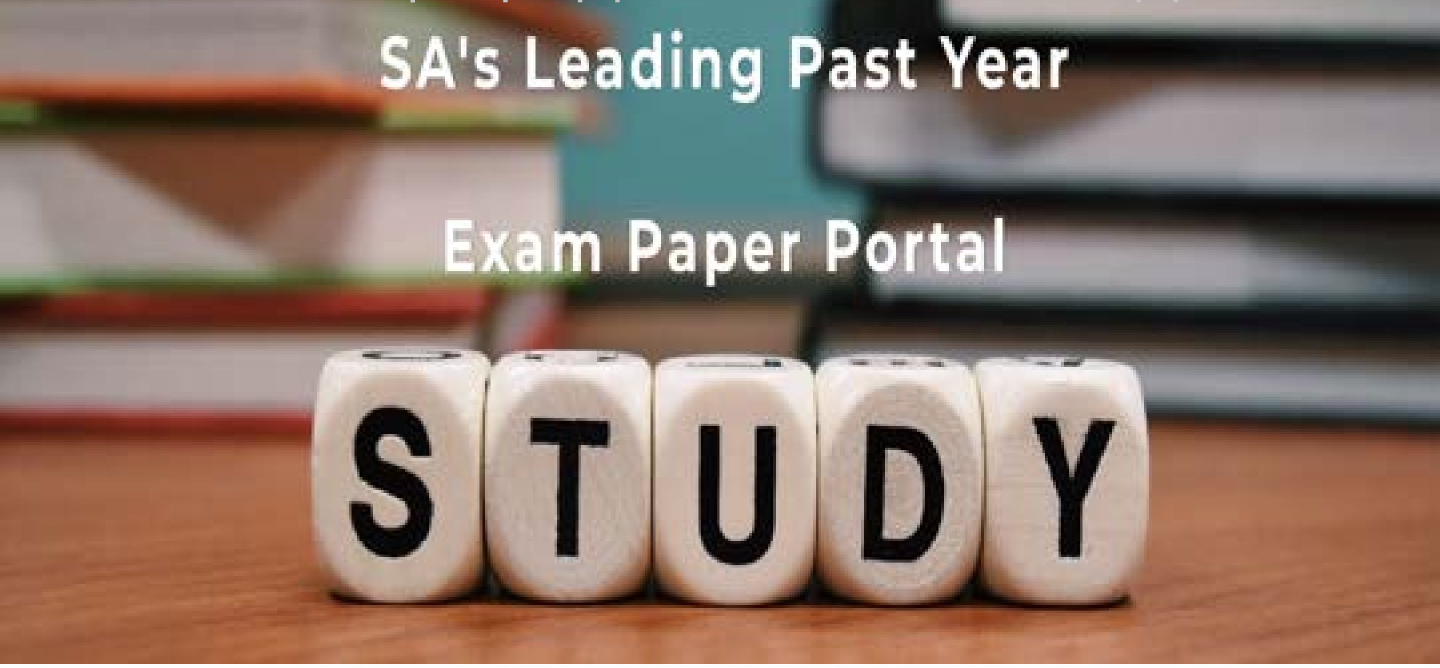

SA's Leading Past Year
Exam Paper Portal
STUDY

DISTRICT PAPER

GRADE 12

TERM 1 CONTROL TEST 2024

NATIONAL SENIOR CERTIFICATE

JOHANNESBURG NORTH

MATHEMATICS

MARKING GUIDELINE

MARKS: 100

This Memorandum consists of 16 pages.

GENERAL NOTES

1. Consistent accuracy applies in this marking guideline.
2. If a learner answers the same question twice, but does not cancel one of the answers, **ONLY** consider the first attempt.
3. If a learner cancels the answer but does not make a second attempt, consider the cancelled attempt.
4. If a learner provided an answer not mentioned in this memorandum, first check/prove it before disqualifying their attempt. Please check through all **OPTIONS** provided in this marking guideline.

QUESTION 1				
1.1				
	1.1.1	$(x-2)(x+9)=0$ Either $x+9=0$ or $x-2=0$ $\therefore x=2$ or $x=-9$	✓ $x=-9$ ✓ $x=2$	(2)
	1.1.2	$5x^2-2=6x$ $5x^2-6x-2=0$ $a=5; b=-6; c=-2$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(5)(-2)}}{2(5)}$ $x=\frac{6\pm\sqrt{76}}{10}$ $\therefore x=1.47$ or $x=-0.27$ **Or** $5x^2-6x-2=0$ $x^2-\frac{6}{5}x-\frac{2}{5}=0$ $x^2-\frac{6}{5}x=\frac{2}{5}$	✓ Standard form ✓ Substitution ✓ $\sqrt{76}$ ✓ $x=1.47$ ✓ $x=-0.27$ A candidate will get a maximum of 4 marks if rounding off is not correct to 2decimal places. **OR** ✓ Standard form ✓ Completion of a square	

		$\left(x-\frac{3}{3}\right)^2=\frac{2}{5}+\frac{9}{25}$ $\left(x-\frac{3}{5}\right)^2=\frac{19}{25}$ $x-\frac{3}{5}=\pm\frac{\sqrt{19}}{5}$ $x=\frac{3}{5}\pm\frac{\sqrt{19}}{5}$ $x=\frac{3\pm\sqrt{19}}{5}$ $\therefore x=1.47\ or\ x=-0.27$	✓ $\sqrt{19}$ ✓ $x=1.47$ ✓ $x=-0.27$ A candidate will get a maximum of 4 marks if rounding off is not correct to 2decimal places	(5)
	1.1.3	$2x^2-5x-3\geq 0$ $(2x+1)(x-3)\geq 0$ Critical values: $x=-\frac{1}{2}\ and\ x=3$ $x\leq -\frac{1}{2}\quad or\ \ x\geq 3$	✓ Factorisation/Other methods ✓ Critical values ✓ $x\leq -\frac{1}{2}$ ✓ $x\geq 3$	(4)
1.2	$2y+x-3=0$ and $x^2-3xy+5y^2=3$ $2y+x-3=0$........................ Equation 1 $x^2-3xy+5y^2=3$.............. Equation 2 $x=3-2y$ .... Equation 3 Substitute equation 1 into equation 2 $(3-2y)^2-3y(3-2y)+5y^2=3$ $9-12y+4y^2-9y+6y^2+5y^2-3=0$ $15y^2-21y+6=0$ $5y^2-7y+2=0$ $(5y-2)(y-1)=0$ $y=\frac{2}{5}\quad or\ y=1$		✓ Equation 3 ✓ Substitution ✓ Standard form ✓ Factorisation ✓ x – Values ✓ y –Values	

	$x = 3-2\left(\frac{2}{5}\right) = \frac{11}{5}$ *or* $x = 3-2(1) = 1$ $\left(\frac{2}{5};\frac{11}{5}\right)$ (1; 1)		(6)
			[17]

QUESTION 2			
2.1,1	$S_n = n^2 + 2n$ $S_{100} = (100)^2 + 2(100)$ $S_{100} = 10\ 200$	✓ Substitution ✓ 10 200	(2)
2.1.2	$S_1 = (1)^2 + 2(1) = 3$ $\therefore T_1 = 3$ $S_2 = (2)^2 + 2(2) = 8$ $T_2 = S_2 - S_1$ $= 8 - 3$ $= 5$ $S_3 = (3)^2 + 2(3) = 15$ $T_3 = S_3 - S_2$ $= 15 - 8$ $= 7$ The first three terms of the original pattern are: 3; 5; 7 …	✓ 3 ✓ 5 ✓ 7	(3)

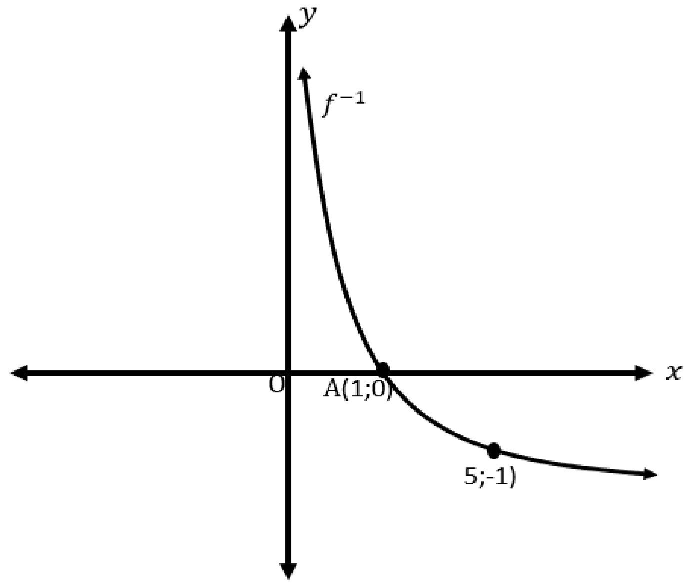
2.2	$\sum_{k=2}^{n}(5-2k) = -\frac{800n}{17}$ $1+(-1)+(-3)+\cdots+(5-2n) = -\frac{800n}{17}$ The number of terms is $(n-2)+1 = n-1$ $S_n = \frac{n}{2}(a+l)$ $-\frac{800n}{17} = \frac{n-1}{2}(1+5-2n)$ $-\frac{800n}{17} = \frac{n-1}{2}(6-2n)$ $-\frac{800n}{17} = (n-1)(3-n)$ $-800n = 17(4n-n^2-3)$ $-800n = 68n-17n^2-51$ $17n^2-868n+51 = 0$ $(17n-1)(n-51) = 0$ $\therefore n = 51$ and $n \neq \frac{1}{17}$ **Or** $1+(-1)+(-3)+\cdots+(5-2n) = -\frac{800n}{17}$ The number of terms is $(n-2)+1 = n-1$ $S_n = \frac{n}{2}[2a+(n-1)d]$ $-\frac{800n}{17} = \frac{n-1}{2}[2+(n-1-1)(-2)]$ $-\frac{800n}{17} = \frac{n-1}{2}[2-2n+4]$ $-\frac{800n}{17} = (n-1)(3-n)$ $-800n = 17(4n-n^2-3)$ $-800n = 68n-17n^2-51$ $17n^2-868n+51 = 0$	✓ Expansion ✓ $n-1$ ✓ Substitution into correct formula ✓ Equation ✓ Factors/method ✓ 51 1 mark penalty for not discarding $\frac{1}{17}$	(6)

	$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $n = \frac{-(-868) \pm \sqrt{(-868)^2 - 4(17)(51)}}{2(17)}$ $n = \frac{868 \pm \sqrt{749\,956}}{34}$ $\therefore n = 51 \; and \; n \neq \frac{1}{17}$ **Or** $1 + (-1) + (-3) + \cdots + (5 - 2n) = -\frac{800n}{17}$ The number of terms is $(n-2) + 1 = n - 1$ $S_n = \frac{n}{2}(a + l)$ $-\frac{800n}{17} = \frac{n-1}{2}(1 + 5 - 2n)$ $-\frac{800n}{17} = \frac{n-1}{2}(6 - 2n)$ $-\frac{800n}{17} = (n-1)(3-n)$ $-800n = 17(4\text{n} - n^2 - 3)$ $-800n = 68\text{n} - 17n^2 - 51$ $17n^2 - 868n + 51 = 0$ $5x^2 - 6x - 2 = 0$ $n^2 - \frac{868}{17}n + 3 = 0$ $n^2 - \frac{868}{17}n = -3$ $\left(x - \frac{434}{17}\right)^2 = -3 + \frac{188\,356}{289}$ $\left(n - \frac{434}{17}\right)^2 = \frac{187\,489}{289}$ $n - \frac{434}{17} = \pm\frac{433}{17}$ $n = \frac{434}{17} \pm \frac{433}{17}$ $x = 17$ $\therefore x = 51 \; and \quad n \neq \frac{1}{17}$		**[11]**

QUESTION 3			
3.1	$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$ $-(rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n)$ $S_n - rS_n = a - ar^n$ $S_n(1-r) = a(1-r^n)$ $\therefore S_n = \frac{a(1-r^n)}{1-r}; r \neq 1$	✓ S_n ✓ rS_n ✓ Subtraction ✓ Factorisation	(4)
3.2.1	$8(x-2) + 4(x-2)^2 + 2(x-2)^3 + \cdots$ $r = \frac{T_2}{T_1} = \frac{4(x-2)^2}{8(x-2)} = \frac{x-2}{2}$ For convergence. $-1 < r < 1$ $-1 < \frac{x-2}{2} < 1$ $-2 < x - 2 < 2$ $0 < x < 4$	✓ Common ratio ✓ Substitution ✓ Answer	(3)
3.2.2	If $x = 3.5$ $r = \frac{3.5-2}{2} = \frac{3}{4}$ $a = 8(3.5-2) = 12$ $S_\infty = \frac{a}{1-r}; r \neq 1$ $S_\infty = \frac{12}{1-\frac{3}{4}} = 48$	✓ $\frac{3}{4}$ ✓ 12 ✓ Substitution into formula ✓ 48	(4)
			[11]

QUESTION 4

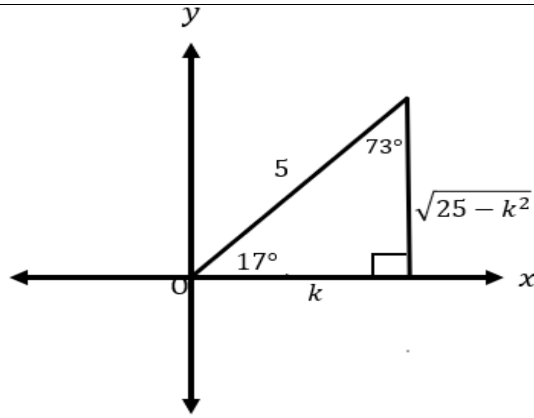
4.1	$y = \left(\frac{1}{5}\right)^0 = 1$ A$(0; 1)$	✓ A$(0; 1)$	(1)
4.2	$f(x) = \frac{a}{x+1} + 3$ Use A $(0; 1)$ $1 = \frac{a}{0+1} + 3$ $1 = a = 3$ $\therefore a = -2$ $f(x) = -\frac{2}{x+1} + 3$	✓ Substitution of asymptotes ✓ Substitution of point A(0;1) ✓ -2 ✓ Equation	(4)
4.3	$y = \left(\frac{1}{5}\right)^x$ Inverse: $x = \left(\frac{1}{5}\right)^y$ $\log x = ylog\frac{1}{5}$ $y = \log_{\frac{1}{5}} x$ or $y = -\log_5 x$	✓ Interchanging the variables. ✓ Introduction of logs. ✓ Answer	(3)

4.4	 	✓ Intercept ✓ Any other point ✓ Shape	(3)
4.5	$f^{-1}(x) =\geq -2$ Consider $f^{-1}(x) = -2$ $\log_{\frac{1}{5}} x = -2$ $x = \left(\frac{1}{5}\right)^{-2} = 25$ $\therefore f^{-1}(x) =\geq -2$ when: $x \leq 25$	✓ $x = \left(\frac{1}{5}\right)^{-2} =$ 25 ✓ $x \leq 25$	(2)
4.6	$h(x) = -\frac{2}{x+1-1} + 3 - 2$ $h(x) = -\frac{2}{x} + 1$ The equations of asymptotes are: $x = 0$ $y = 1$	✓ $x = 0$ ✓ $y = 1$	(2)
			[15]

QUESTION 5				
5.1				
		$y=-\frac{1}{5}x+c$ $12=-\frac{1}{5}(5)+c$ $12=-1+c$ $\therefore c=13$	✓ Substitution ✓ 13	(2)
5.2		$f(x)=a(x-3)^2+16$ $12=a(x-3)^2+16$ $-4=4a$ $\therefore a=-1$	✓ Substitution ✓ Simplification ✓ $a=-1$	(3)
5.3		$m_1\times m_2=-1$ Lines are perpendicular. $-\frac{1}{5}\times m_2=-1$ $m_2=5$ $k(x)=5x+c$ Use (0;7) $7=5(0)+c$ $c=7$ $k(x)=5x+7$	✓ 5 ✓ Substitution using point (0;7) ✓ Equation	(3)
5.4		$-x^2+6x+7=5x+7$ $x^2-x=0$ $x(x-1)=0$ $x=0$ or $x=1$ $0\le x\le 1$	✓ Equating ✓ Factorisation ✓ Answer	(3)

5.5		$h(x) = 2^{-f(x)}$ The minimum value of $h(x)$ is; $y = 2^{-16} = \frac{1}{2^{16}}$	✓ Substitution ✓ $\frac{1}{2^{16}}$	(2)
5.6		Average gradient $= \frac{f(x+h) - f(x)}{h}$ $= \frac{f(1)-f(-3)}{1-(-3)}$ $= \frac{12-(-20)}{4}$ $= \frac{32}{4}$ $= 8$	✓ 12 ✓ -20 ✓ 4 ✓ 8	(4)
				[17]

QUESTION 6

6.1	$\cos 17° = \frac{k}{5}$			
	6.1.1	 Use Pythagoras's Theorem. $x^2 + y^2 = r^2$ $k^2 + y^2 = 5^2$ $y^2 = 25 - k^2$ $y = \pm\sqrt{25-k^2}$ $\therefore y = \sqrt{25-k^2}$ $\therefore \sin 17° = \frac{\sqrt{25-k^2}}{5}$	✓ Application of Pythagoras theorem ✓ $\sqrt{25-k^2}$ ✓ $\frac{\sqrt{25-k^2}}{5}$	(3)
	6.1.2	$\tan 253°$ $\tan(180° + 73°)$ $\tan 73° = \frac{k}{\sqrt{25-k^2}}$	✓ Compound angle. ✓ $\tan 73°$ ✓ $\frac{k}{\sqrt{25-k^2}}$	(3)
	6.1.3	$\sin 124° = \sin(90° + 34°)$ $= cos 34°$ $cos 34° = 2cos^2 17° - 1$ $= 2\left(\frac{k}{5}\right)^2 - 1$ $= \frac{2k^2}{25} - 1$ $= \frac{2k^2 - 25}{25}$	✓ $(90° + 34°)$ ✓ $cos 34°$ ✓ Substitution into correct formula. ✓ $\frac{2k^2-25}{25}$	(4)

		Or $\sin 124° = \sin(90° + 34°)$ $= \cos 34°$ $\cos 34° = \cos^2 17° - \sin^2 17°$ $= \frac{k^2}{25} - \left(\frac{\sqrt{25-k^2}}{5}\right)^2$ $= \frac{k^2}{25} - \frac{25-k^2}{25}$ $= \frac{2k^2-25}{25}$ **Or** $\sin 124° = \sin(90° + 34°)$ $= \cos 34°$ $\cos 34° = 1 - 2\sin^2 17°$ $= 1 - 2\left(\frac{\sqrt{25-k^2}}{5}\right)^2$ $= 1 - 2\left(\frac{25-k^2}{25}\right)$ $= \frac{2k^2-25}{25}$		

6.2	$\cos(\alpha - \beta) = \sin[90° - (\alpha - \beta)]$ $= \sin[90° - \alpha + \beta]$ $= \sin[(90° - \alpha) - (-\beta)]$ $= \sin(90° - \alpha)\cos(-\beta) - \cos(90° - \alpha)\sin(-\beta)$ $= \cos\alpha\cos\beta - \sin\alpha(-\sin\beta)$ $= \cos\alpha\cos\beta + \sin\alpha\sin\beta$	✓ $\sin[90° - \alpha + \beta]$ ✓ $\sin[(90° - \alpha) - (-\beta)]$ ✓ Substitution into sine formula.	(3)
6.3.1	$\cos(60° + x) + \sin(30° - x)$ $\cos 60° \cos x - \sin 60° \sin x + [\sin 30° \cos x - \cos 30° \sin x]$ $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + [\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x]$ $\cos x - \sqrt{3}\sin x$	✓ Expansion ✓ Substitution of special angles. ✓ Answer.	(3)

6.3.2	$cos45° - \sqrt{3}sin45°$ $\frac{\sqrt{2}}{2} - \sqrt{3}\left(\frac{\sqrt{2}}{2}\right)$ $\frac{\sqrt{2} - \sqrt{6}}{2}$ **Or** $\cos(60° + 45°) + \sin(30° - 45°)$ $\cos(60° + 45°) - \sin(45° - 30°)$ $cos60°cos45° - sin60°sin45° + [sin30°cos45° - cos30°sin45°$ $\frac{1}{2}.\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2} + [\frac{1}{2}.\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2}]$ $\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ $\frac{2\sqrt{2}}{4} - \frac{2\sqrt{6}}{4}$ $\frac{2(\sqrt{2} - \sqrt{6})}{4}$ $\frac{\sqrt{2} - \sqrt{6}}{2}$	✓ Substitution ✓ Answer ✓ Substitution ✓ Answer	(2)
6.4	$\frac{1 - cos2\theta + sin2\theta}{1 + cos2\theta + sin2\theta} = tan\theta$ **LHS** $\frac{1 - (1 - 2sin^2\theta) + 2sin\theta cos\theta}{1 + (2cos^2\theta - 1) + 2sin\theta cos\theta}$ $\frac{2sin^2\theta + 2sin\theta cos\theta}{2cos^2\theta + 2sin\theta cos\theta}$ $\frac{2sin\theta(sin\theta + cos\theta)}{2cos\theta(cos\theta + sin\theta)}$ $\frac{sin\theta}{cos\theta}$ $tan\theta = RHS$	✓ $1 - 2sin^2\theta$ ✓ $2sin\theta cos\theta$ ✓ $2cos^2\theta - 1$ ✓ Simplification ✓ factorization	(5)

	Or $\frac{1-cos2\theta+sin2\theta}{1+cos2\theta+sin2\theta}=tan\theta$ LHS $\frac{cos^2\theta+sin^2\theta-(cos^2\theta-sin^2\theta)+2sin\theta cos\theta}{cos^2\theta+sin^2\theta+(cos^2\theta-sin^2\theta)+2sin\theta cos\theta}$ $\frac{2sin^2\theta+2sin\theta cos\theta}{2cos^2\theta+2sin\theta cos\theta}$ $\frac{2sin\theta(sin\theta+cos\theta)}{2cos\theta(cos\theta+sin\theta)}$ $\frac{sin\theta}{cos\theta}$ $tan\theta=RHS$	✓ $cos^2\theta+sin^2\theta$ ✓ $cos^2\theta-sin^2\theta$ ✓ $2sin\theta cos\theta$ ✓ Simplification ✓ Factorization	

6.5	$cos2x+5sinx=-2$ $1-2sin^2x+5sinx=-2$ $2sin^2x-5sinx-3=0$ $(2sinx+1)(sinx-3)=0$ $2sinx+1=0$ or $sinx\neq 3$ $sinx=-\frac{1}{2}$ Ref angle $=\sin^{-1}\left(\frac{1}{2}=30°\right)$ $\therefore x=210°+k.360°;k\in Z$ **Or** $x=330°+k.360°;k\in Z$	✓ $1-2sin^2x$ ✓ Equation in standard form ✓ Factorization ✓ $210°+k.360°$ ✓ $330°+k.360°$ ✓ $k\in Z$	(6)
			[29]