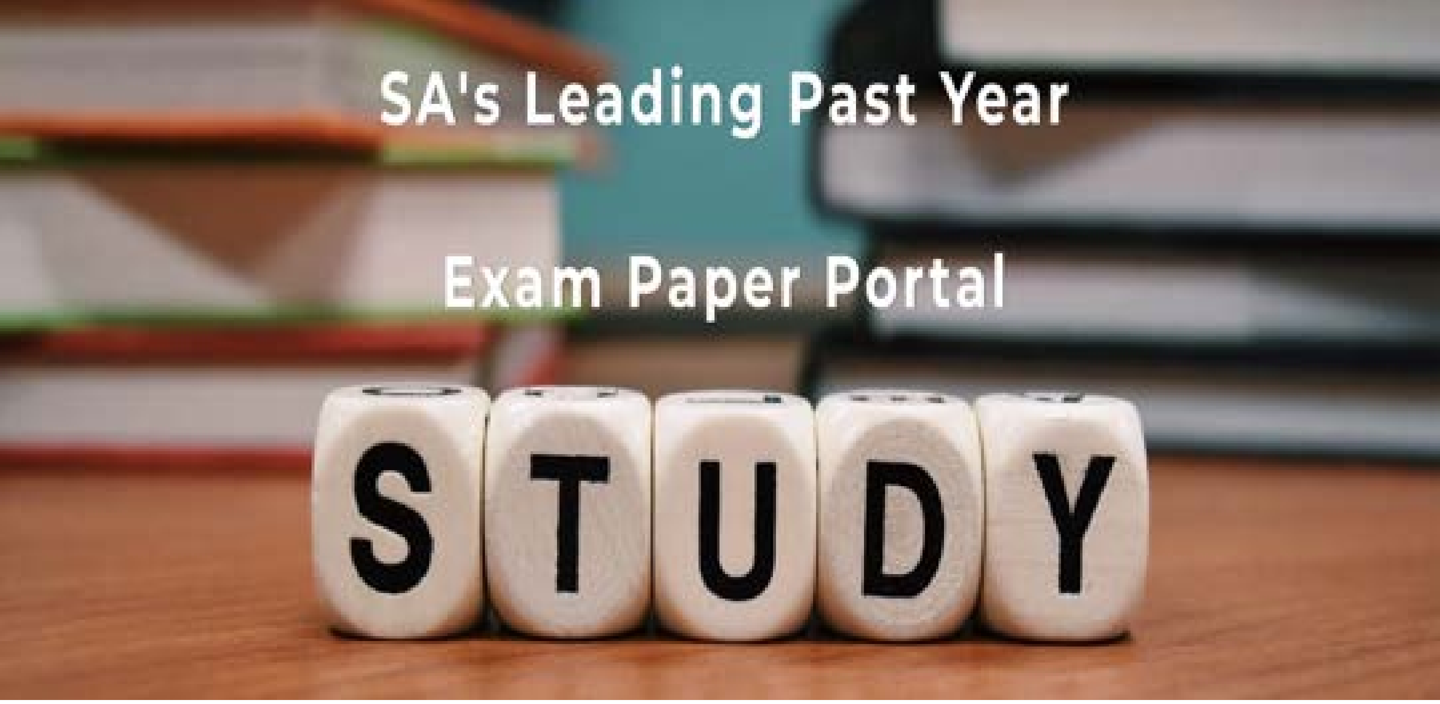
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STUDY

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2016

10612

MATHEMATICS

SECOND PAPER

TIME: **3 hours**

MARKS: **150**

13 pages + 1 information sheet

MATHEMATICS: Paper 2
1061E

10612E

X10

3

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION – 2016

MATHEMATICS
(Second Paper)

TIME: 3 hours

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. A INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 The table below shows the recorded monthly rainfall (in mm) measured at Silver Lakes, a residential area in Pretoria, for the months of December. The statistics was taken for the period 2004 to 2015.

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Rainfall for December (in mm)	286	68	150	147	176	132	255	174	172	197	172	39

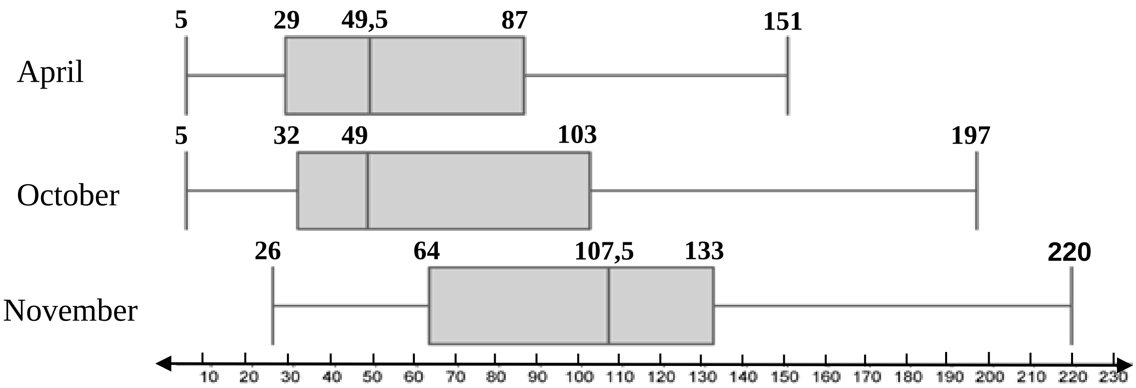
Source: Prof. Francois Swanepoel from Silver Lakes

1.1.1 Which year's rainfall will decrease the average December rainfall the most? (1)

1.1.2 Calculate the standard deviation for the rainfall in December. (2)

1.1.3 Determine the percentage of the data points for the rainfall in December, which lie within one standard deviation of the mean. (3)

1.2 The diagram below shows a comparison of the recorded monthly rainfall statistics for the months of April, October and November, for the years 2004 to 2015.



1.2.1 Determine the range of the rainfall for the November months. (1)

1.2.2 Comment on the skewness of the data for the October months. (1)

1.2.3 If the maximum rainfall for October was incorrectly recorded and is now increased, what effect will this have on

(a) the mean and

(b) the median. (2)

1.2.4 For how many years was November's rainfall less than 64 mm? (2)

[12]

QUESTION 2

A company called Mathπrates sent out a number of advertising pamphlets. The table below shows the number of pamphlets sent out to a certain place. The table also shows the orders that were received after the distribution of the pamphlets.

Number of Pamphlets	Number of Orders
600	350
1000	550
500	300
700	300
600	330
100	200
500	350
800	450
300	250
900	500

2.1 Represent the above information in a scatter plot on the grid provided in the ANSWER BOOK. (2)

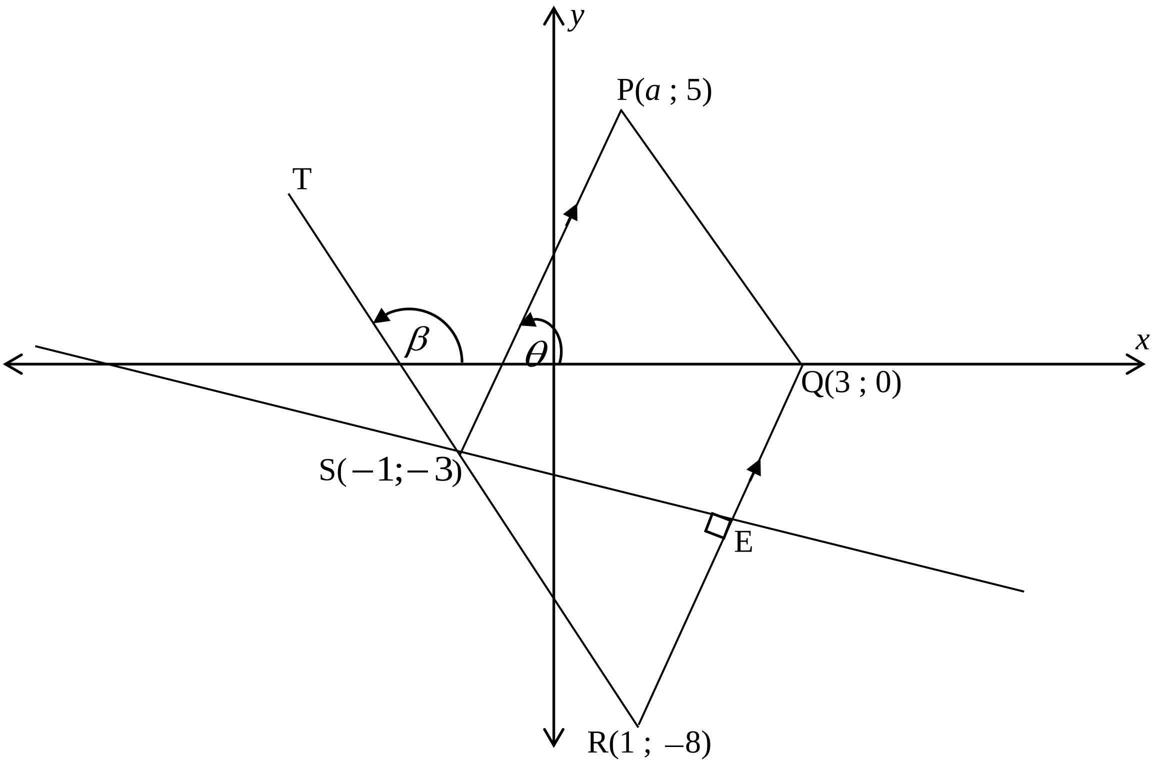
2.2 Determine, using your calculator, the equation of the least squares regression line. (2)

2.3 Draw the least squares regression line on the scatter plot drawn for QUESTION 2.1. (2)

2.4 Calculate the possible number of orders if 200 pamphlets were sent out. (2)
[8]

QUESTION 3

In the diagram below, PQRS is a trapezium with SP || QR and vertices P(a ; 5), Q(3 ; 0), R(1 ; −8) and S(−1;−3). SE ⊥ QR.



3.1 Determine the length of QR. (2)

3.2 Determine the gradient of QR. (2)

3.3 Calculate the value of y if $\text{E}\left(\frac{35}{17}\,;y\right)$ lies on the line QR. (2)

3.4 Determine the equation of SE. (3)

3.5 Calculate the value of a. (3)

3.6 Calculate the value of:

3.6.1 θ (2)

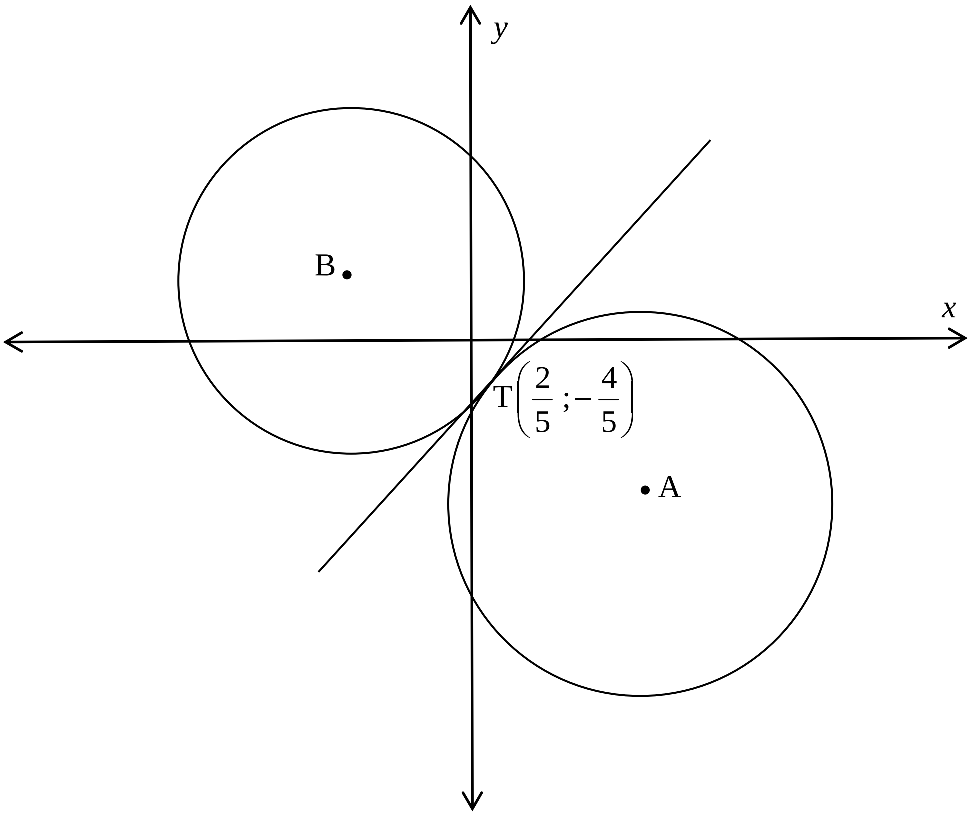
3.6.2 $\text{S}\hat{\text{R}}\text{Q}$ (5)

3.7 Determine the length of the radius of circle ESR. (3)

[22]

QUESTION 4

The diagram below shows circle centre A with equation $(x-2)^2+(y+2)^2=4$ and a circle centre B with equation, $x^2+y^2+4x-2y+p=0.$ The two circles touch externally at $\mathrm{T}\left(\frac{2}{5};-\frac{4}{5}\right)$.



4.1 Write down the coordinates of the centre of circle A. (1)

4.2 Calculate:

4.2.1 The coordinates of the centre of circle B (2)

4.2.2 The radius of circle B (in terms of *p*) (2)

4.2.3 The length of AB (2)

4.2.4 The value of *p* (3)

4.3 If the two circles touch at the point $\mathrm{T}\left(\frac{2}{5};-\frac{4}{5}\right)$, determine the equation of the tangent in the form $y = mx + c$. (5)

[15]

QUESTION 5

5.1 If $5\tan\theta + 2\sqrt{6} = 0$ and $0^\circ < \theta < 270^\circ$, determine with the aid of a sketch and without the use of a calculator, the value of:

5.1.1 $\sin\theta$ (2)

5.1.2 $\cos\theta$ (1)

5.1.3 $\dfrac{14\cos\theta + 7\sqrt{6}\sin\theta}{\cos(-240^\circ).\tan 225^\circ}$ (4)

5.2 Prove the identity:

$$\frac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta} = \frac{1+\cos\theta}{\sin\theta}$$ (5)

5.3 Determine the general solution of $\sin\theta\sin\dfrac{3\theta}{2} + \cos\dfrac{3\theta}{2}\cos\theta = -\dfrac{\sqrt{3}}{2}$. (4)

5.4 Given: $\sin\theta \cdot \cos\beta = -1$

5.4.1 Write down the maximum and minimum value of $\cos\beta$. (1)

5.4.2 Solve for $\theta \in [0^\circ; 270^\circ]$ and $\beta \in [-180^\circ; 90^\circ]$. (4)

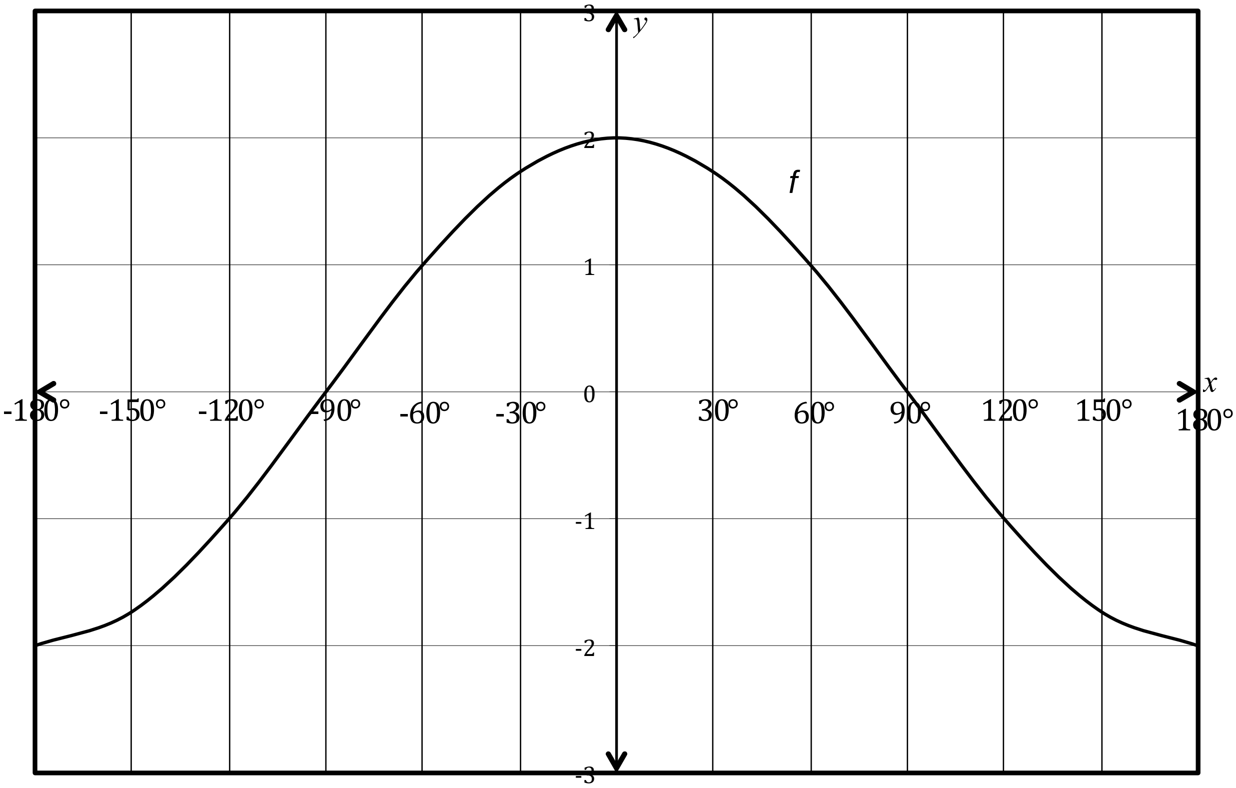
[21]

QUESTION 6

6.1 Show that the equation $2\cos x = \sin(x+30^\circ)$ is equivalent to $\sqrt{3}\sin x = 3\cos x$. (3)

6.2 Hence or otherwise, calculate the value of x for $x \in [-180^\circ; 180^\circ]$ if $2\cos x = \sin(x+30^\circ)$. (4)

6.3 In the diagram below, the graph of $f(x) = 2\cos x$ is drawn for $x \in [-180^\circ; 180^\circ]$



6.3.1 Write down the range of f. (1)

6.3.2 Draw the graph of g, where $g(x) = \sin(x+30^\circ)$ for $x \in [-180^\circ; 180^\circ]$. Use the grid provided in your ANSWER BOOK. (3)

6.3.3 Write down the period of g. (1)

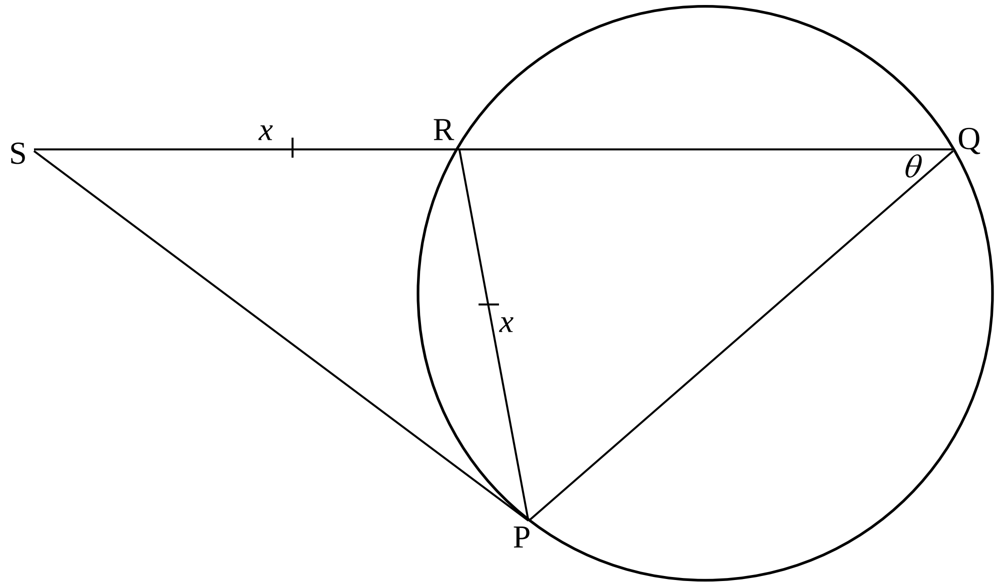
6.3.4 For which values of x is g strictly increasing as x increases, for $x \in [-180^\circ; 180^\circ]$? (2)

6.3.5 For which values of x will $f(x).g(x) > 0$ for $x \in [-90^\circ; 90^\circ]$? (2)

[16]

QUESTION 7

In the diagram below, PS is a tangent to the circle through P, Q and R. QRS is a straight line.
PR = RS = x and $P\hat{Q}R = \theta$.



Prove that $PS = 2x\cos\theta$. (6)

[6]

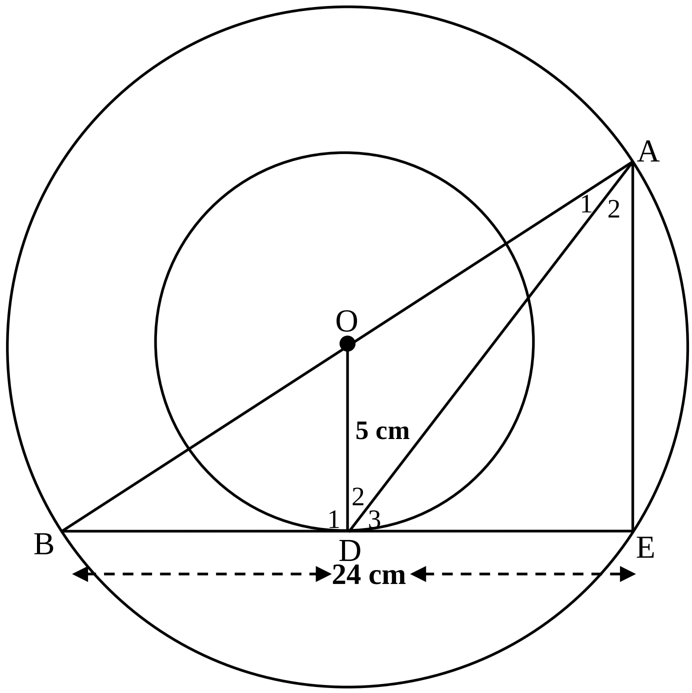
Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 Complete the following statement.

The line drawn from the centre of a circle, perpendicular to a chord, ... the chord. (1)

8.2 In the diagram below, two concentric circles with centre O are drawn. Chord BDE of the larger circle has a length of 24 cm and is a tangent to the smaller circle at D. The radius of the smaller circle is 5 cm. AB is the diameter of the larger circle.



8.2.1 Calculate the length of OB. (4)

8.2.2 Write down the reason why $\hat{E} = 90°$. (1)

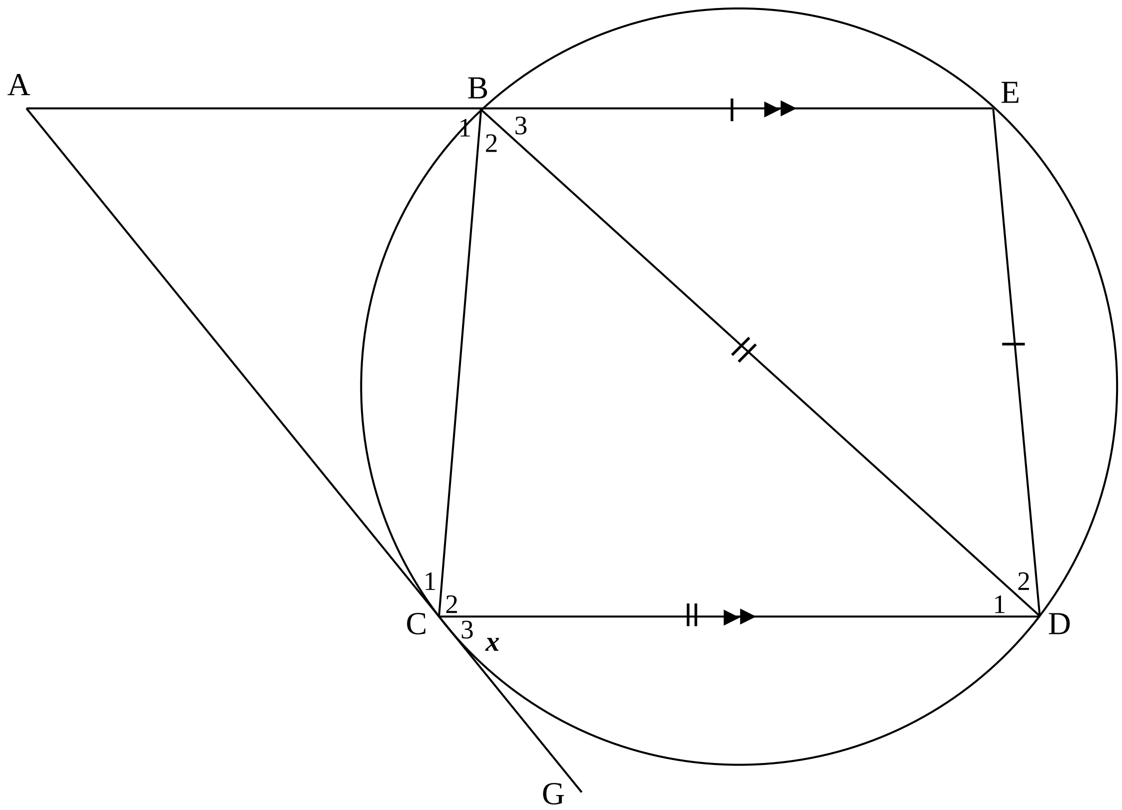
8.2.3 Calculate the length of AE. (3)

8.2.4 Calculate the length of AD. (2)

[11]

QUESTION 9

In the diagram below, BCDE is a cyclic quadrilateral with $BE = ED$, $BD = CD$ and $AE \parallel CD$. ACG is a tangent at C and meets EB produced at A.



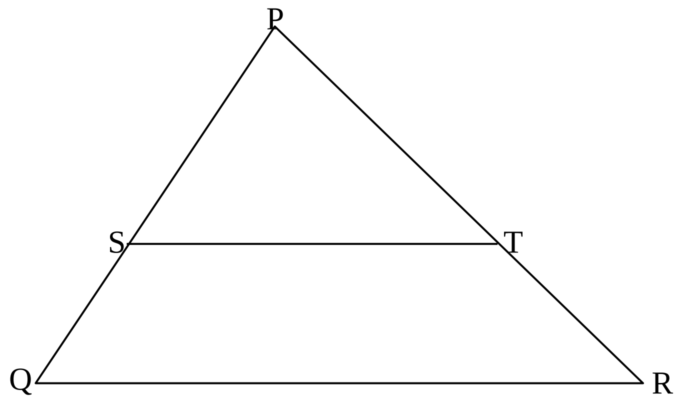
9.1 If $\hat{C}_3 = x$, determine, with reasons, FIVE other angles each equal to x. (7)

9.2 Prove that $BC = ED$. (2)

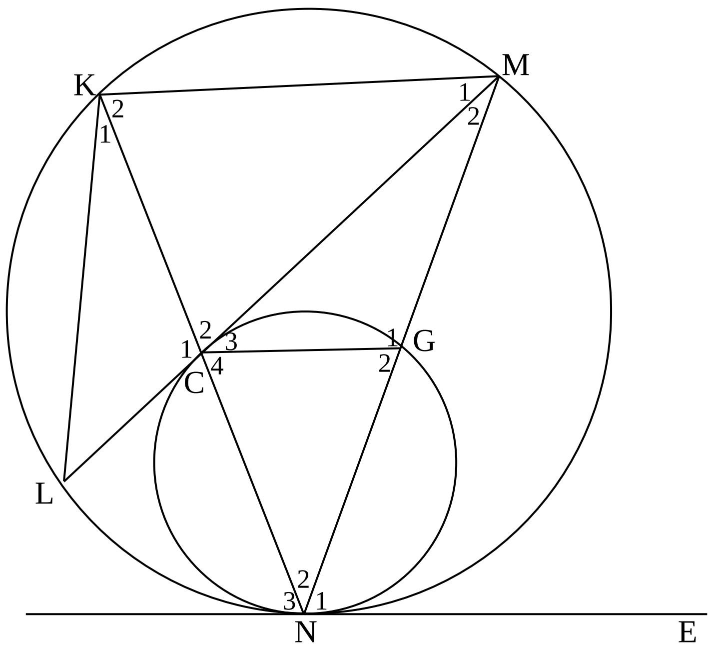
[9]

QUESTION 10

10.1 Use the diagram given in the ANSWER BOOK, to prove the theorem which states that if $ST \parallel QR$, then $\frac{PS}{SQ} = \frac{PT}{TR}$. Show all construction lines. (5)



10.2 In the diagram below NE is a common tangent to the two circles. NCK and NGM are double chords. Chord LM of the larger circle is a tangent to the smaller circle at point C. KL, KM and CG are drawn.



Prove that:

10.2.1 $\frac{KC}{KN} = \frac{MG}{MN}$ (4)

10.2.2 KMGC is a cyclic quadrilateral if CN = NG. (3)

10.2.3 $\Delta MCG \parallel\!\mid \Delta MNC$ (3)

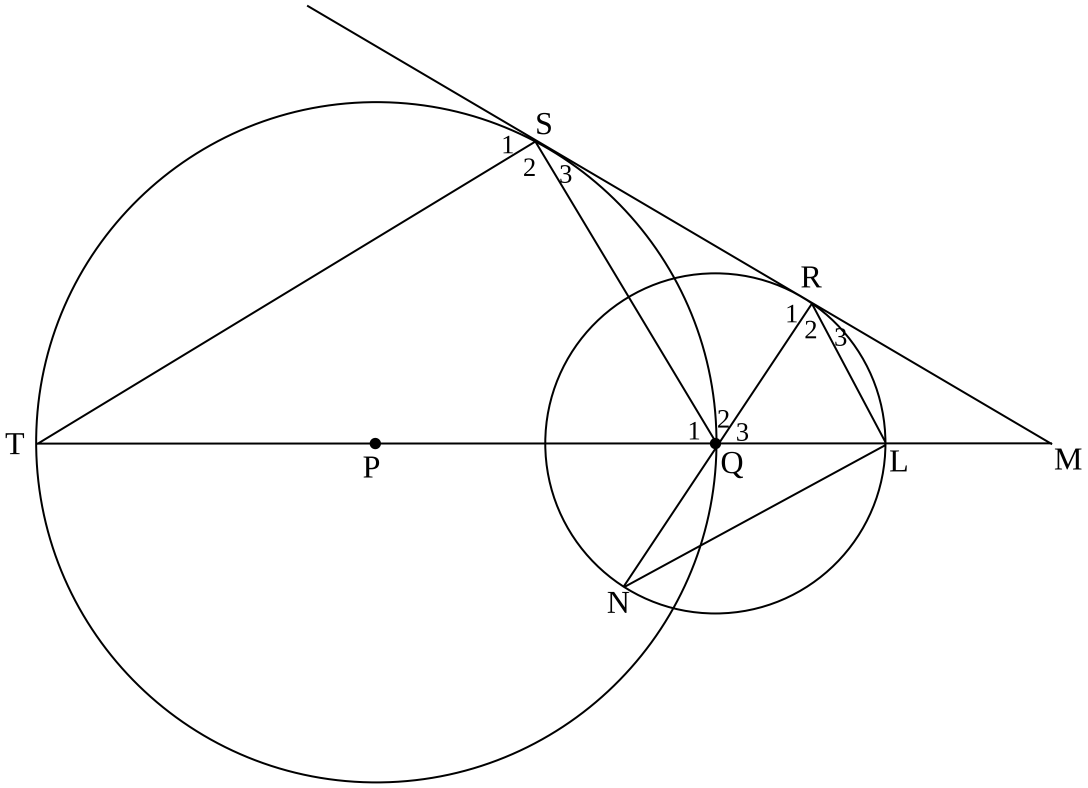
10.2.4 $\frac{MC^2}{MN^2} = \frac{KC}{KN}$ (4)

[19]

P.T.O.

QUESTION 11

In the diagram below, the length of the radius of the larger circle, with centre P, is twice the length of the radius of the smaller circle, with centre Q. SRM is a tangent to both circles.



11.1 Calculate the size of $\hat{R}_3$ if $\hat{Q}_3 = 60°$. (4)

11.2 If the length of the radius of the smaller circle is r and $\Delta TSQ \,|||\, \Delta SRQ$, show that the length of SQ is equal to $2r$. (3)

11.3 Calculate the size of $T\hat{S}R$. (4)

[11]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n - 1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1 + i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1 + i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$