basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

TECHNICAL MATHEMATICS P1

EXEMPLAR 2018

MARKS: 150

TIME: 3 hours

**This question paper consists of 8 pages, 1 answer sheet and
an information sheet consisting of 2 pages**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 9 questions.

2.   Answer ALL the questions.

3.   Answer QUESTION 4.2.4 and QUESTION 7.4 on the ANSWER SHEET provided.

4.   Number the answers correctly according to the numbering system used in this question paper.

5.   Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

6.   Answers only will not necessarily be awarded full marks.

7.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

9.   Diagrams are NOT necessarily drawn to scale.

10.   Write neatly and legibly.

QUESTION 1

1.1 Given: $f(x) = x(x+2)$

Solve for x if:

1.1.1 $f(x) = 0$ (2)

1.1.2 $f(x) \geq 0$ and then represent the solution on a number line. (4)

1.2 Solve for x if $5x^2 = 2 + x$ (rounded off to TWO decimal places) (4)

1.3 Solve algebraically for m and t simultaneously if:
$m - t - 1 = 0$ and $m^2 + t^2 = 5$ (6)

1.4 The two diagrams below represent a metal rod with an initial length L_1 which is then stretched (elongated) to a length L_2.

$\vdash\!\!-\!\!L_1\!\!-\!\!\dashv$ $\vdash\!\!-\!\!-\!\!L_2\!\!-\!\!-\!\!\dashv$

The strain measure (ε) is defined as the ratio of elongation with respect to the original length and is given by the formula:

$$\varepsilon = \frac{L_2 - L_1}{L_1}$$

1.4.1 Express L_1 as the subject of the formula. (3)

1.4.2 Hence, or otherwise, calculate the value of L_1 if $\varepsilon = 0,8$ and $L_2 = 18$ cm. (2)

1.4.3 Convert the value obtained in QUESTION 1.4.2 to a binary number. (2)

1.5 Write the simplified value of $12 \times 0,00361$ in scientific notation without any rounding. (2)

[25]

QUESTION 2

2.1   Given: $A = \dfrac{\sqrt{9-3p}}{p+1}$

Determine the value(s) of p for which A will be:

2.1.1   Undefined   (1)

2.1.2   Non-real   (2)

2.1.3   Rational (give only ONE integer)   (1)

2.2   Determine the value of k for which the equation $x^2 - 4x + (k-1) = 0$ will have equal roots.   (4)

[8]

QUESTION 3

3.1   Simplify the following without the use of a calculator (show ALL steps):

3.1.1 $\dfrac{5 \times 2^{n-1} - 2^n}{2^n}$ (3)

3.1.2 $\sqrt{64+16} - \sqrt{20}$ (4)

3.1.3 $\log_6 216 \times \log 0{,}001$ (4)

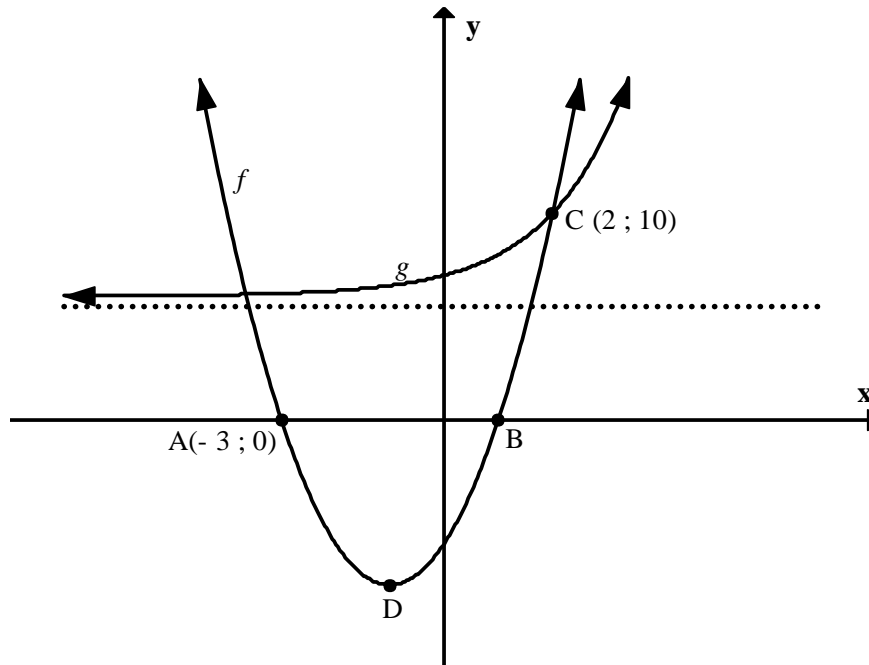3.2   Solve for x: $\log(x-18) - \log x = 1$ (4)

3.3   Express the complex number $z = 3 + \sqrt{3}\,i$ in trigonometric (polar) form.   (5)

3.4   Solve for x and y if $x + yi = (3+5i)(2-7i)$ (5)

[24]

QUESTION 4

4.1   The graphs of the functions defined by $f(x) = 2x^2 + 4x - 6$ and $g(x) = k^x + 6$
        are shown in the figure below. $C(2;10)$ is a point of intersection of f and g.
        Points $A(-3;0)$ and B are the x-intercepts and D is the turning point of f.

        Determine:

        4.1.1   The coordinates of B   (2)

        4.1.2   The coordinates of turning point D   (3)

        4.1.3   The numerical value of k (3)

        4.1.4   The equation of the asymptote of g (1)

        4.1.5   The values of x for which $f(x) \times g(x) < 0$ (2)

4.2   Given: $g(x) = \sqrt{4 - x^2}$ and $h(x) = \dfrac{3}{x} + 1$

        4.2.1   Write down the equations of the asymptotes of h.   (2)

        4.2.2   Determine the x-intercept of h.   (2)

        4.2.3   Write down the length of the radius of g.   (1)

        4.2.4   On the ANSWER SHEET provided, draw neat sketch graphs of g and
                    h on the same set of axes. Clearly show ALL the asymptotes and the
                    intercepts with the axes.   (7)

        4.2.5   Determine the range of g.   (2)
                                                                                                  [25]

QUESTION 5

5.1   The nominal interest rate charged on an investment is 7,2 % compounded half yearly.
        Calculate the annual effective interest rate for the investment.   (3)

5.2   The air pressure of a punctured tyre deflated from 220 kPa to 70 kPa at a decreasing
        rate of 8% per minute. Determine (to the nearest minute) how long it took the tyre
        to deflate from 220 kPa to 70 kPa.   (5)

5.3   Mrs Rethabile invested an amount of R150 000 to buy a drilling machine for her
        engineering company. Interest, compounded quarterly, is calculated at a rate of
        10,5% p.a. for 5 years. At the end of the third year, Mrs Rethabile withdrew an
        amount of R30 000 from the investment account and then continued investing the
        balance for the remaining period.

        Determine the value of the investment at the end of the investment period.   (6)
                                                                                                           [14]

QUESTION 6

6.1   Determine the average gradient of $f(x) = 2x^2 - 3$ between the points where $x = -2$
        and $x = 1$.   (4)

6.2   Determine $f^{/}(x)$ from FIRST PRINCIPLES if $f(x) = 4 - 3x$.   (5)

6.3   Determine $\dfrac{dy}{dx}$ if $y = \dfrac{2}{x^3} + \sqrt{x}$ (4)

6.4   Determine the equation of the tangent to the curve defined by $g(x) = -x^2 - x$ at
        the point where $x = 2$.   (5)
                                                                                                           [18]

QUESTION 7

Given: $f(x) = x^3 + 2x^2 - 7x + 4$

7.1   Show that $(x - 1)$ is a factor of $f(x)$.   (2)

7.2   Hence, or otherwise, find the x-intercepts of f.   (3)

7.3   Determine the coordinates of the turning points of f.   (5)

7.4   Sketch the graph of f on the ANSWER SHEET provided. Clearly show ALL
        the intercepts with the axes and the turning points.   (4)
                                                                                                           [14]

QUESTION 8

8.1 An industrial open water tank, as shown in the picture below, has an inlet pipe and an outlet pipe. The depth of the water in the tank changes continually.

The equation $D(t) = 4 + 0,5t^2 - 0,25t^3$ gives the depth (in metres) of the water, where t represents the time (in hours) that has lapsed since the depth reading was taken at 09:00.
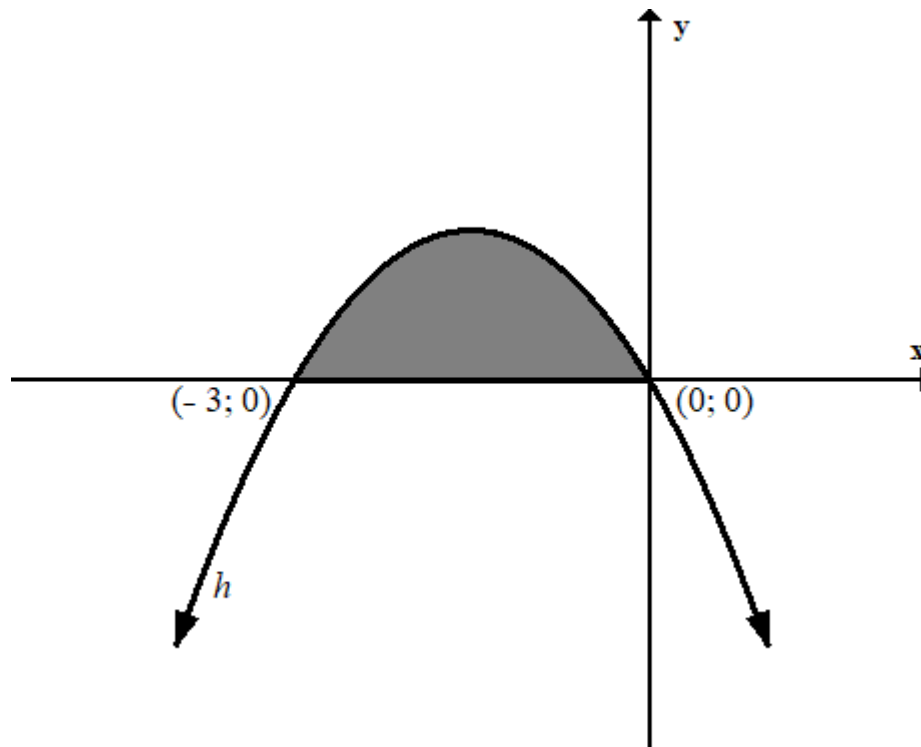
Determine:

8.1.1 The depth of the water in the tank at 11:00 (2)

8.1.2 The rate of change of the depth of the water in the tank at 12:00 (3)

8.2 The profit (in R1000s) yielded by a company, using a machine that produces bottle caps, is dependent on the average speed at which the machine runs.

The profit (P) is calculated using the formula:

$P = -3v^2 + 30v$,

where v is the average speed (in kilometres per hour) and $v > 0$.

8.2.1 Calculate the average speed at which neither a profit, nor a loss is yielded. (3)

8.2.2 Determine at what average speed the machine should run so that the maximum profit is obtained. (3)

8.2.3 Hence, or otherwise, calculate the resulting maximum profit. (2)

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QUESTION 9

9.1   Determine the following integral: $\int \left(x^{-4} + \dfrac{7}{x} -1 \right) dx$ (4)

9.2   The sketch below represents the graph of the function defined by $h(x) = -2x^2 - 6x$.

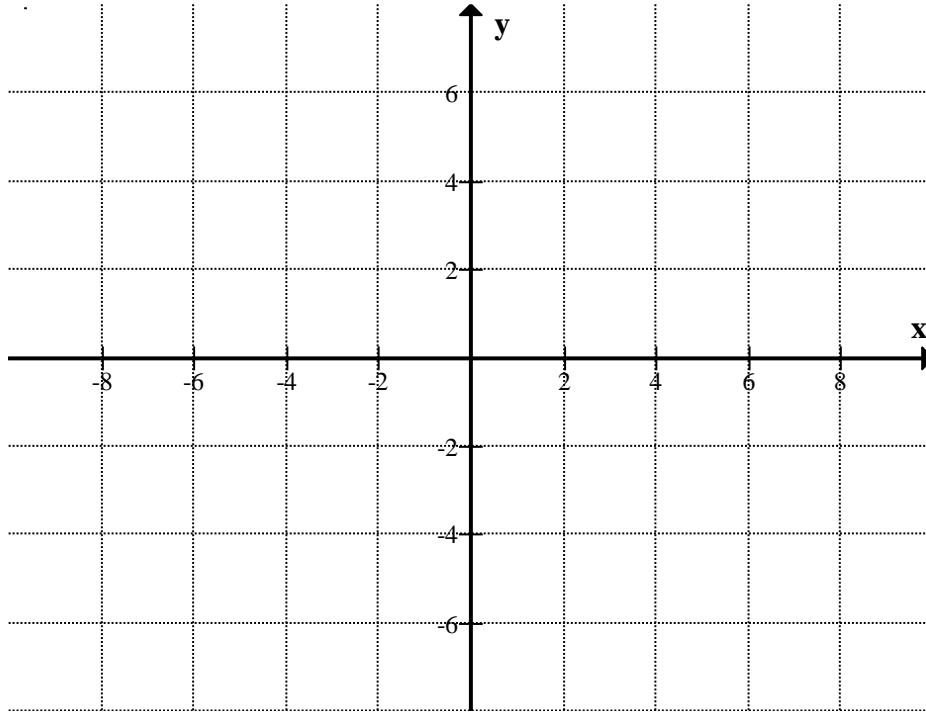Determine the shaded area bounded by a curve defined by $h(x) = -2x^2 - 6x$ and the x-axis.   (5)
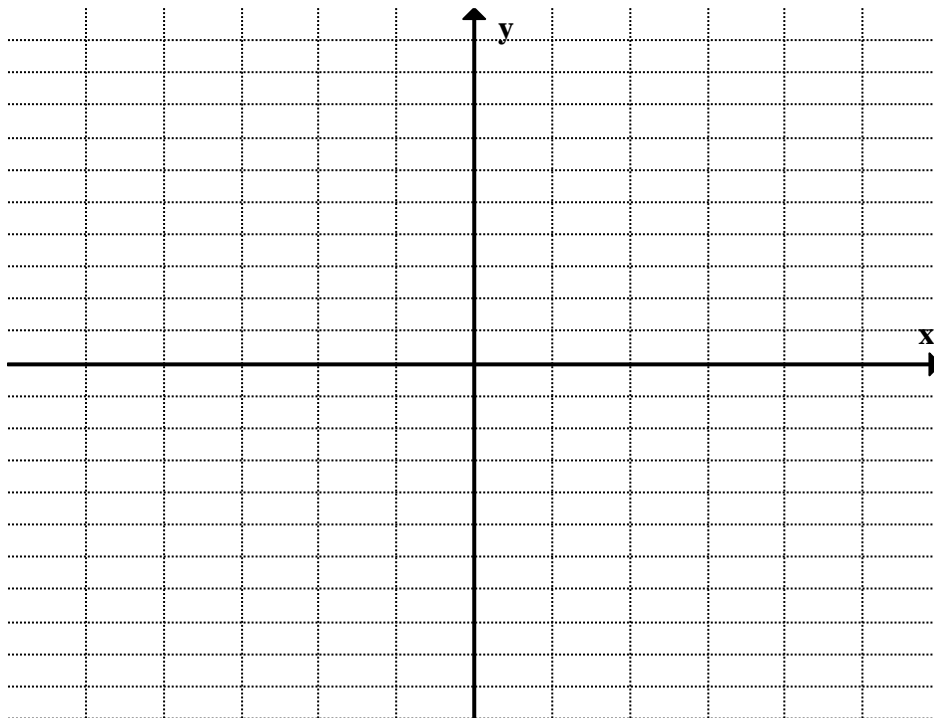
[9]

TOTAL: 150

ANSWER SHEET

NAME:……………………………………SCHOOL:…………………………………

QUESTION 4.2.4

QUESTION 7.4

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad\qquad x = -\frac{b}{2a} \qquad\qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b \ , \qquad a > 0 \ , \ a \neq 1 \ \text{and} \ b > 0$$

$$A = P(1 + ni) \qquad\qquad A = P(1 - ni) \qquad\qquad A = P(1 - i)^n \qquad\qquad A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \qquad x > 0 \qquad\qquad\qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In $\triangle ABC$: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,of\ \triangle ABC = \dfrac{1}{2}\,ab\cdot\sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad\qquad \cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ $\qquad$ where n = rotation frequency

Circumferencial velocity $= v = \pi Dn$ $\qquad$ where D = diameter and n = rotation frequency

$s = r\theta$ $\qquad$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2} = \dfrac{r^2\theta}{2}$ $\qquad$ where r = radius, s = arc length and

θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ $\qquad$ where h = height of segment, d = diameter of circle and

x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ $\qquad$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ $\qquad$ where a = equal parts, $o_i = i^{th}$ ordinate and

n = number of ordinates