Write down the gradient of this tangent. (1)

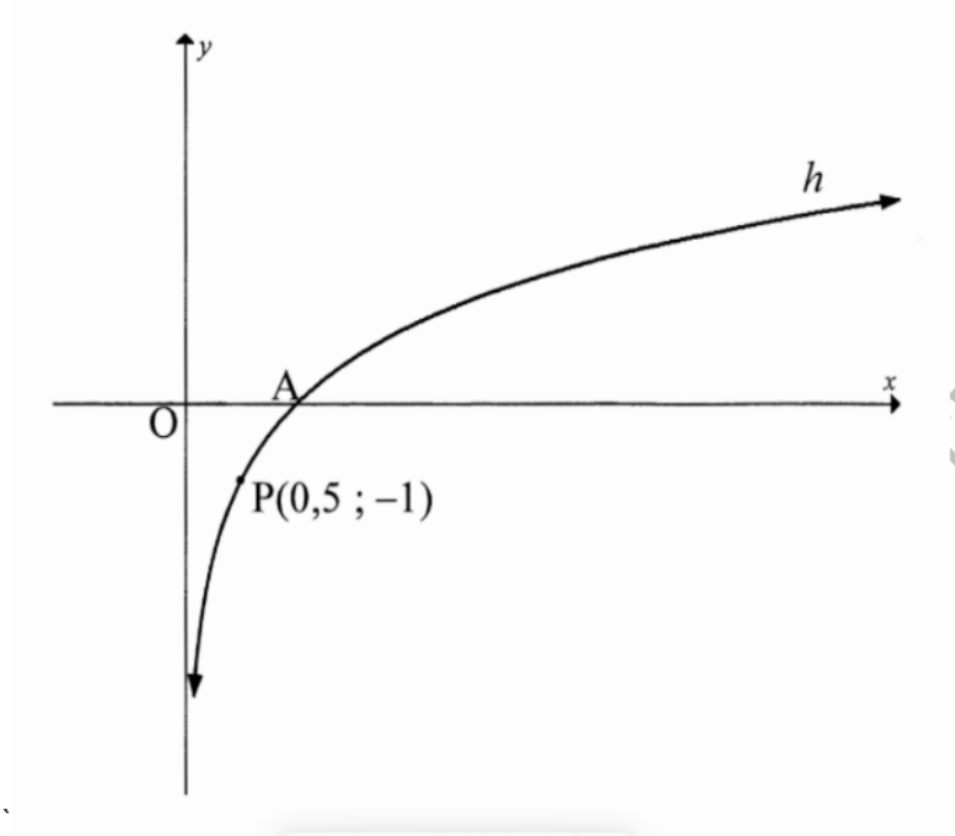
6.6.2 Determine the coordinates of E. (2)

6.7 Determine the value of k for which $y = -x + k$ is a tangent to f. (5)

[20]

QUESTION 7

In the diagram, the graph $h(x) = \log_a x$ is drawn. $P(0,5\ ;\ -1)$ lies on h.



7.1 Calculate the value of a. (3)

7.2 Write down the equation of h^{-1}, the inverse of h, in the form $y = \ldots$ (2)

7.3 Write down the domain of h^{-1}. (1)

7.4 Determine the values of x if $h(x) \leq -1$. (2)

[8]

QUESTION 8

8.1 Determine $f'(x)$ from first principle given $f(x) = x^2 + 5x + 6$ (5)

8.2 Determine:

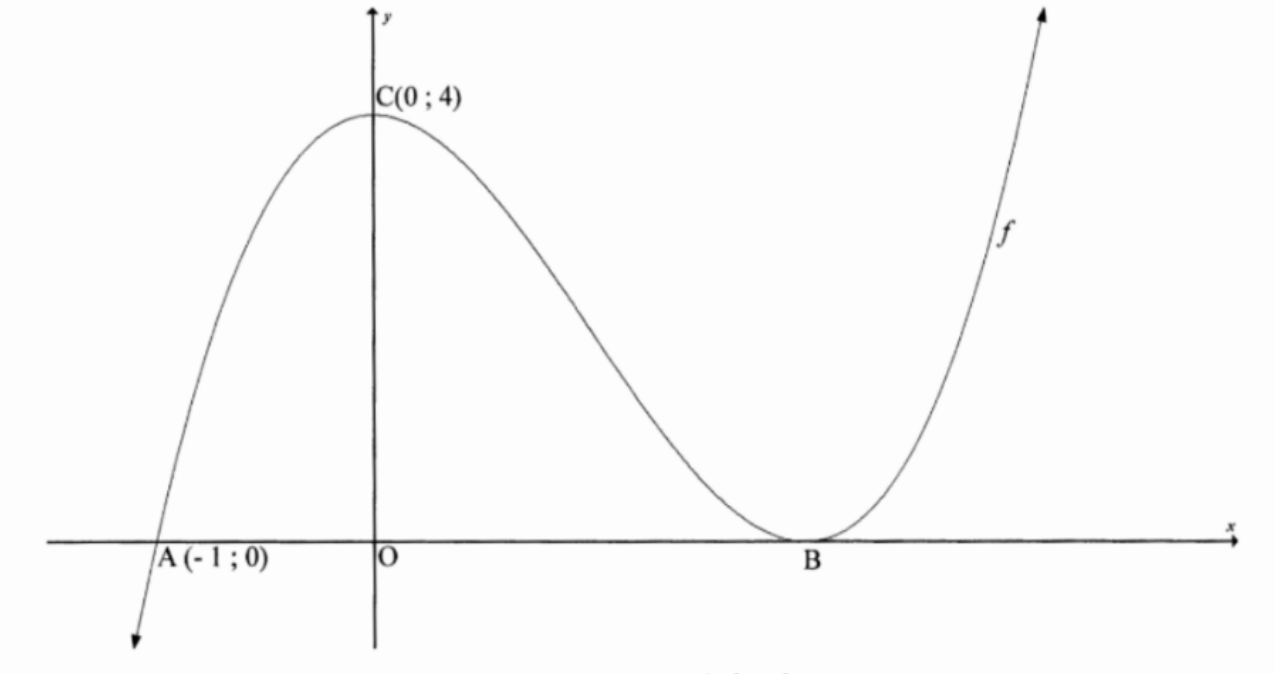
8.2.1 $f'(x)$ if $f(x) = 3x(\sqrt{x} - 4)$ (3)

8.2.2 $\frac{dy}{dx}$ *if* $y = \frac{x^3 - 4x}{2 - x}$ (4)

[12]

QUESTION 9

The graph of $f(x) = x^3 + bx^2 + cx + d$; $a \neq 0$ is sketched below.
A(–1; 0) is an x – intercept. C(0 ; 4) is a turning point and B is both a local minimum and x – intercept of f.



9.1 Write down the value of d. (1)

9.2 Show that $b = -3$ and $c = 0$. (4)

9.3 Determine the equation of the tangent to f at $x = 5$. (4)

9.4 For which values of k will $f(x) = k$ have 2 unequal positive roots and 1 negative roc simultaneously. (2)

9.5 Determine the coordinates of the local minimum of g if $g(x) = f(-x) + 3$. (4)

[15]

QUESTION 10

10.1 Use the information below to draw a graph of the function defined by

$f(x) = ax^3 + bx + cx + d.$

Indicate the intercepts with the axes as well as the coordinates of the turning points.

- $f(0) = 3$ *and* $f(-3) = 0$
- $f'(-2) = f'(1) = 0$
- $f(-2) = 5$ *and* $f(1) = 1$ (5)

10.2 Use the graph to answer the questions below:

10.2.1 Determine the value of x for which $f(x) < 0.$ (2)

10.2.2 If $g(x) = -f(x)$, write down the coordinates of the local minimum point of g. (2)

[9]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 + i)^n$ $A = P(1 - i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

In $\triangle ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc\cos A$

$$\text{area } \triangle ABC = \frac{1}{2}ab\sin C$$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$(x; y) \to (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta)$ $(x; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$