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NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

MODEL 2014

MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 13 bladsye.

LET WEL:

- *Indien ‘n kandidaat ‘n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.*
- *Indien ‘n kandidaat ‘n poging om ‘n vraag te beantwoord gekanselleer het en die vraag nie weer gedoen het nie, merk die gekanselleerde poging.*
- *Volgehoue akkuraatheid is van toepassing in **ALLE** aspekte van die nasien-memorandum.*
- *Aanvaar van antwoorde/waardes om ‘n probleem op te los, is ONAANVAARBAAR.*

VRAAG 1

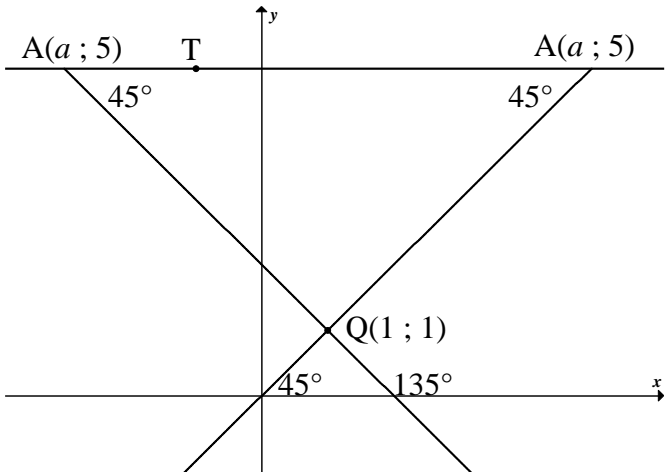
1.1	<p>Indien die aantal dae wat ‘n atleet oefen toeneem, neem die tyd waarin die 100m naelloop afgelê word, af.</p> <p style="text-align: center;">OF</p> <p>Indien die aantal dae wat ‘n atleet oefen afneem, neem die tyd waarin die 100m naelloop afgelê word, toe.</p> <p style="text-align: center;">OF</p> <p>Hoe meer dae ‘n atleet oefen, hoe korter is die tyd wat hy die 100m naelloop aflê.</p>	<p>✓ verduideliking</p> <p style="text-align: right;">(1)</p>
1.2	(60 ; 18,1)	<p>✓</p> <p style="text-align: right;">(1)</p>
1.3	$a = 17,81931464\dots$ $b = - 0,070685358\dots$ $\therefore \hat{y} = - 0,07x + 17,82$	<p>✓✓ <i>a</i></p> <p>✓ <i>b</i></p> <p>✓ vergelyking</p> <p style="text-align: right;">(4)</p>
1.4	$\therefore \hat{y} \approx - 0,07(45) + 17,82$ $\approx 14,67$ sekondes	<p>✓ substitusie</p> <p>✓ antwoord</p> <p style="text-align: right;">(2)</p>
1.5	$r = - 0,74 (-0,740772594\dots)$	<p>✓✓ <i>r</i></p> <p style="text-align: right;">(2)</p>
1.6	Daar is ‘n redelike sterk verwantskap tussen die veranderlikes.	<p>✓ redelik sterk</p> <p style="text-align: right;">(1)</p> <p style="text-align: right;">[11]</p>

VRAAG 2

<p>2.1</p>		<p>✓ anker by 0 ✓ plot by boonste limiete ✓ gladde kurwe</p> <p>(3)</p>
<p>2.2</p>	<p>$40 \leq t < 60$</p>	<p>✓ klas</p> <p>(1)</p>
<p>2.3</p>	<p>(96 ; 164) ∴ 172 – 164 = 8 leerdere</p>	<p>✓ 164 ✓ 8</p> <p>(2)</p>
<p>2.4</p>	<p>Frekwensie: 25; 44; 60; 28; 9; 6</p> $\text{gemiddelde} = \frac{25 \times 10 + 44 \times 30 + 60 \times 50 + 28 \times 70 + 9 \times 90 + 6 \times 110}{172}$ $= \frac{8000}{172}$ $= 46,51 \text{ uur}$	<p>✓ frekwensie ✓ middelpunte</p> <p>✓ $\frac{8000}{172}$ ✓ antwoord</p> <p>(4) [10]</p>

VRAAG 3

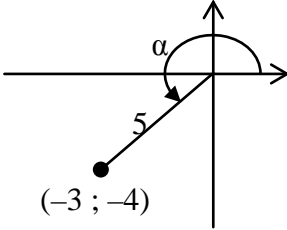
3.1	$K(7 ; 0)$	✓ antwoord (1)
3.2	$1 = \frac{x_M + 7}{2}$ en $1 = \frac{y_M + 3}{2}$ $\therefore M(-5 ; -1)$	✓ x ✓ y (2)
3.3	$m_{PM} = \frac{3-1}{7-1}$ $= \frac{1}{3}$	✓ substitusie ✓ antwoord (2)
3.4	$\tan \hat{P}\hat{S}\hat{K} = m_{PM} = \frac{1}{3}$ $\hat{P}\hat{S}\hat{K} = \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$ $\therefore \theta = 180^\circ - 90^\circ - 18,43^\circ = 71,57^\circ$	✓ $\tan \hat{P}\hat{S}\hat{K} = m_{PM}$ ✓ $\hat{P}\hat{S}\hat{K}$ ✓ θ (3)
3.5	$\cos 71,57^\circ = \frac{PK}{PS} = \frac{3}{PS}$ $PS = \frac{3}{\cos 71,57^\circ}$ $= 9,49$ eenhede OF $\sin 18,43^\circ = \frac{PK}{PS} = \frac{3}{PS}$ $PS = \frac{3}{\sin 18,43^\circ}$ $= 9,49$ eenhede	✓ korrekte verhouding ✓ PS onderwerp ✓ antwoord (3) ✓ korrekte verhouding ✓ PS onderwerp ✓ antwoord (3)
3.6	$N(x ; -2x + 17)$ $m_{TN} = m_{PM}$ (TN PM) $\frac{-2x + 17 - 5}{x - (-1)} = \frac{1}{3}$ $-6x + 36 = x + 1$ $-7x = -35$ $x = 5$ $\therefore y = -2(5) + 17 = 7$ $\therefore N(5 ; 7)$ OF	✓ N in terme van x ✓ gelyke gradiënte ✓ substitusie ✓ x -waarde ✓ y -waarde (5)

	$m_{TM} = \frac{1}{3} \quad (\text{TN} \parallel \text{PM})$ <p>vergelyking van TM:</p> $y - y_1 = \frac{1}{3}(x - x_1)$ $y - 5 = \frac{1}{3}(x - (-1))$ $y - 5 = \frac{1}{3}x + \frac{1}{3}$ $y = \frac{1}{3}x + 5\frac{1}{3}$ <p style="text-align: center;">OF</p> $y = \frac{1}{3}x + c$ $5 = \frac{1}{3}(-1) + c$ $5\frac{1}{3} = c$ $y = \frac{1}{3}x + 5\frac{1}{3}$ $-2x + 17 = \frac{1}{3}x + 5\frac{1}{3}$ $-2\frac{1}{3}x = -11\frac{2}{3}$ $x = 5$ $\therefore y = -2(5) + 17 = 7$ $\therefore N(5 ; 7)$	<p>✓ m_{TM}</p> <p>✓ vergelyking van TM</p> <p>✓ stel gelyk aan mekaar</p> <p>✓ x-waarde</p> <p>✓ y-waarde</p> <p style="text-align: right;">(5)</p>
<p>3.7.1</p>	<p>$y = 5$</p>	<p>✓ vergelyking</p> <p style="text-align: right;">(1)</p>
<p>3.7.2</p>	 <p>gradiënt van AQ = $\tan 45^\circ$ of $\tan 135^\circ$ = 1 of -1</p> $m_{AQ} = \frac{5-1}{a-1} = \pm 1$ $\therefore a - 1 = 4 \text{ of } -4$ $\therefore a = 5 \text{ of } -3$	<p>✓ $m_{AQ} = 1$ of</p> <p>✓ $m_{AQ} = -1$</p> <p>✓ substitusie in gradiëntformule</p> <p>✓ x-waarde</p> <p>✓ y-waarde</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[22]</p>

VRAAG 4

4.1	$M(-1 ; -1)$	✓ antwoord (1)
4.2	$m_{NT} = \frac{2-1}{3-4} = -1$ $\therefore m_{AT} = 1 \quad (\text{radius} \perp \text{raaklyn})$ $y - 1 = 1(x - 4)$ $y = x - 3$	✓ m_{NT} ✓ m_{AT} ✓ rede ✓ substitusie van m en $(4 ; 1)$ ✓ vergelyking (5)
4.3	$MR \perp AB$ (lyn vanaf midpt na midpt van koord) $MB^2 = MR^2 + RB^2$ (Stelling van Pythagoras) $9 = \left(\frac{\sqrt{10}}{2}\right)^2 + RB^2$ $RB^2 = \frac{13}{2}$ $RB = \sqrt{\frac{13}{2}}$ $AB = 2\left(\sqrt{\frac{13}{2}}\right) = \sqrt{26}$ eenhede	✓ $MR \perp AB$ ✓ $MB = 3$ ✓ substitusie in stelling van Pythagoras ✓ AB in wortelvorm (4)
4.4	$MN^2 = (-1 - 3)^2 + (-1 - 2)^2$ $= 16 + 9$ $= 25$ $MN = 5 \text{ eenhede}$	✓ substitusie in afstandformule ✓ antwoord (2)
4.5	$r = 5 - 3 = 2$ eenhede $\therefore (x - 3)^2 + (y - 2)^2 = 4$ $\therefore x^2 + y^2 - 6x - 4y + 9 = 0$	✓ r ✓ substitusie in sirkelvergelyking ✓ vergelyking (3) [15]

VRAAG 5

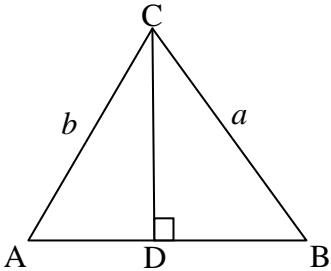
5.1.1	$-\sin \alpha$ $= -\left(-\frac{4}{5}\right) = \frac{4}{5}$	✓ reduksie ✓ antwoord (2)
5.1.2	$(-4)^2 + b^2 = 5^2$ $b^2 = 25 - 16 = 9$ $b = -3$ $\cos \alpha = \frac{-3}{5}$ 	✓ $b = -3$ ✓ antwoord (2)
5.1.3	$\sin(\alpha - 45^\circ)$ $= \sin \alpha \cos 45^\circ - \cos \alpha \sin 45^\circ$ $= -\frac{4}{5} \cdot \frac{1}{\sqrt{2}} - \left(-\frac{3}{5}\right) \cdot \frac{1}{\sqrt{2}}$ $= -\frac{1}{5\sqrt{2}}$ <p style="text-align: center;">OF</p> $\sin(\alpha - 45^\circ)$ $= \sin \alpha \cos 45^\circ - \cos \alpha \sin 45^\circ$ $= -\frac{4}{5} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{3}{5}\right) \cdot \frac{\sqrt{2}}{2}$ $= -\frac{\sqrt{2}}{10}$	✓ uitbreiding ✓ $\frac{1}{\sqrt{2}}$ ✓ antwoord in eenvoudigste vorm (3)
5.2.1	$LHS = \frac{8 \sin x \cdot \cos x}{\sin^2 x - \cos^2 x}$ $= \frac{4(2 \sin x \cdot \cos x)}{\sin^2 x - \cos^2 x}$ $= \frac{4 \sin 2x}{-(\cos^2 x - \sin^2 x)}$ $= \frac{4 \sin 2x}{-\cos 2x}$ $= -4 \tan 2x$	✓ $\sin x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ $4 \sin 2x$ ✓ faktoriseer ✓ $-\cos 2x$ (6)
5.2.2	Ongedefinieer as $\cos 2x = 0$ of $\tan 2x = \infty$: $x = 45^\circ$ en $x = 135^\circ$	✓ 45° ✓ 135° (2)

5.3	$1 - 2\sin^2 \theta + 4\sin^2 \theta - 5\sin \theta - 4 = 0$ $2\sin^2 \theta - 5\sin \theta - 3 = 0$ $(2\sin \theta + 1)(\sin \theta - 3) = 0$ <p>$\therefore \sin \theta = -\frac{1}{2}$ of $\sin \theta = 3$ (geen oplossing)</p> <p>$\therefore \theta = 210^\circ + 360^\circ k$ of $\theta = 330^\circ + 360^\circ k ; k \in \mathbb{Z}$</p> <p>OF</p> <p>$\therefore \theta = 210^\circ + 360^\circ k$ of $\theta = 30^\circ + 360^\circ k ; k \in \mathbb{Z}$</p>	<ul style="list-style-type: none"> ✓ $1 - 2\sin^2 \theta$ ✓ standaardvorm ✓ faktore ✓ geen oplossing ✓ 210° ✓ 330° ✓ $+ 360^\circ k ; k \in \mathbb{Z}$ <p style="text-align: right;">(7) [22]</p>
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VRAAG 6

6.1	$b = \frac{1}{2}$	✓ waarde van b (1)
6.2	A($30^\circ ; 1$)	✓ 30° ✓ 1 (2)
6.3	$x = 160^\circ$	✓ $x = 160^\circ$ (1)
6.4	$h(x) = 2\cos(x - 30^\circ) + 1$ $y \in [-1 ; 3]$ OF $-1 \leq y \leq 3$	<ul style="list-style-type: none"> ✓ kritiese waardes ✓ notasie <p style="text-align: right;">(2) [6]</p>

VRAAG 7

<p>7.1</p>	<p>Trek $CD \perp AB$ In $\triangle ACD$: $\sin A = \frac{CD}{b} \therefore CD = b \cdot \sin A$ In $\triangle CBD$: $\sin B = \frac{CD}{a} \therefore CD = a \cdot \sin B$ $\therefore b \cdot \sin A = a \cdot \sin B$ $\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$</p> 	<p>✓ konstruksie ✓ sin A ✓ maak CD die onderwerp ✓ sin B ✓ $b \cdot \sin A = a \cdot \sin B$ (5)</p>
<p>7.2.1</p>	<p>$\widehat{SPQ} = 180^\circ - 2x$ (teenoorst $\angle e$ van koordevierh) $\widehat{PSQ} + \widehat{PQS} = 2x$ (som van $\angle e$ in \triangle) $\widehat{PSQ} = \widehat{PQS} = x$ ($\angle e$ teenoor gelyke sye)</p>	<p>✓ $\widehat{SPQ} = 180^\circ - 2x$ (S/R) ✓ rede (2)</p>
<p>7.2.2</p>	$\frac{\sin \widehat{SPQ}}{\sin(180^\circ - 2x)} = \frac{\sin \widehat{PSQ}}{\sin x}$ $\frac{SQ}{\sin(180^\circ - 2x)} = \frac{PQ}{\sin x}$ $SQ = \frac{k \sin 2x}{\sin x}$ $SQ = \frac{k(2 \sin x \cdot \cos x)}{\sin x} = 2k \cos x$ <p style="text-align: center;">OF</p> $SQ^2 = PQ^2 + PS^2 - 2PQ \cdot PS \cdot \cos \widehat{SPQ}$ $= k^2 + k^2 - 2 \cdot k \cdot k \cdot \cos(180^\circ - 2x)$ $= 2k^2 + 2k^2 \cos 2x$ $= 2k^2 + 2k^2(2\cos^2 x - 1)$ $= 4k^2 \cos^2 x$ $SQ = 2k \cos x$	<p>✓ substitusie in korrekte formule ✓ sin 2x ✓ SQ onderwerp ✓ $2 \sin x \cdot \cos x$ (4) ✓ substitusie in korrekte formule ✓ $-\cos 2x$ ✓ $2\cos^2 x - 1$ ✓ vereenvoudig (4)</p>
<p>7.2.3</p>	$\tan y = \frac{3}{k}$ $k = \frac{3}{\tan y}$ $SQ = 2 \cos x \left(\frac{3}{\tan y} \right)$ $\therefore = \frac{6 \cos x}{\tan y}$	<p>✓ tan-verhouding ✓ k onderwerp en substitusie (2) [13]</p>

VRAAG 8

8.1	die hoek onderspan in die teenoorstaande sirkelsegment	✓korrekte stelling (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68^\circ$ (rkl-koordst)	✓ $\hat{E}_1 = 68^\circ$ ✓ rede (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68^\circ$ (verwiss \angle e; AE BC)	✓ $\hat{B}_3 = 68^\circ$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68^\circ$ (buite \angle v koordevh)	✓ $\hat{D}_1 = 68^\circ$ ✓ rede (2)
8.2.4	$\hat{E}_2 = 20^\circ + 68^\circ$ $= 88^\circ$ (buite \angle v Δ)	✓ $\hat{E}_2 = 88^\circ$ (S/R) (1)
8.2.5	$\hat{C} = 180^\circ - 88^\circ$ $= 92^\circ$ (tos \angle e v koordevh)	✓ $\hat{C} = 92^\circ$ ✓ rede (2) [9]

VRAAG 9

9.1	$\hat{D}_4 = \hat{A} = x$ (rkl-koordstelling) $\hat{A} = \hat{D}_2 = x$ (\angle e tos gelyke sye)	$\checkmark \hat{A} = x$ \checkmark rede $\checkmark \hat{A} = \hat{D}_2 = x$ (S/R) (3)
9.2	$\hat{M}_1 = 2x$ (buite \angle v Δ) OF (\angle by midpt = $2\angle$ by omtr) $\hat{M}\hat{D}\hat{E} = 90^\circ$ (radius \perp rkl) $\hat{M}_2 = 90^\circ - 2x$ $\therefore \hat{E} = 180^\circ - (90^\circ + 90^\circ - 2x)$ (som v \angle e in Δ MDE) $= 2x$ \therefore CM is 'n rkl (omgek rkl-koordst)	$\checkmark \hat{M}_1 = 2x$ (S/R) $\checkmark \hat{M}\hat{D}\hat{E} = 90^\circ$ (S/R) $\checkmark \hat{E} = 2x$ \checkmark rede (4)
9.3	$\hat{M}_3 = 90^\circ$ (EM \perp AC) $\hat{A}\hat{D}\hat{B} = 90^\circ$ (\angle in halfsirkel) \therefore FMBD is koordevh (buite \angle v vh = tos binne \angle) OF $\hat{E}\hat{M}\hat{C} = 90^\circ$ (EM \perp AC) $\hat{A}\hat{D}\hat{B} = 90^\circ$ (\angle in halfsirkel) \therefore FMBD is koordevh (tos \angle e v vh suppl)	$\checkmark \hat{M}_3 = 90^\circ$ $\checkmark \hat{A}\hat{D}\hat{B} = 90^\circ$ (S/R) \checkmark rede (3) $\checkmark \hat{E}\hat{M}\hat{C} = 90^\circ$ $\checkmark \hat{A}\hat{D}\hat{B} = 90^\circ$ (S/R) \checkmark rede (3)
9.4	$DC^2 = MC^2 - MD^2$ (Pythagoras) $= (3BC)^2 - (2BC)^2$ (MB = MD = radii) $= 9BC^2 - 4BC^2$ $= 5BC^2$	\checkmark Pythagoras \checkmark substitusie $\checkmark 9BC^2 - 4BC^2$ (3)
9.5	In Δ DBC en Δ DFM: $\hat{D}_4 = \hat{D}_2 = x$ (bewys in 9.1) $\hat{B}_1 = \hat{F}_2$ (buite \angle v koordevh) $\hat{C} = \hat{M}_2$ $\therefore \Delta$ DBC $\parallel\parallel\parallel$ Δ DFM (\angle ; \angle ; \angle)	$\checkmark \hat{D}_4 = \hat{D}_2$ $\checkmark \hat{B}_1 = \hat{F}_2$ \checkmark rede $\checkmark \hat{C} = \hat{M}_2$ of (\angle ; \angle ; \angle) (4)
9.6	$\frac{DM}{FM} = \frac{DC}{BC}$ (Δ DBC $\parallel\parallel\parallel$ Δ DFM) $= \frac{\sqrt{5}BC}{BC}$ $= \sqrt{5}$	\checkmark S \checkmark antwoord (2) [19]

VRAAG 10

10.1		<p>✓ konstruksie</p> <p>✓ $\frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEB} = \frac{AD}{DB}$</p> <p>✓ rede</p> <p>✓ $\frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEC} = \frac{AE}{EC}$</p> <p>✓ Area $\triangle DEB = \text{Area } \triangle DEC$ (S/R)</p> <p>✓</p> <p>$\frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEB} = \frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEC}$</p>
	<p>Konstruksie: Verbind DC en BE en trek hoogtes k en h</p> $\frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEB} = \frac{\frac{1}{2} \cdot AD \cdot k}{\frac{1}{2} \cdot DB \cdot k} = \frac{AD}{DB} \quad (\text{gelyke hoogtes})$ $\frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEC} = \frac{\frac{1}{2} \cdot AE \cdot h}{\frac{1}{2} \cdot EC \cdot h} = \frac{AE}{EC} \quad (\text{gelyke hoogtes})$ <p>Maar $\text{Opp } \triangle DEB = \text{Opp } \triangle DEC$ (dies basis, dies hoogte)</p> $\therefore \frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEB} = \frac{\text{opp } \triangle ADE}{\text{opp } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	(6)

<p>10.2.1</p>	$\frac{AB}{BE} = \frac{AC}{CD}$ <p>(Ewered st; BC ED)</p> $\frac{1}{3} = \frac{3}{CD}$ $\therefore CD = 9 \text{ eenhede}$	<p>✓ $\frac{AB}{BE} = \frac{AC}{CD}$ (S/R) ✓ substitusie ✓ antwoord (3)</p>
<p>10.2.2</p>	$\frac{DG}{GA} = \frac{FD}{FE}$ <p>(Ewered st; FG EA)</p> $\frac{9-x}{3+x} = \frac{3}{6}$ $54 - 6x = 9 + 3x$ $-9x = -45$ $x = 5$	<p>✓ $\frac{DG}{GA} = \frac{FD}{FE}$ (S/R) ✓ substitusie ✓ vereenvoudig ✓ antwoord (4)</p>
<p>10.2.3</p>	<p>In $\triangle ABC$ en $\triangle AED$:</p> <p>\hat{A} is gemeen $\hat{A}BC = \hat{E}$ (ooreenk \angles; BC ED) $\hat{A}CB = \hat{D}$ (ooreenk \angles; BC ED) $\triangle ABC \sim \triangle AED$ (\angle, \angle, \angle) $\therefore \frac{BC}{ED} = \frac{AC}{AD}$ $\frac{BC}{9} = \frac{3}{12}$ $BC = 2\frac{1}{4}$ eenhede</p>	<p>✓ \hat{A} is gemeen ✓ $\hat{A}BC = \hat{E}$ (S/R) ✓ $\hat{A}CB = \hat{D}$ (S/R) of ($\angle; \angle; \angle$) ✓ $\frac{BC}{ED} = \frac{AC}{AD}$ ✓ antwoord (5)</p>
<p>10.2.4</p>	$\frac{\text{opp } \triangle ABC}{\text{opp } \triangle GFD} = \frac{\frac{1}{2} AC \cdot BC \cdot \sin \hat{A}CB}{\frac{1}{2} GD \cdot FD \cdot \sin \hat{D}}$ $= \frac{\frac{1}{2} (3) (2\frac{1}{4}) \sin \hat{D}}{\frac{1}{2} (4) (3) \sin \hat{D}}$ <p>(ooreenk \angles; BC ED)</p> $= \frac{9}{16}$	<p>✓ gebruik v opp reël ✓ korrekte sye en \anglee ✓ substitusie v waardes ✓ $\sin \hat{A}CB = \sin \hat{D}$ (S/R) ✓ antwoord (5) [23]</p>

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