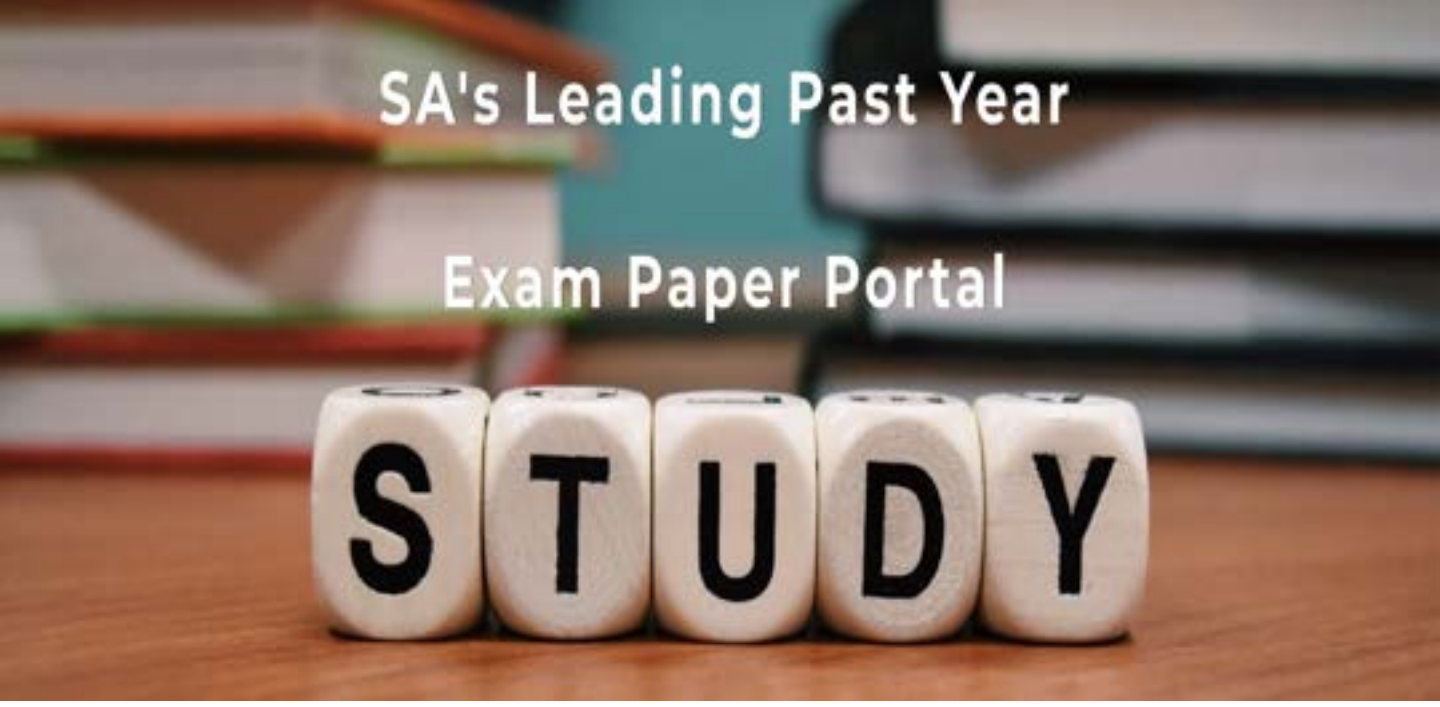

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GRADE 12

MATHEMATICS

PAPER 2

SEPTEMBER 2015

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an ANSWER BOOK

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Write neatly and legibly.

QUESTION 1

The data in the table below represents the score in percentage of 12 Mathematics learners in their Grade 12 trial examination and their corresponding final examination.

Trial Exam	76	64	90	68	70	79	52	64	61	71	84	70
Final Exam	82	69	94	75	80	88	56	81	76	78	90	76

1.1 Determine the equation of the least squares regression line for this set of data. (3)

1.2 Hence, predict the final percentage for a learner obtaining 73% in the trial examination. Give your answer to the nearest percentage. (2)

1.3 Calculate the correlation coefficient for the above data. (2)

1.4 Do you think that by using the least squares regression line one can accurately predict a learner's final percentage? Provide Mathematical justification for your answer. (2)
[9]

QUESTION 2

The time taken (to the nearest minute) for a certain task to be completed was recorded on 48 occasions and the following data was obtained:

Time (in minutes)	**Cumulative Frequency**
$11 \leq t < 15$	6
$15 \leq t < 19$	15
$19 \leq t < 23$	28
$23 \leq t < 27$	40
$27 \leq t < 31$	48

2.1 Write down the modal class. (1)

2.2 Draw an ogive (cumulative frequency curve) for the given data. (4)

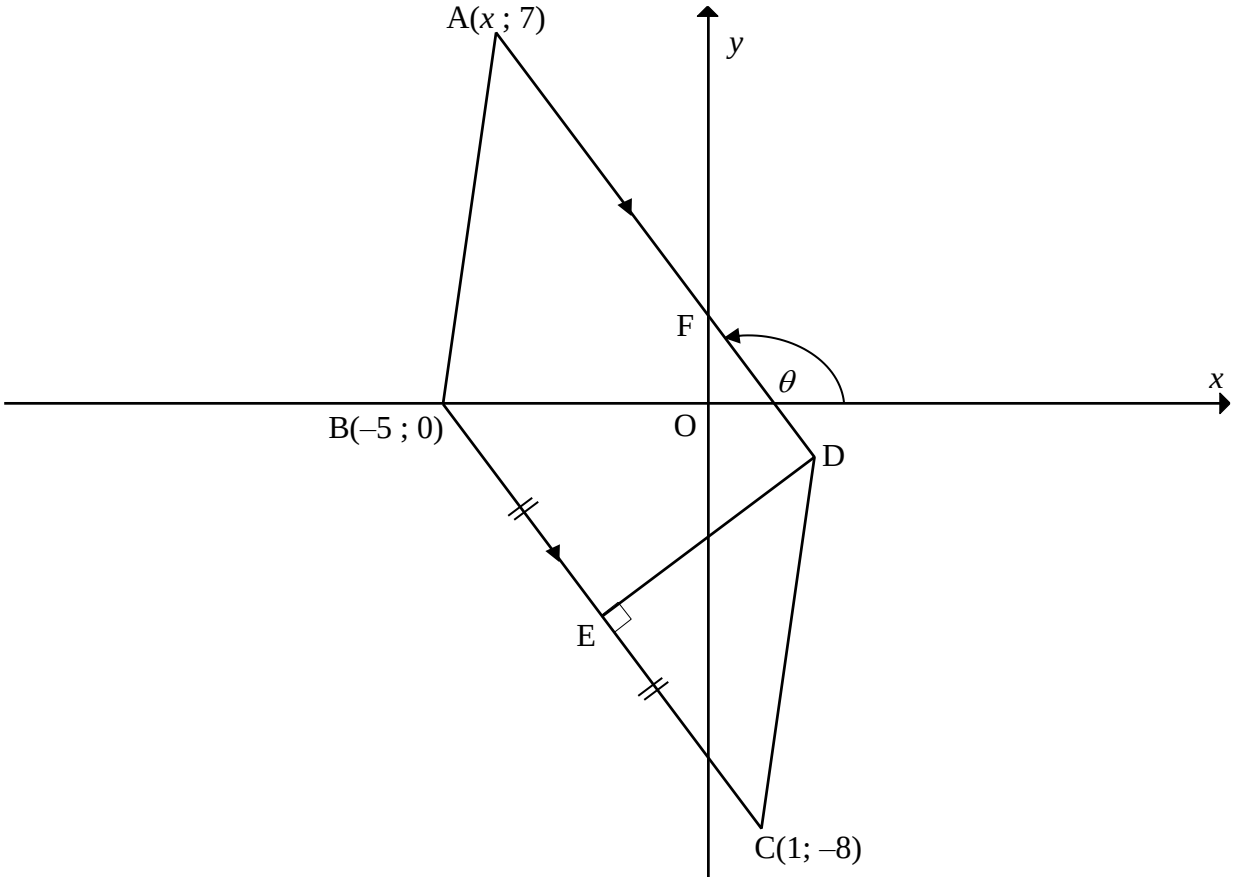
2.3 Determine, using the ogive, the interquartile range for the data. (3)

2.4 Use your graph to estimate in how many of the 48 occasions was this task completed in more than 20 minutes? (2)

[10]

QUESTION 3

In the diagram, ABCD is a trapezium with AD | | BC and vertices A(x ; 7), B(–5 ; 0), C(1 ; –8) and D. DE ⊥ BC with E on BC such that BE = EC. The inclination of AD with the positive x-axis is θ and AD cuts the y-axis in F.



3.1 Calculate the gradient of BC. (2)

3.2 Calculate the coordinates of E. (2)

3.3 Determine the equation of DE in the form $y = mx + c$. (3)

3.4 Calculate the size of θ. (3)

3.5 Calculate the size of $\hat{OFD}$. (2)

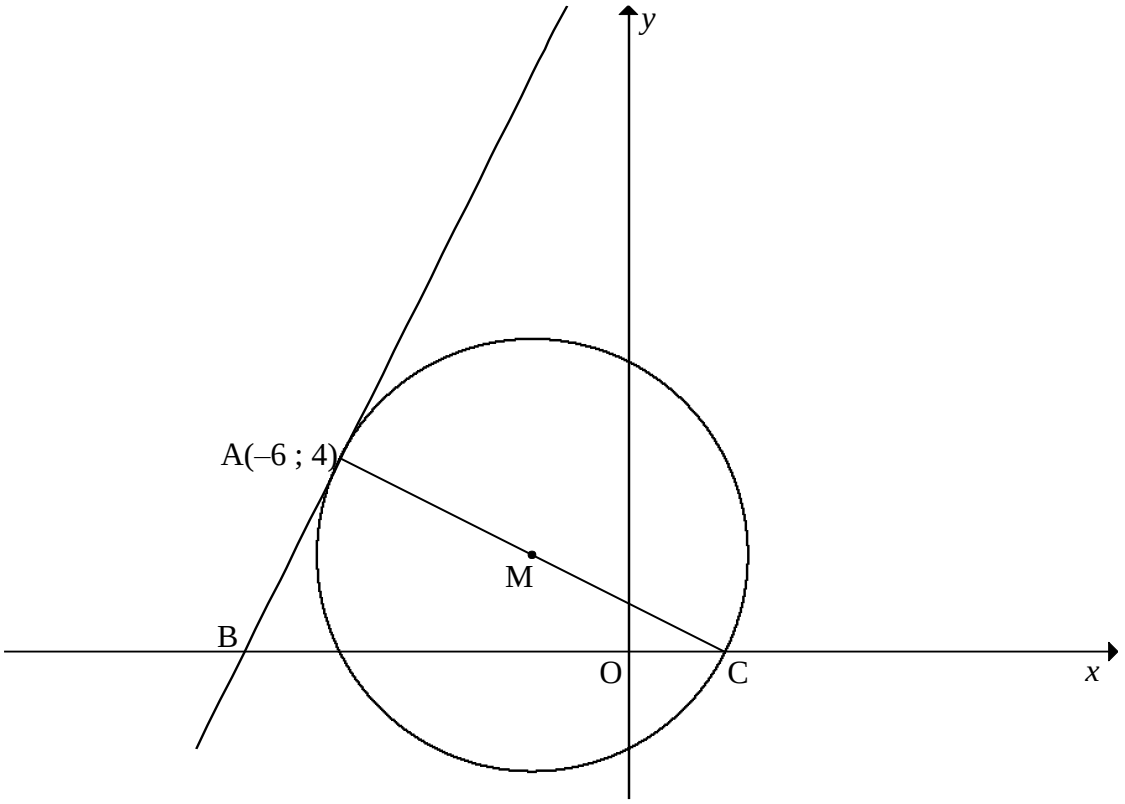
3.6 Calculate the value of x if the length of $AB = 5\sqrt{2}$. (5)

3.7 Determine the equation of the circle with diameter BC in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

QUESTION 4

In the diagram, the circle with centre M and equation $x^2 + y^2 + 4x - 4y - 12 = 0$ is drawn. C is the *x*-intercept of the circle. The tangent AB touches the circle at A(–6 ; 4) and cuts the *x*-axis at B.



4.1 Calculate the

4.1.1 coordinates of M. (2)

4.1.2 coordinates of C. (3)

4.2 Determine, giving reasons, the equation of the tangent AB in the form $y = mx + c$ if it is given that the gradient of MC is $-\frac{1}{2}$. (4)

4.3 Calculate the area of ΔABC. (5)

4.4 Determine for which values of *k* the line $y = 2x + k$ will intersect the circle at two points. (5)

[19]

QUESTION 5

5.1 If sin 31° = *p*, determine the following, without using a calculator, in terms of *p*:

5.1.1 sin 149° (2)

5.1.2 cos (–59°) (2)

5.1.3 cos 62° (2)

5.2 Simplify the following expression to a single trigonometric ratio:

$$\tan(180° - \theta).\sin^2(90° + \theta) + \cos(\theta - 180°).\sin\theta$$ (6)

5.3 Consider: $\dfrac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$

5.3.1 Prove the identity. (5)

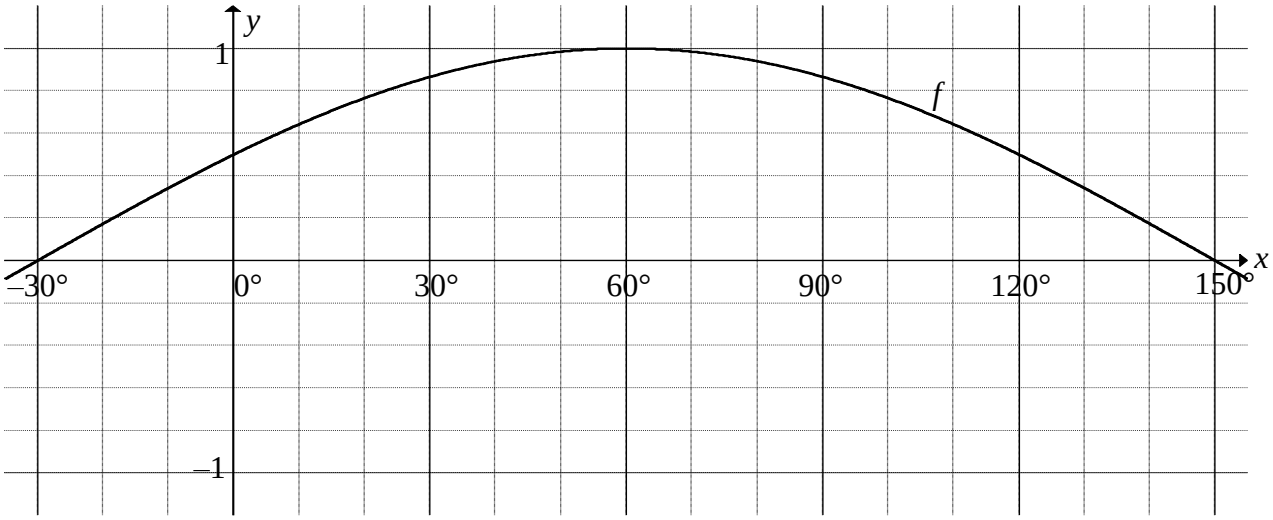
5.3.2 Determine the values of *x*, where $x \in [180° ; 360°]$, for which the above identity will be invalid/undefined. (2)

[19]

QUESTION 6

6.1 Determine the general solution of : $\sin(x + 30°) = \cos 3x$. (6)

6.2 In the diagram below, the graph of $f(x) = \sin(x + 30°)$ is drawn for the interval $x \in [-30° ; 150°]$.



6.2.1 On the same system of axes sketch the graph of g, where $g(x) = \cos 3x$, for the interval $x \in [-30° ; 150°]$. (3)

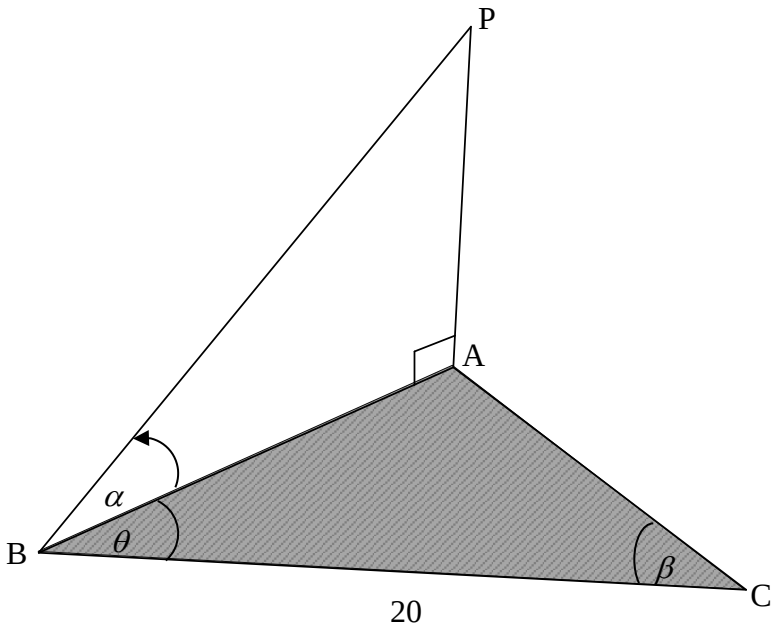
6.2.2 Write down the period of g. (1)

6.2.3 For which values of x will $f(x) \geq g(x)$ in the interval $x \in (-30° ; 150°)$? (3)

[13]

QUESTION 7

In the diagram below, A, B and C are in the same horizontal plane. P is a point vertically above A. The angle of elevation from B to P is α.
$A\hat{C}B = \beta$, $A\hat{B}C = \theta$ and BC = 20 units.



7.1 Write AP in terms of AB and α. (2)

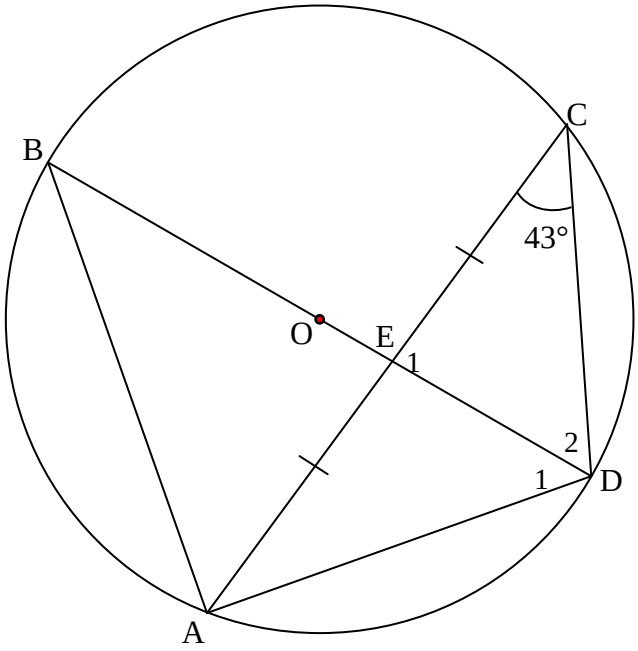
7.2 Prove that $AP = \dfrac{20\sin\beta\tan\alpha}{\sin(\theta+\beta)}$ (3)

7.3 Given that AB = AC, determine AP in terms of α and β in its simplest form. (3)
[8]

PROVIDE REASONS FOR ALL YOUR STATEMENTS AND CALCULATIONS IN QUESTION 8, 9 10 AND 11.

QUESTION 8

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Diameter BD bisects chord AC at E. Chords AB, CD and AD are drawn. $\hat{C} = 43°$.



8.1 Give a reason for $DE \perp AC$. (1)

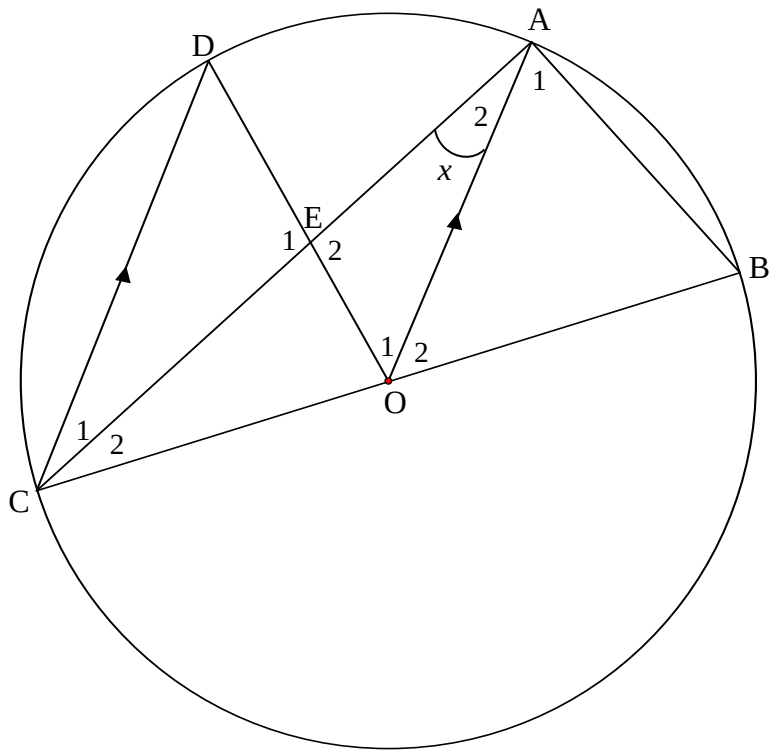
8.2 Calculate, giving reasons, the size of $\hat{B}$. (2)

8.3 Prove that $\hat{E}_1 = B\hat{A}D$. (2)

8.4 The length of the diameter of the circle is 28 units. Calculate the length of AB. (2)
[7]

QUESTION 9

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersect radius OD at E. AB is drawn. CD | | OA and $\hat{A}_2 = x$.



9.1 Give reasons why

9.1.1 $\hat{C}_1 = x$ (1)

9.1.2 $\hat{C}_2 = x$ (1)

9.2 Determine, giving reasons, the size of the following angles in terms of *x*.

9.2.1 $\hat{A}_1$ (3)

9.2.2 $\hat{O}_1$ (2)

9.2.3 $\hat{O}_2$ (2)

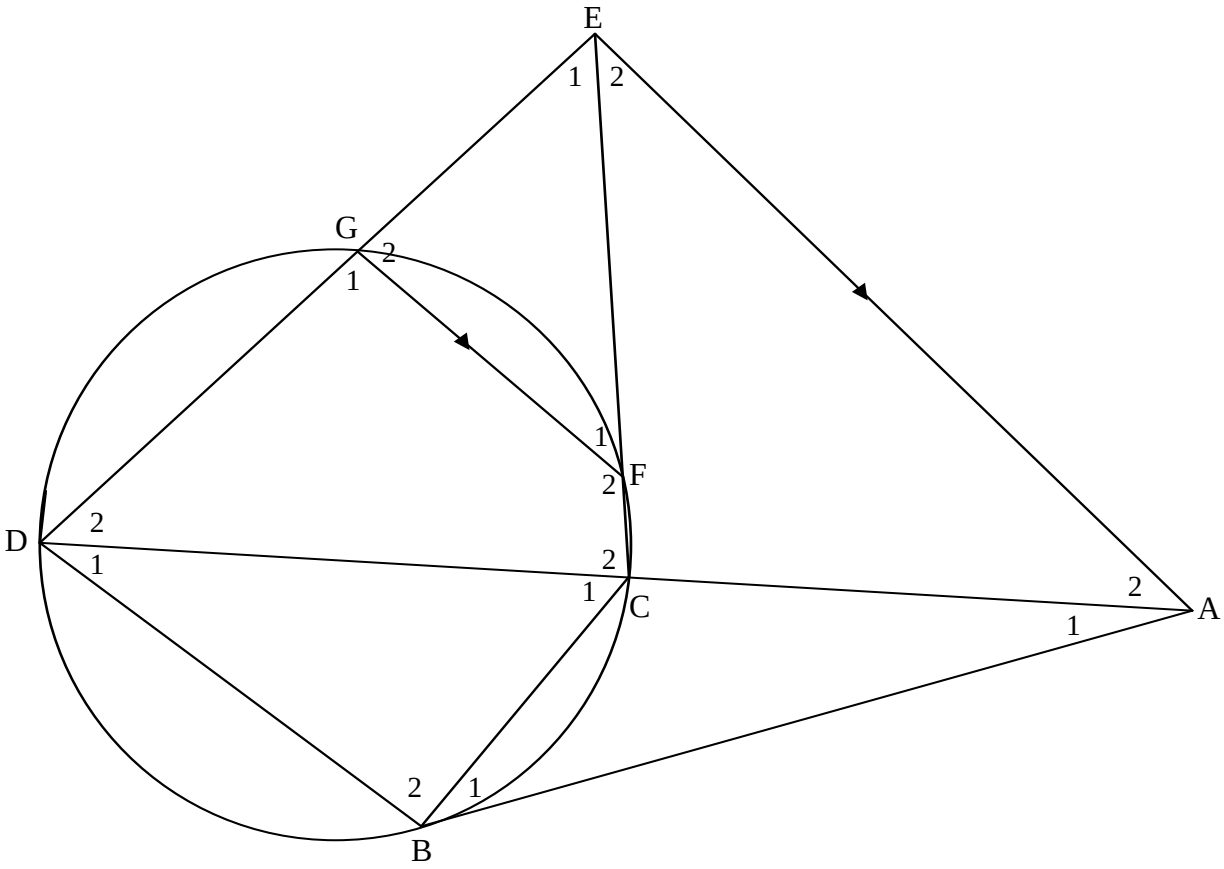
9.3 For which value of *x* will ABOE be a cyclic quadrilateral? (3)

[12]

QUESTION 10

10.1 Complete the following statement of the theorem in the ANSWER BOOK:
If two triangles are equiangular, then the corresponding sides are … (1)

10.2 In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG and CF produced meet at E and DC is produced to A. EA | | GF.



10.2.1 Give a reason why $\hat{B}_1 = \hat{D}_1$. (1)

10.2.2 Prove $\Delta ABC \,|||\, \Delta ADB$. (3)

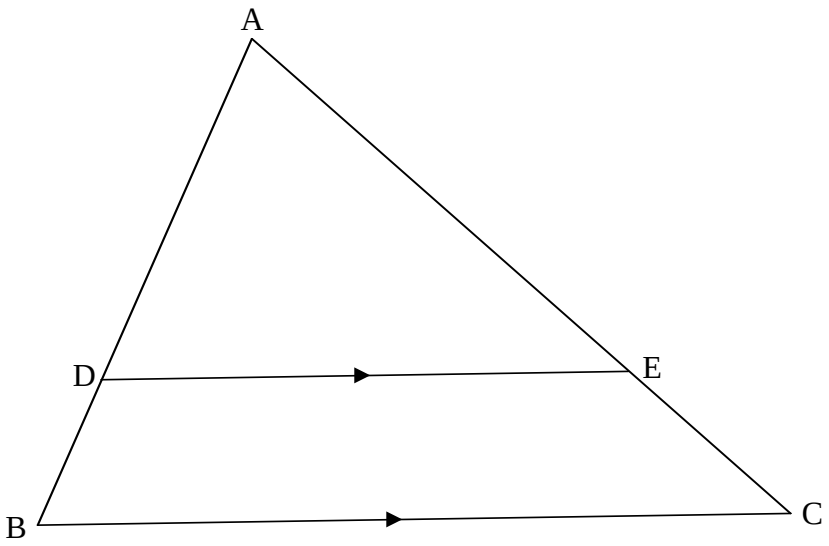
10.2.3 Prove $\hat{E}_2 = \hat{D}_2$. (4)

10.2.4 Prove $AE^2 = AD \times AC$. (4)

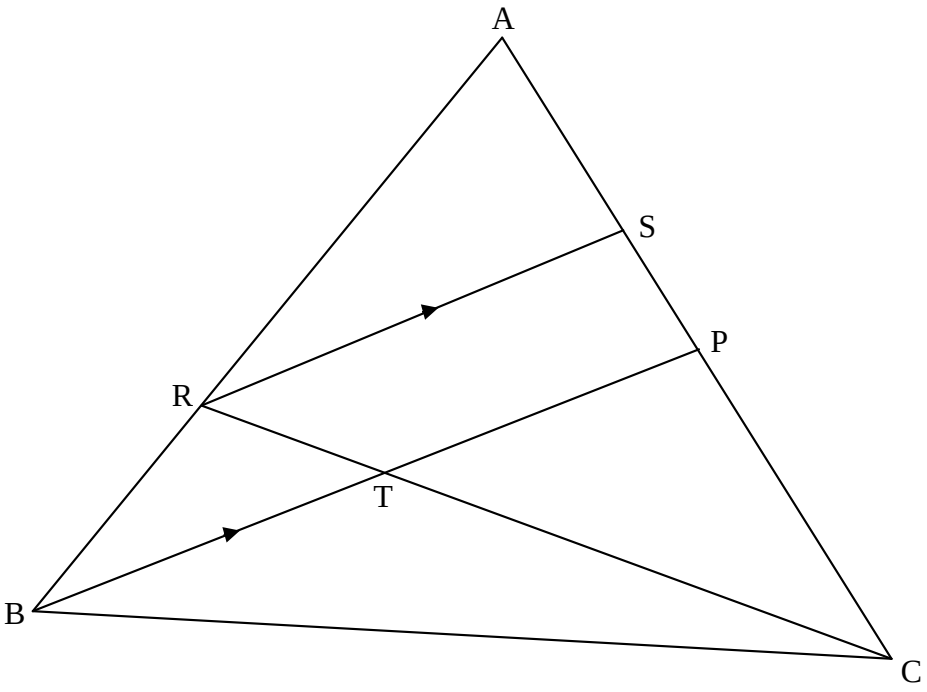
10.2.5 Hence, deduct that AE = AB. (3)
[16]

QUESTION 11

11.1 In ΔABC below, D and E are points on AB and AC respectively such that DE | | BC. Prove the theorem that states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



11.2 In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that RS | | BP and $\frac{AR}{AB} = \frac{3}{5}$. RC cuts BP in T.



Determine, giving reasons, the following ratios:

11.2.1 $\frac{AS}{SC}$ (4)

11.2.2 $\frac{RT}{TC}$ (3)

11.2.3 $\frac{\text{Area of } \Delta TPC}{\text{Area of } \Delta RSC}$ (4)

[17]

TOTAL : 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\mathrm{A} = \mathrm{P}(1+ni)$ $\mathrm{A} = \mathrm{P}(1-ni)$ $\mathrm{A} = \mathrm{P}(1-i)^n$ $\mathrm{A} = \mathrm{P}(1+i)^n$

$T_n = a + (n-1)d$ $\mathrm{S}_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$\mathrm{F} = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\mathrm{P} = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$ $a^2 = b^2 + c^2 - 2bc.\cos \mathrm{A}$

area $\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$