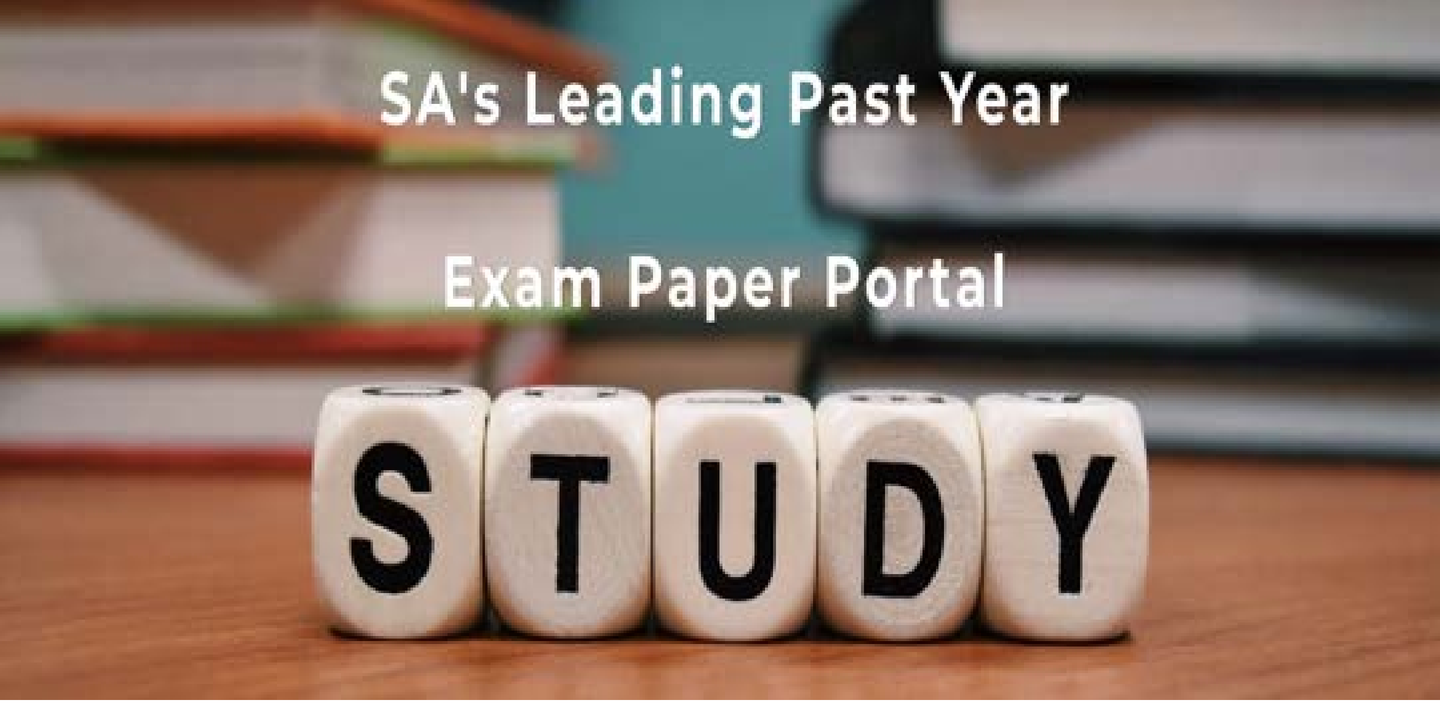
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STUDY

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2015

10611

MATHEMATICS

FIRST PAPER

MARKS: 150
TIME: 3 hours

11 Pages and 1 Information sheet

MATHEMATICS: Paper 1
1061E

10611E

X10

3

P.T.O.

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

MATHEMATICS
(First Paper)

TIME: 3 hours

MARKS: 150

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 13 questions.
2. Answer ALL the questions.
3. Show ALL calculations, diagrams, graphs, etc. that you have used in obtaining your answer.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable, non-graphic) may be used, unless stated otherwise.
6. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are not necessarily drawn to scale.
8. A formula sheet is attached at the end of the question paper.
9. Number your answers exactly as the questions are numbered.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x. Leave the answer in the simplest surd form where necessary:

1.1.1 $(2x+5)(x^2-2)=0$ (3)

1.1.2 $x^2-4\geq 5$ (4)

1.1.3 $12^{2x}=8.36^x$ (4)

1.2 Solve for x, correct to two decimal places:

$2(x+1)^2=9$ (4)

1.3 Solve for x and y simultaneously:

$y=-2x+7$ and $\dfrac{y+5}{x-1}=\dfrac{1}{2}$ (4)

1.4 Determine the values of p, for which the equation $3^x=1-p$ will have real solutions. (2)

[21]

QUESTION 2

The first two terms of an arithmetic series, A, and an infinite geometric series, B, are the same.

A: $-2+x+$ .... and

B: $-2+x+$ .... are given.

2.1 Write down in terms of x

2.1.1 The third term of the geometric series, B. (2)

2.1.2 The third term of the arithmetic series, A. (2)

2.2 If the sum of the first three terms in the arithmetic series A is equal to the third term of the geometric series B, then calculate the value of x. (5)

2.3 If $x=-6$, does the geometric series B converge? Show calculations to support your answer. (3)

[12]

QUESTION 3

Given: $\sum_{k=1}^{n} T_k = n^2 + 4n$, where T_k is the general term of a series.

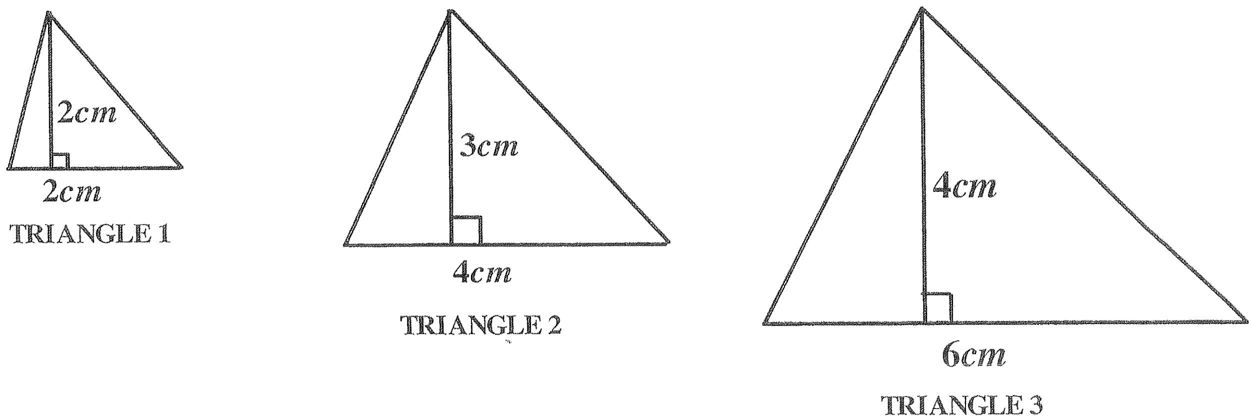
3.1 Calculate $\sum_{k=1}^{250} T_k$ (2)

3.2 Calculate T_{100} (3)

3.3 How many terms of the sequence must be added to give a sum of 1 440? (3)
[8]

QUESTION 4

A pattern of triangles is formed by increasing the base of the triangle by 2 cm and the perpendicular height by 1 cm, in each successive triangle.
The first triangle has a base of 2 cm and a height of 2 cm. The pattern continues in this manner.



4.1 Calculate the areas of the first four triangles. (2)

4.2 Calculate the area of the hundredth triangle in the pattern. (4)
[6]

QUESTION 5

5.1 5.1.1 Dumisani establishes a trucking company. The first truck he buys costs R650 000 and it depreciates at 30% per annum on a reducing balance method.

Calculate the depreciated value of the truck after 4 years. (2)

5.1.2 The price of a new truck appreciates, as a result of inflation, at a rate of 15% per annum.

Calculate the value of a new truck in 4 years' time. (2)

5.1.3 Dumisani plans to replace the original truck in 4 years' time.
He plans to use the current truck as a trade-in and pay the remaining amount in cash.
To save up for the required cash amount, he sets up a sinking fund, making equal monthly instalments into a fund earning interest of 9,5% per annum, compounded monthly.
The first instalment of the sinking fund is paid at the end of the first month of the 4-year period and the last instalment is made at the end of the 4 years.

Calculate the monthly instalment Dumisani pays into the sinking fund. (4)

5.2 5.2.1 Harry buys a property for R1 500 000. After paying a deposit, he takes a loan for the remaining amount of R1 275 000, at an interest rate of 9,2% per annum, compounded monthly over a period of 20 years. The monthly instalment on the loan is R11 636,02.

Calculate the outstanding balance of the loan after 7 years. (3)

5.2.2 After 7 years, due to financial difficulty, Harry misses 5 consecutive payments. Thereafter he continues making monthly payments into the loan account until the end of the 20 year period.

Calculate the value of the new monthly instalment to settle the loan. (4)
[15]

QUESTION 6

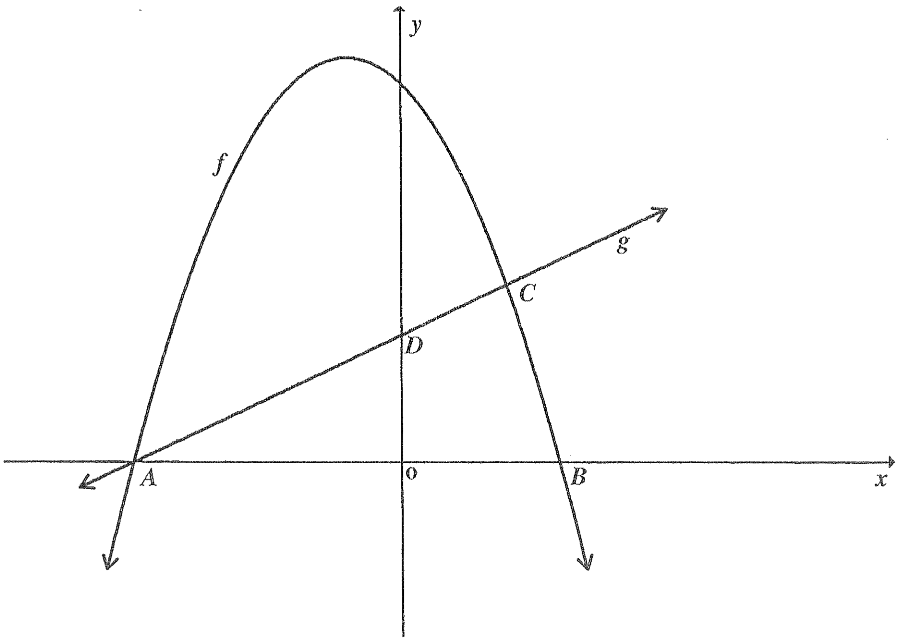
In your ANSWER BOOK, draw a clear sketch graph of function h defined by the equation $h(x) = a.b^x + q$, where $a < 0$; $b > 1$ and $q < 0$; a, b and q are real numbers.
Indicate all intercepts with the axes and asymptotes. (3)
[3]

QUESTION 7

The figure below represents the sketch graphs of f and g, defined by

$$f(x) = -2x^2 - 4x + 30 \quad \text{and} \quad g(x) = 2x + 10.$$

A and B are the x-intercepts of f. The graph of g passes through A.
C is the point of intersection of f and g. The graph of g intersects the y-axis at D.



Referring to the sketch above:

7.1 Determine the coordinates of A and B. (4)

7.2 Write the function of f in the form $f(x) = a(x-p)^2 + q$ and hence write down the coordinates of the turning point. (5)

7.3 Determine whether (1;12) are the coordinates of C. Show ALL calculations. (2)

7.4 The straight line with equation $y = mx + 32$ is a tangent to the graph of f. Calculate the possible values of m. (6)

7.5 Determine the values of x for which $f(x).g(x) > 0$. (2)

[19]

QUESTION 8

Given: $f(x)=\dfrac{1}{x-2}-1$

8.1 Write down the equation of the horizontal and vertical asymptotes of f. (2)

8.2 Determine the x- and y-intercepts of f. (4)

8.3 Draw a neat sketch of f in your ANSWER BOOK. Show ALL asymptotes and intercepts with the axes. (3)

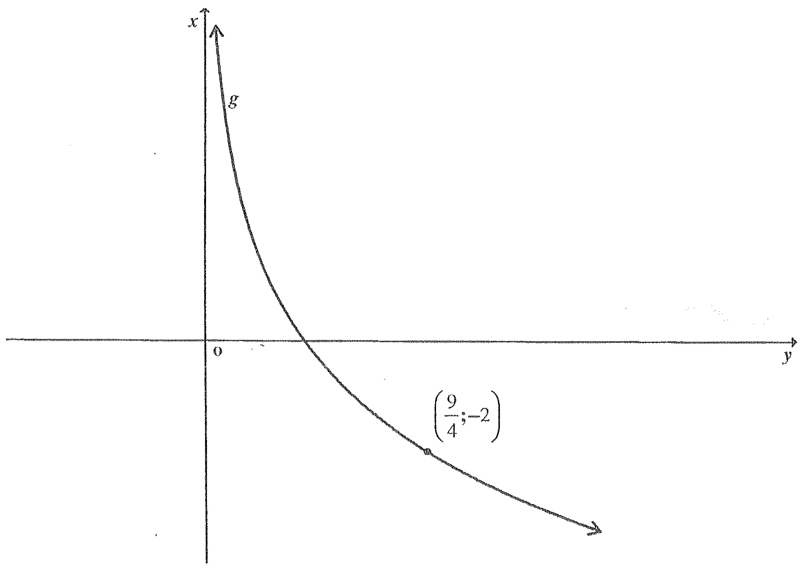
8.4 Calculate value of k if $(k;2)$ is a point on f. (2)

[11]

QUESTION 9

The sketch shows the graph of $g(x)=\log_a x$, where a is a positive real number.

The graph passes through $\left(\dfrac{9}{4};-2\right)$.



9.1 Calculate the value of a. (3)

9.2 Determine the equation of the function h, the reflection of g in the x-axis. (1)

9.3 Use the graph to determine the values of $g(x)$ for which $x<\dfrac{9}{4}$. (1)

9.4 Write down the domain of g^{-1}. (1)

[6]

QUESTION 10

10.1 Given: $f(x) = -2x^2 + 1$.

10.1.1 Show that the average gradient of the graph of f between the points where $x = 3$ and $x = 3 + h$, $(h \neq 0)$, is $-12 - 2h$. (4)

10.1.2 Use your answer in QUESTION 10.1.1 to calculate $f'(3)$ from first principles. (2)

10.1.3 Determine the numerical value of the gradient of the graph of f at $x = 0$. (1)

10.2 Differentiate y with respect to x. Leave your answer with positive exponents.

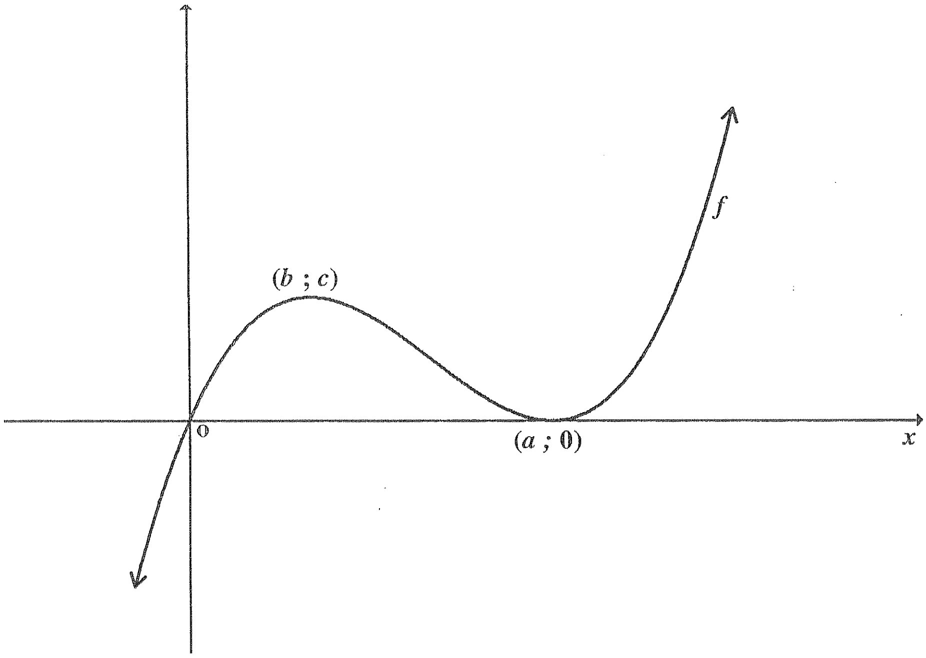
10.2.1 $y = (4 - 2x)^2$ (3)

10.2.2 $y = \sqrt{x}(\sqrt{x} - a) + a$ (4)

[14]

QUESTION 11

The sketch graph below represents a cubic function f with equation $f(x) = x^3 - 4x^2 + 4x + k$.
The graph passes through the origin, has a local maximum at $(b;c)$ and a local minimum at $(a;0)$.



11.1 Explain why $k = 0$. (1)

11.2 Using this value of k, determine the values of

11.2.1 a. (2)

11.2.2 b. (4)

11.3 The graph of g with equation $g(x) = mx$ is a tangent to f at the point $(0;0)$.
Calculate the value of m. (2)

11.4 Make use of the graph, or any other way, to determine the value of p for which
$x^3 - 4x^2 + 4x + 2 = p$ will have only one negative solution. (2)

[11]

QUESTION 12

The profit P in rand/day which the owner of a taxi makes when the taxi is driven at an average speed of x km/hour, is given by the formula

$$P = x^2\left(40 - \frac{x}{3}\right)$$

12.1 Calculate the average speed at which the owner will suffer a loss in revenue. (3)

12.2 Calculate the most economical average speed of the taxi and the corresponding daily profit. (6)
[9]

QUESTION 13

13.1 A bag contains 3 purple and 6 white balls. A second bag contains 7 purple and 1 white balls.
A ball is taken randomly from one of the bags.
Calculate the probability of the ball being purple.
(4)

13.2 For a sample space S and events A and B, it is given that:

$$\text{P([not A] and B)} = \text{P(A' and B)} = \frac{5}{12}$$

$$\text{P(A and B)} = \frac{1}{6}$$

$$\text{P(1-[A or B])} = \frac{1}{3}$$

13.2.1 Represent this information on a Venn diagram. (3)

13.2.2 Calculate P(A). (3)

13.3 A team of class representatives, consisting of 4 learners, needs to be elected at the beginning of the school year. The learners are chosen from Grades 11A, 11B, 11C and 11D. One learner from each class is randomly chosen and each learner has an equal chance to be chosen.

The number of learners in each class is:

Grade 11A – 20 learners;
Grade 11B – 15 learners;
Grade 11C – 12 learners;
Grade 11D – 10 learners.

Calculate in how many different ways this team of representatives can be elected. (1)

13.4 Two friends, Albert and James, take part in a swimming gala. There are in total 10 competitors and ten lanes. Only one swimmer is allowed in each lane.

If all swimmers take part, calculate the possible arrangements in the starting line-up under the following conditions:

13.4.1 Albert and James are placed next to each other. (2)

13.4.2 Albert is placed in the first lane and James is NOT placed next to Albert. (2)
[15]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x\ ;\ y) \rightarrow (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$$

$\overline{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \overline{x})(y - \overline{y})}{\sum(x - \overline{x})^2}$

END