education
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GRADE 12

MATHEMATICS

13 March 2024

Marks: 100
Time : 2hrs

This paper consists of 7 pages with 1 information sheet.

Instructions and Information

Read the following instructions carefully before answering this question paper:

1 This question paper consists of 6 questions.

2 Answer ALL questions.

3 Answers only will not necessarily be awarded full marks.

4 Diagrams are NOT necessarily drawn to scale.

5 Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.

6 An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

7 If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

8 Number your answers correctly according to the numbering system used in this question paper.

9 It is in your own interest to write legibly and to present your work neatly.

10 An information sheet with formulae is included at the end of the question paper.

Question 1

1.1.1 Solve for x

1.1.1 $x^2 - 2x - 3 = 0$ (2)

1.1.2 $2x^2 - x = 5$ (Correct to two decimal places) (4)

1.1.3 $(x-3)(x+1) \leq 12$ (4)

1.1.4 $x - \sqrt{5+x} = 7$ (4)

1.2 Solve for x and y simultaneously

$x + 2y = 3$ And $3x^2 + 4xy + 9y^2 - 16 = 0$ (5)

1.3 Given: $2mx^2 = 3x - 8$ where $m \neq 0$

Determine the value(s) of m for which the roots of the equation are non-real (4)

[23]

Question 2

2.1 Calculate the number of terms in the following Arithmetic sequence

$6;\ 1;\ -4;\ -9, \ldots\ldots\ldots; -239$ (3)

2.2 2; m; 12; p;....... are the first four terms of a quadratic sequence.

If the second difference is 6, calculate the values of m and p. (5)

2.3 Consider the geometric series: $2(3m-1) + 2(3m-1)^2 + 2(3m-1)^3 + \ldots\ldots\ldots$

2.3.1 For which values of m will the series converge? (3)

2.3.2 Calculate the sum of infinite of the series if $m = \frac{1}{2}$ (4)

[15]

Question 3

Given that: $\sum_{n=1}^{\infty} 63m^{n-1} = \frac{189}{2}$

3.1 Solve for m. (4)

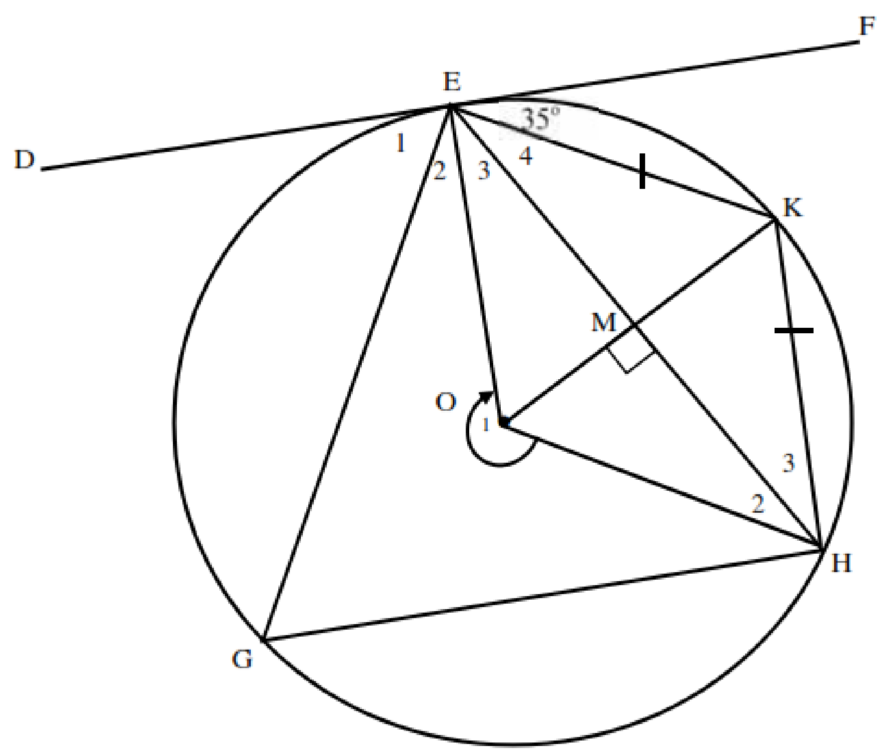
3.2 If it is further given that $m = \frac{1}{3}$, determine the smallest value of n such that

$T_n < \frac{1}{6561}$ (5)

[9]

Question 4

DF is a tangent to the circle at E. EKHG is a cyclic quadrilateral. $K\hat{E}F = 35^\circ$. O is the Centre of the circle. OK $\perp$ EH and EK=HK



4.1 Determine, with reasons the size of each of the following:

4.1.1 $\hat{E}_4$ (3)

4.1.2 $E\hat{K}H$ (2)

4.1.3 $\hat{G}$ (2)

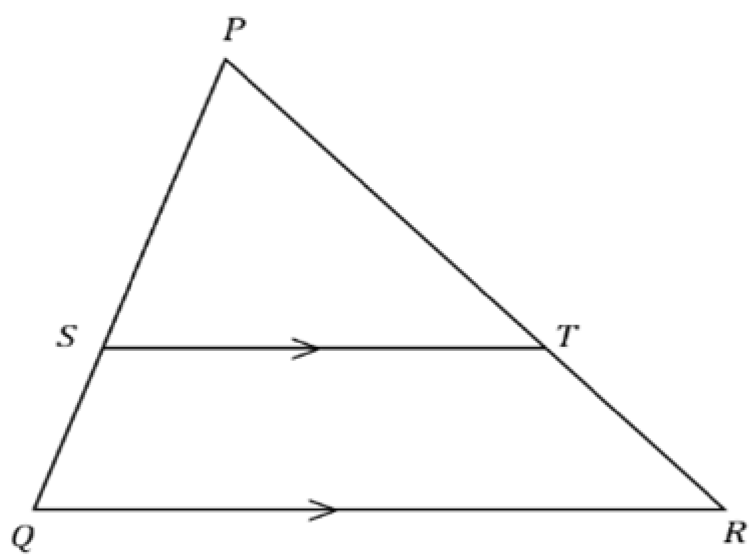
4.1.4 $\hat{O}_1$ (2)

4.2 It is further given that EH=24 units.KM=4 units and the radius of the circle EKHG is x . Determine the value of x . (4)

[14]

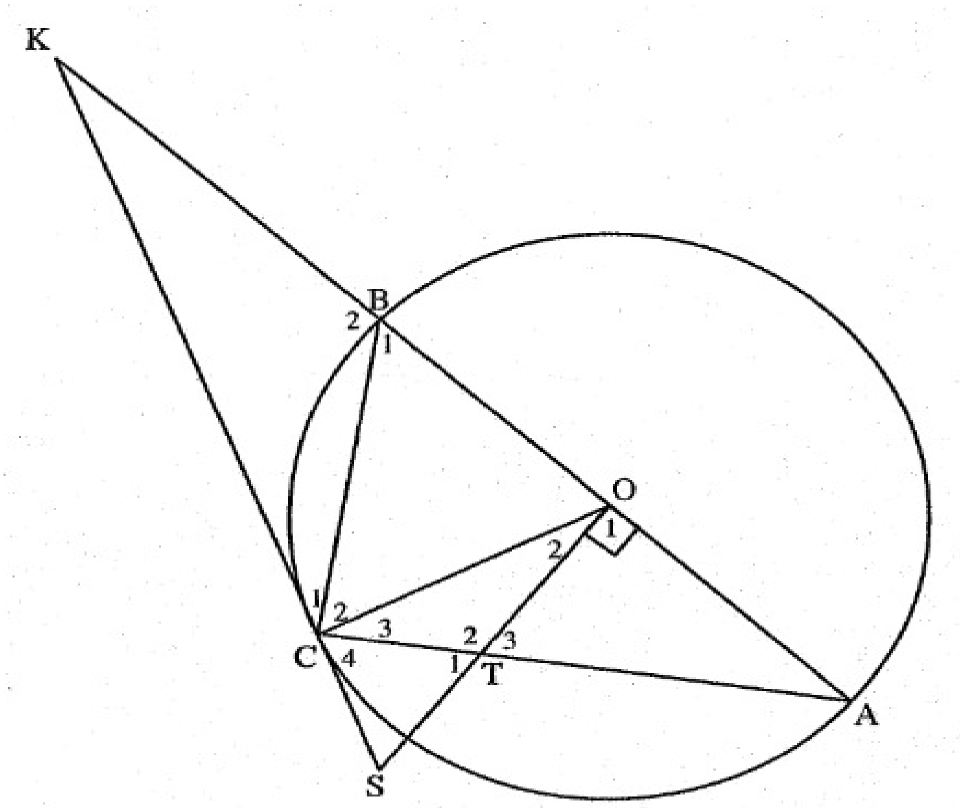
Question 5

5.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that ST //QR.



Prove the theorem that state that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

5.2 In the diagram below, AB is the diameter of the circle with Centre O. AB is produced to K such that SK is a tangent to the circle at C. SO $\perp AB$. CA and SO intersect at T.



Prove that:

5.2.1 $\Delta CKB \,/\!/\!/\, \Delta AKC$ (3)

5.2.2 $K\hat{C}T = \hat{T}_2$ (4)

5.2.3 $\Delta COT \,/\!/\!/\, \Delta AKC$ (3)

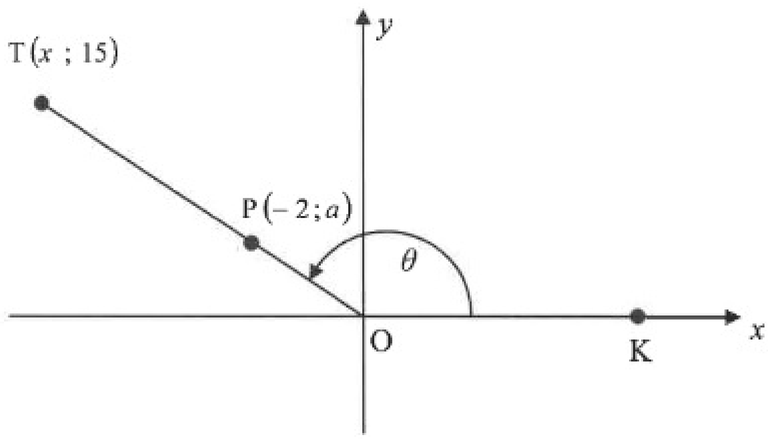
5.2.4 $BK.AK = \dfrac{OT^2.CA^2}{CT^2}$ (4)

[20]

Question 6

6.1 In the diagram below, T(x; 15) is a point in the Cartesian plane such that OT = 17 units.

P(-2;a) lies on OT. K is a point on the positive x-axis and $T\hat{O}K = \theta$.



Determine, with the aid of the diagram, the following:

6.1.1 The value of x (2)

6.1.2 $\tan\theta$ (1)

6.1.3 $\cos(180-\theta)$ (2)

6.1.4 $\sin^2\theta$ (2)

6.1.5 The value of a (3)

6.2 Simplify WITHOUT using a calculator.

$$\frac{\sin 120^\circ . \cos 210^\circ . \tan 315^\circ . \cos 27^\circ}{\sin 63^\circ . \cos 540^\circ}$$ (6)

6.3 Prove the identity.

$$\frac{1}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta} = \tan\theta$$ (4)

[20]

TOTAL=100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$