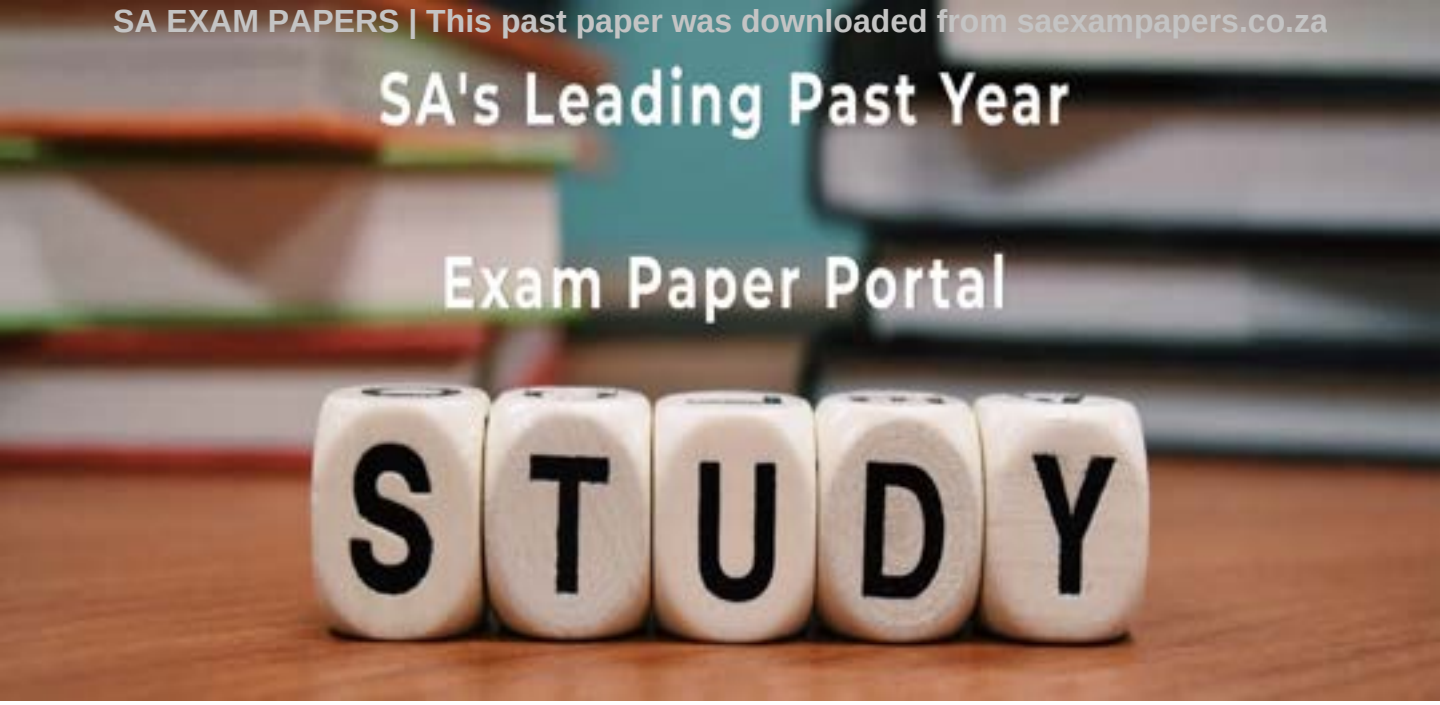


LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

VHEMBE WEST DISTRICT

GRADE 12

MATHEMATICS P1

PRE-MIDYEAR EXAMINATION 2024

MARKS: 120
TIME. : 2,5 *HOURS*

This question paper consists of 7 pages **including the cover page**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 5x - 6 = 0$ (2)

1.1.2 $(3x-1)(x-4) = 16$ (correct to TWO decimal places) (4)

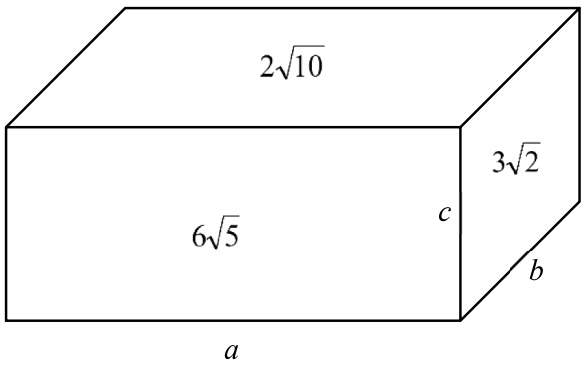
1.1.3 $4x - x^2 \geq 0$ (3)

1.1.4 $\dfrac{5^{2x} - 1}{5^x + 1} = 4$ (3)

1.2 Solve simultaneously for x and y:

$x + 3y = 2$ and $x^2 + 4xy - 5 = 0$ (5)

1.3 A rectangular box has dimensions a, b and c. The area of the surfaces are $2\sqrt{10}$; $3\sqrt{2}$ and $6\sqrt{5}$, as shown in the diagram below.



Calculate, **without using a calculator**, the volume of the rectangular box. (5)
[22]

QUESTION 2

2.1 The first FOUR terms of a quadratic pattern are: 15 ; 29 ; 41 ; 51

2.1.1 Write down the value of the 5th term. (1)

2.1.2 Determine an expression for the n^{th} term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Determine the value of T_{27} (2)

2.2 Given a geometric sequence: $36\ ;\ -18\ ;\ 9\ ;\ \ldots$

2.2.1 Determine the value of r, the common ratio. (1)

2.2.2 Calculate n if $T_n = \dfrac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\dfrac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$ (4)

[17]

QUESTION 3

3.1 The first three terms of an arithmetic sequence are: $2p + 3\ ;\ p + 6$ and $p - 2$.

3.1.1 Show that $p = 11$. (2)

3.1.2 Calculate the smallest value of n for which $T_n < -55$. (3)

3.2 Given that $\sum_{k=1}^{6}(x - 3k) = \sum_{k=1}^{9}(x - 3k)$, prove that $\sum_{k=1}^{15}(x - 3k) = 0$. (5)

[10]

QUESTION 4

Given the exponential function: $g(x) = \left(\dfrac{1}{2}\right)^x$

4.1 Write down the range of g. (1)

4.2 Determine the equation of g^{-1} in the form $y = \ldots$ (2)

4.3 Is g^{-1} a function? Justify your answer. (2)

4.4 The point $M(a\ ;\ 2)$ lies on g^{-1}.

4.4.1 Calculate the value of a. (2)

4.4.2 M', the image of M, lies on g. Write down the coordinates of M'. (1)

4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of M' on h. (3)

[11]

QUESTION 5

5.1 Given: $f(x)=\frac{1}{x+2}+3$

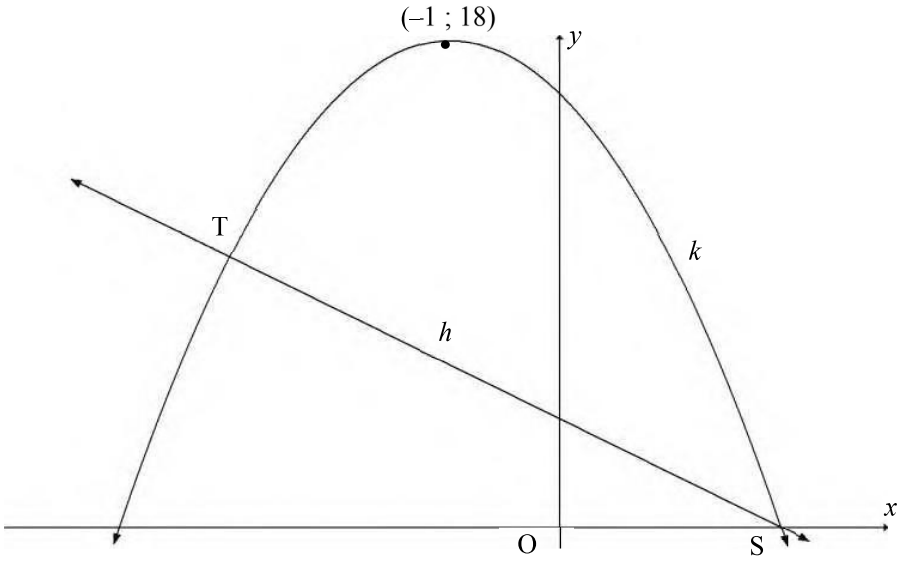
5.1.1 Determine the equations of the asymptotes of f. (2)

5.1.2 Write down the y-intercept of f. (1)

5.1.3 Calculate the x-intercept of f. (2)

5.1.4 Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes. (3)

5.2 Sketched below are the graphs of $k(x)=ax^2+bx+c$ and $h(x)=-2x+4$. Graph k has a turning point at (–1 ; 18). S is the x-intercept of h and k. Graphs h and k also intersect at T.



5.2.1 Calculate the coordinates of S. (2)

5.2.2 Determine the equation of k in the form $y=a(x+p)^2+q$ (3)

5.2.3 If $k(x)=-2x^2-4x+16$, determine the coordinates of T. (4)

5.2.4 Determine the value(s) of x for which $k(x)<h(x)$. (2)

5.2.5 It is further given that k is the graph of $g'(x)$.

(a) For which values of x will the graph of g be concave up? (2)

(b) Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection. (3)

[24]

QUESTION 6

6.1 Given $f(x)=x^2+2$.

Determine $f'(x)$ from first principles. (5)

6.2 Determine $\frac{dy}{dx}$ if:

6.2.1 $y=4x^3+\frac{2}{x}$ (3)

6.2.2 $y=4.\sqrt[3]{x}+(3x^3)^2$ (4)

6.3 If g is a linear function with $g(1)=5$ and $g'(3)=2$, determine the equation of g in the form $y=\ldots..$ (3)
[15]

QUESTION 7

A cubic function $h(x)=-2x^3+bx^2+cx+d$ cuts the x-axis at $(-3\ ;\ 0)$; $\left(-\frac{3}{2}\ ;\ 0\right)$ and $(1\ ;\ 0)$.

7.1 Show that $h(x)=-2x^3-7x^2+9$. (3)

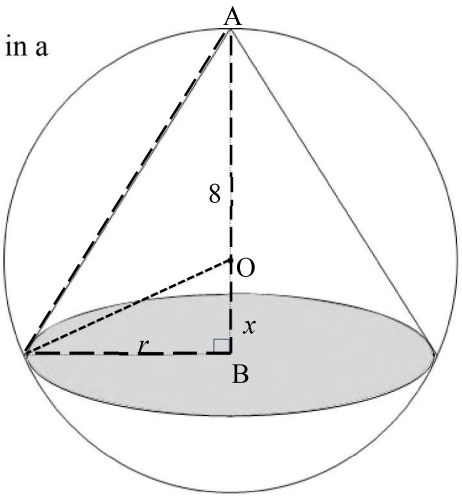
7.2 Calculate the x-coordinates of the turning points of h. (3)

7.3 Determine the value(s) of x for which h will be decreasing. (2)

7.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y-4x=7$. (4)
[12]

QUESTION 8

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. OB = x.



Volume of sphere = $\frac{4}{3}\pi r^3$
Volume of cone = $\frac{1}{3}\pi r^2 h$

8.1 Calculate the volume of the sphere. (1)

8.2 Show that $r^2 = 64 - x^2$. (1)

8.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)
[9]

TOTAL: 120

INFORMATION SHEET: MATHEMATICS

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad A=P(1+ni) \quad A=P(1-ni) \quad A=P(1-i)^n$$

$$A=P(1+i)^n \quad \sum_{i=1}^{n}1=n \quad \sum_{i=1}^{n}i=\frac{n(n+1)}{2} \quad T_n=a+(n-1)d$$

$$S_n=\frac{n}{2}\left(2a+(n-1)d\right) \; T_n=ar^{n-1} \; S_n=\frac{a\left(r^n-1\right)}{r-1} \; ; \quad r\neq 1 \quad S_\infty=\frac{a}{1-r}\; ; \; -1<r<1$$

$$F=\frac{x\left[(1+i)^n-1\right]}{i} \quad P=\frac{x[1-(1+i)^{-n}]}{i} \quad f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

$$y=mx+c \quad y-y_1=m(x-x_1) \quad m=\frac{y_2-y_1}{x_2-x_1} \quad m=\tan\theta$$

$$(x-a)^2+(y-b)^2=r^2 \; In\; \Delta ABC: \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad a^2=b^2+c^2-2bc.\cos A$$

$$area\,\Delta ABC=\frac{1}{2}ab.\sin C$$

$$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta \quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta \quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases} \quad \sin 2\alpha=2\sin\alpha.\cos\alpha$$

$$\bar{x}=\frac{\sum fx}{n} \quad \sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$$

$$P(A)=\frac{n(A)}{n(S)} \quad P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$$

$$\hat{y}=a+bx \quad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$