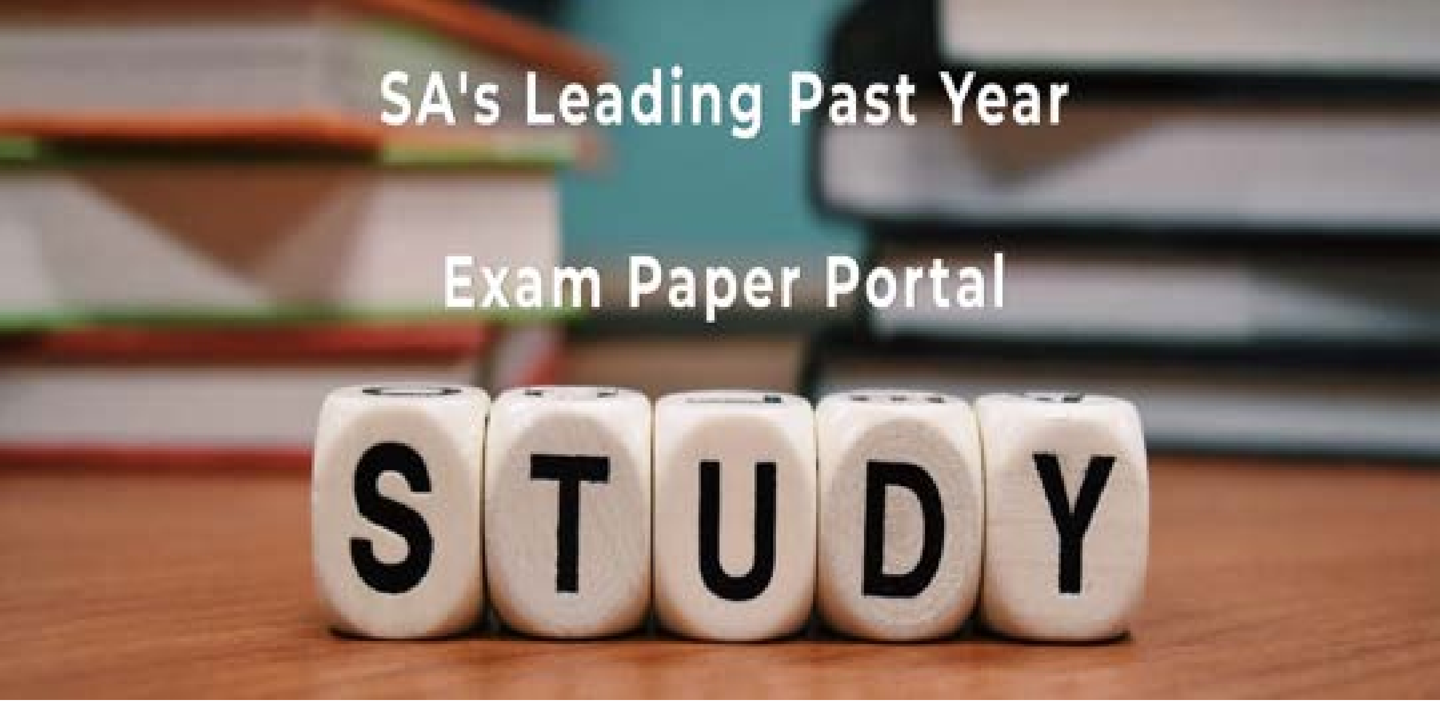
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WESTERN CAPE EDUCATION DEPARTMENT

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS
PAPER 1

SEPTEMBER 2015

MARKS: 150

TIME: 3 hours

This paper consists of 9 pages, 2 diagram sheets and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5. Answers only will not necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. TWO diagram sheets for QUESTION 5.3 and QUESTION 9.1 are attached at the end of the question paper. Write your name and class in the space provided and hand it in with you answer script.

10. An information sheet with formulae is included at the end of the question paper.

11. Write neatly and legibly.

QUESTION 1

1.1 Given $x^2 + 2x = 0$

1.1.1 Solve for x (2)

1.1.2 Hence, determine the positive values of x for which
$x^2 \geq -2x$ (3)

1.2 Solve for x:
$2x^2 - 3x - 7 = 0$ (correct to TWO decimal places) (4)

1.3 Given: $k + 5 = \frac{14}{k}$

1.3.1 Solve for k. (3)

1.3.2 Hence, or otherwise, solve for x if $\sqrt{x+5} + 5 = \frac{14}{\sqrt{x+5}}$. (3)

1.4 Solve for x and y simultaneously if:

$x - 2y - 3 = 0$ and

$4x^2 - 5xy + y^2 = 0$ (7)

1.5 The roots of a quadratic equation is given by $x = \frac{-2 \pm\sqrt{4-20k}}{2}$. Determine the value(s) of k for which the equation will have real roots. (2)

[24]

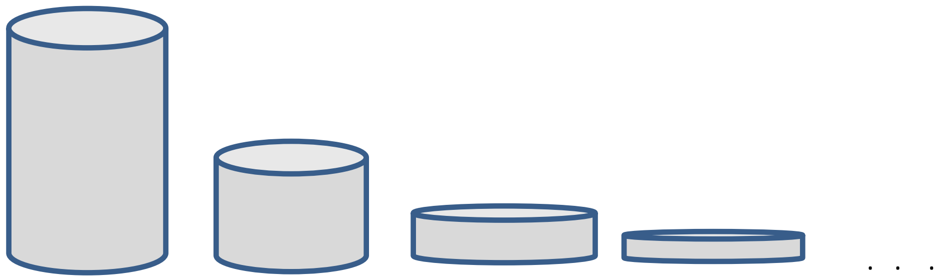
QUESTION 2

2.1 The sequence 3; 5; 7; ... is given.

2.1.1 Which term is equal to 71? (2)

2.1.2 Determine the sum of the first 40 terms. (2)

2.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is ½ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water. Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer through calculations.



(5)

2.3 Prove that $2^x + 2^{x+1} + 3.2^x + 2^{x+2} + \cdots (15 \text{ terms}) = 15.2^{x+3}$ (4)

[13]

QUESTION 3

3.1 The quadratic pattern $-3\,;4\,;x\,;30\ \ldots$ is given. Determine the value of x. (4)

3.2 Given: $x + x\left(\frac{x-1}{2}\right) + x\left(\frac{x-1}{2}\right)^2 + \cdots$

3.2.1 For what values of x will the series converge? (3)

3.2.2 Hence determine:

$$\sum_{k=1}^{\infty} x\left(\frac{x-1}{2}\right)^{k-1} \text{ if } x = \frac{3}{4}$$

(4)

[11]

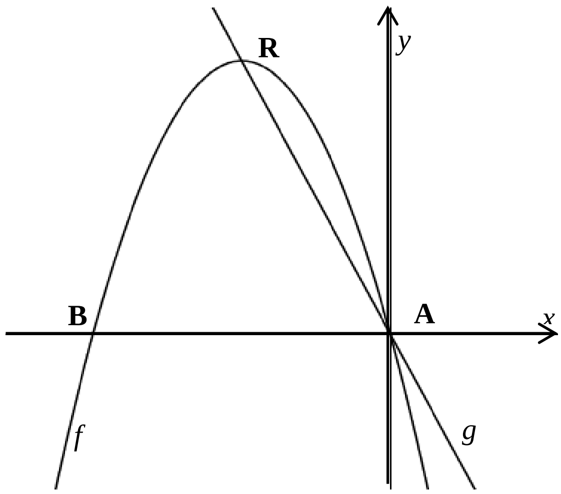
QUESTION 4

Sketched alongside are the graphs of

$f(x) = -(x+2)^2 + 4$ and

$g(x) = ax + q$

R is the turning point of *f*



4.1 Give the coordinates of R. (2)

4.2 Calculate the length of AB. (2)

4.3 Determine the equation of *g*. (2)

4.4 For which values of *x* is $g(x) > f(x)$? (2)

4.5 Write down the equation of the axis of symmetry of *h* if $h(x) = f(-x)$. (2)

4.6 Give the range of *p* if $p(x) = -f(x)$. (2)
[12]

QUESTION 5

Given: $h(x) = \frac{2}{x-2} + 1$

5.1 Give the equations of the asymptotes of *h* (2)

5.2 Determine the *x*- and *y*-intercepts of the graph of *h*. (3)

5.3 Sketch the graph of *h* using the grid on the DIAGRAM SHEET. (3)

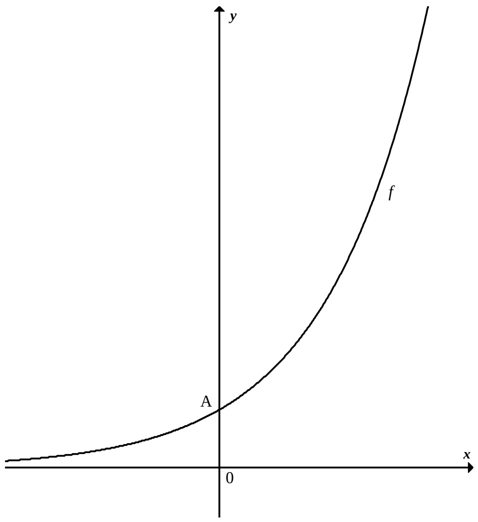
5.4 Give the domain of *h*. (2)

5.5 Describe the transformation of *h* to *f* if $f(x) = h(x+3)$ (2)

5.6 Determine the equation of the symmetry axis of the hyperbola with a negative gradient. (2)
[14]

QUESTION 6

6.1 The graph of $f(x) = 3^x$ is sketched alongside.



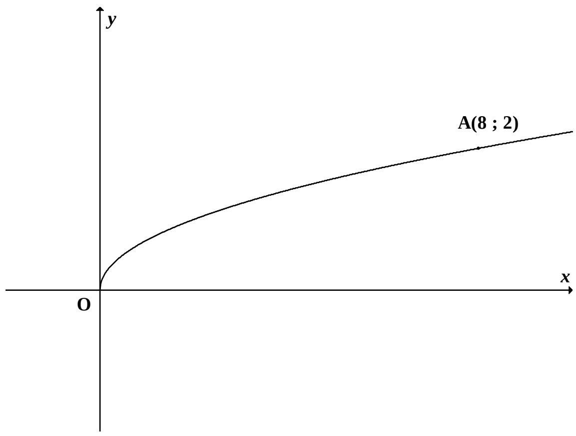
6.1.1 Give the coordinates of A. (1)

6.1.2 Write down the equation of f^{-1} in the form $y = \ldots$ (1)

6.1.3 For which value(s) of x will $f^{-1} \leq 0$? (2)

6.1.4 Write down the equation of the asymptote of $f(x-1)$ (1)

6.2 Sketched is the graph of f, the inverse of a restricted parabola. The point A(8 ; 2) lies on the graph of f.



6.2.1 Determine the equation of f in the form $y = \ldots$ (2)

6.2.2 Hence, write down the equation of f^{-1} in the form $y = \ldots$ (2)

6.2.3 Give the coordinates of the turning point of $g(x) = f^{-1}(x+3)-1$. (1)

[10]

QUESTION 7

7.1 Hein invests R12 500 for k years at a compound interest rate of 9% p.a. compounded quarterly. At the end of the k years his investment is worth R30 440. Calculate the value of k. (4)

7.2 Matt bought a car for R500 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 5 years. Interest is charged at 18% p.a. compounded monthly.

7.2.1 Calculate the annual effective interest rate of the loan. (3)

7.2.2 Calculate Matt's monthly instalments. (4)

7.2.3 Matt decided to pay R12 700 each month as his repayment. Calculate the outstanding balance of the loan after 2 years. (4)

7.2.4 At the end of the 2 years, the market value of Matt's car had reduced to R304 200. Determine the annual interest rate of depreciation on the reducing value. (2)

[17]

QUESTION 8

8.1 If $f(x) = -2x^2$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \frac{2x^2 - 1}{\sqrt{x}}$ (3)

8.2.2 $D_x[(3x-2)^2]$ (3)

8.3 Given: $y = \frac{1}{x^2}$.

Prove that the gradient of the curve is negative at each point on the curve where $x > 0$. (3)

[14]

QUESTION 9

The following information about a cubic polynomial, $y = f(x)$, is given:

- $f(-1) = 0$
- $f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(-1) = f'(1) = 0$
- if $x < -1$ then $f'(x) > 0$
- if $x > 1$ then $f'(x) > 0$

9.1 Use this information to draw a neat sketch graph of f using the grid on the DIAGRAM SHEET (5)

9.2 For which value(s) of x is f strictly decreasing? (2)

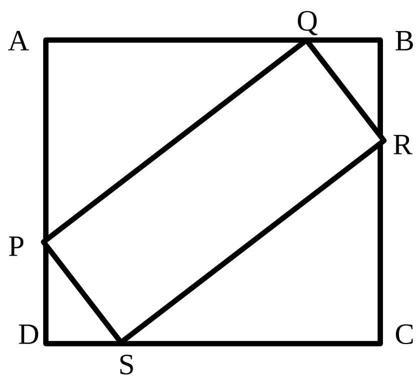
9.3 Use your graph to determine the x-coordinate of the point of inflection of f . (2)

9.4 For which value(s) of x is f concave up? (2)

[11]

QUESTION 10

ABCD is a square with sides 20 mm each. PQRS is a rectangle that fits inside the square such that $QB = BR = DS = DP = k$ mm.



10.1 Prove that the area of $PQRS = -2k(k-20) = 40k - 2k^2$ (4)

10.2 Determine the value of k for which the area of $PQRS$ is a maximum. (4)

[8]

QUESTION 11

11.1 If $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{4}$, find:

11.1.1 $P(A \text{ or } B)$ if A and B are mutually exclusive events. (1)

11.1.2 $P(A \text{ or } B)$ if A and B are independent events. (3)

11.2 A car park has 14 VOLKSWAGEN cars and 18 BMW's. There are no other cars. During the afternoon two cars are stolen – one early afternoon, the other later. Determine, using a tree diagram, the probability that:

11.2.1 Both cars were BMW's. (4)

11.2.2 The first one stolen was a BMW and the second one a Volkswagen. (2)

11.3 Eight boys and seven girls are to be seated randomly in a row. What is the probability that:

11.3.1 The row has a girl at each end? (3)

11.3.2 The row has girls and boys sitting in alternate positions? (3)
[16]

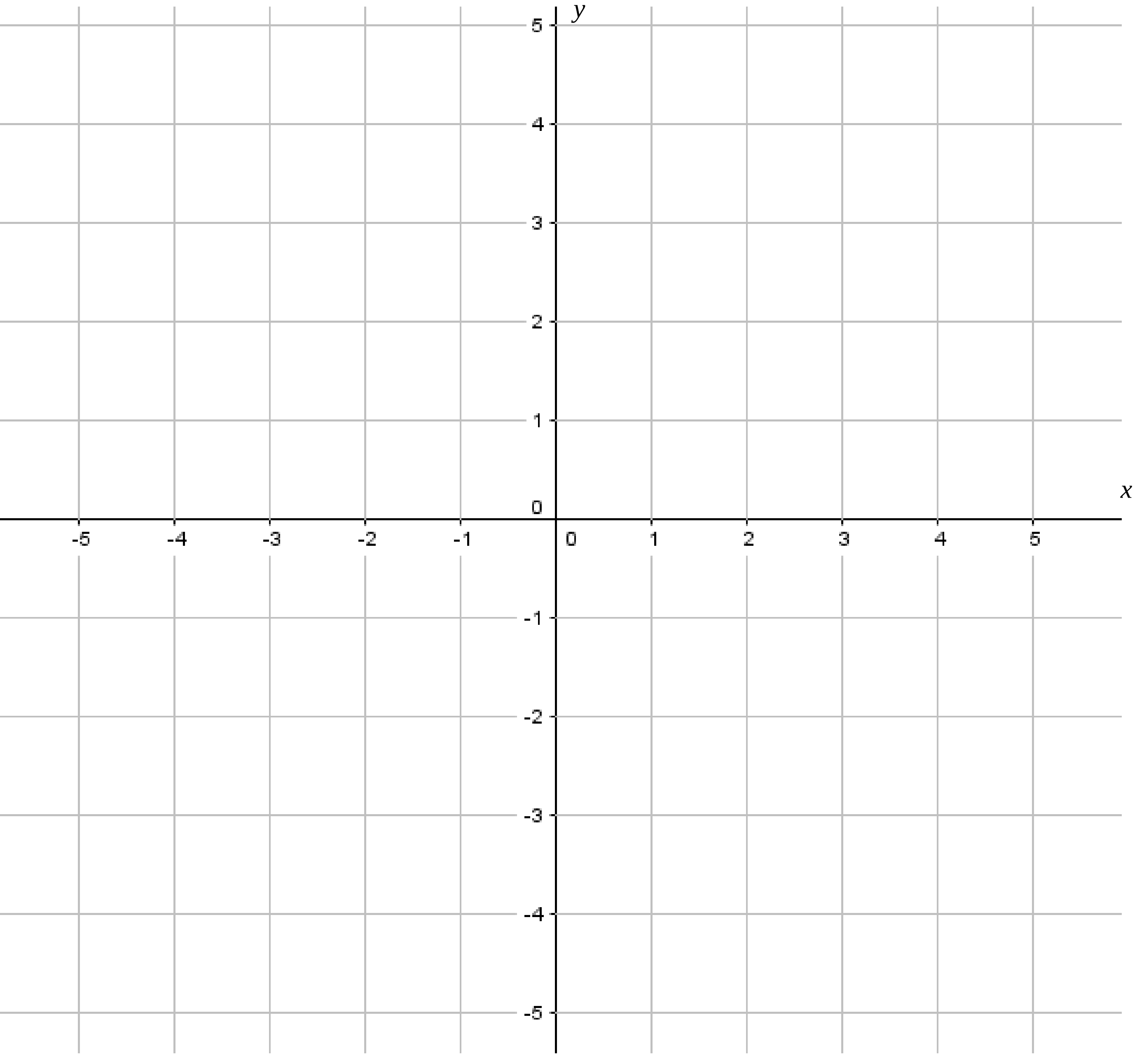
TOTAL: 150

Name and Surname: ……………………………………………………………….

Class: ....

DIAGRAM SHEET

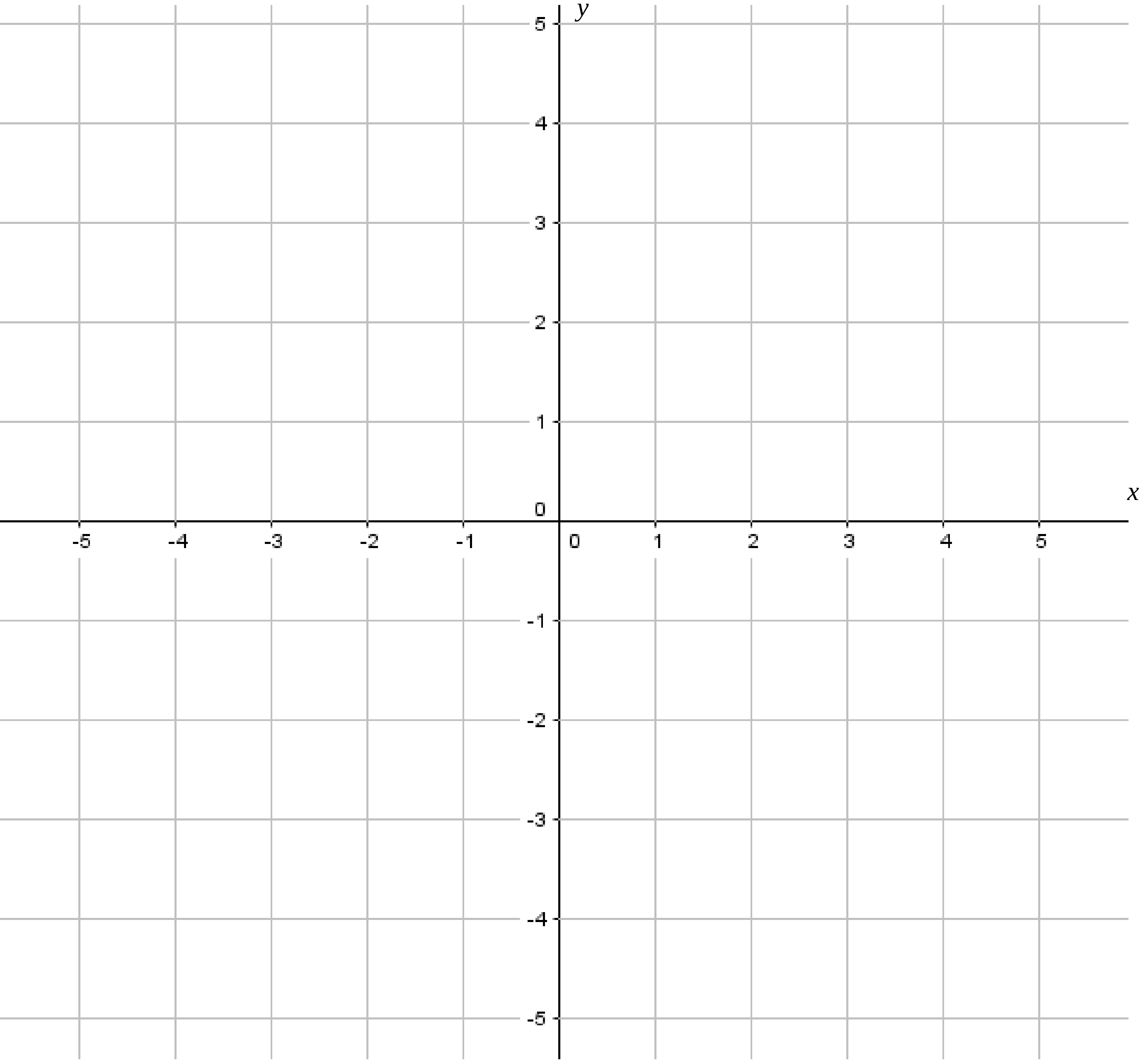
QUESTION 5.3



Name and Surname: ……………………………………………………………….

Class: ....

QUESTION 9.1



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 + i)^n$ $A = P(1 - i)^n$

$T_n = a + (n - 1)d$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$ $S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x[(1 + i)^n - 1]}{i}$ $P = \frac{x[1 - (1 + i)^{-n}]}{i}$

$$f'(x) = \lim_{n \to 0}\left(\frac{f(x + h) - f(x)}{h}\right)$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

$In\ \Delta ABC$: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ $a^2 = b^2 + c^2 - 2bc.cosA$

$Area\ \Delta ABC = \frac{1}{2}ab.sinC$

$\sin(\alpha + \beta) = sin\alpha.cos\beta + cos\alpha.sin\beta$ $\sin(\alpha - \beta) = sin\alpha.cos\beta - cos\alpha.sin\beta$

$\cos(\alpha + \beta) = cos\alpha.cos\beta - sin\alpha.sin\beta$ $\cos(\alpha - \beta) = cos\alpha.cos\beta + sin\alpha.sin\beta$

$$cos2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$sin2\alpha = 2sin\alpha.cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$